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# Resilient dynamic output feedback control for leader-following consensus of high-order uncertain multi-agent systems under sensor–actuator attacks

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ABSTRACT

This paper investigates the resilient leader-following consensus problem for a class of high-order multi-agent systems subject to unmatched lower triangular uncertainty dynamics and sensor–actuator attacks. Compared with previous work, the sensor and actuator attacks are modeled to be both time-varying and state-dependent, which makes the resilient consensus control more challenging. To deal with the unmatched uncertainties and sensor–actuator attacks, a new compensator with two online tuned dynamic gains is proposed relying only on the relative output measurements of neighboring agents. Then, using only the local compensation information, a fully distributed output feedback resilient protocol is designed to guarantee the leader-following consensus. Sufficient conditions in terms of matrix inequalities are derived to determine control gains and compensator parameters and simulation studies are presented to demonstrate the proposed theoretical results.

## 1. Introduction

Multi-agent systems (MASs) are a significant subclass of cyber–physical systems that incorporate collaborated agents as physical entity and communication as cyber component [1]. Given that cyber–physical systems demand system robustness, resiliency, reliability, safety and security for addressing the constantly changing and reconfiguring system dynamics [2], it is essential to provide secure and resilient control for MASs whose information sharing and control processing are actually tightly coupled with the physical process. In particular, the resilient consensus control of MASs involves not only ensuring the high performance consensus but also preventing and mitigating the effects of attackers in adversarial environments, wherein attackers may inject the false data into the measurements of sensors and control input commands of actuators(usually known as false data injection attacks [3–5]) to severely compromise system performance and even overall stability. Hence, the pervasive security and resilience challenges underlying MASs place additional burdens on standard consensus control methods.

To address the resilient control of MASs under malicious attacks, two mainly prevention and mitigation approaches have been reported in existing literatures. The first one is detecting and identifying attacked agents based on the discrepancy between their neighbors and themselves, and then discard their information [6–8]. Despite such approach can counteract various attacks including sensor–actuator attacks and communication link attack, additional assumptions regarding to the maximum number of agents under attack and the connectivity of communication network are commonly required. Moreover, the rejecting of neighbors' information

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may decrease the speed of convergence to the desired consensus and harm the connectivity of communication network. The second approach is designing distributed resilient control protocols to directly mitigate attack without identifying and isolating attacked agents' information [9,10]. In this direction, new techniques extended from adaptive control theory are proposed in [11] to mitigate the effects of sensor and actuator attacks instead of removing compromised agents, and thus the leader-following consensus of heterogeneous MASs is achieved. Resilient distributed control protocols are designed in [12] for MASs under sensor–actuator attacks, in which the adaptive compensator injects control inputs to reach synchronization while attenuating the attack effects. In [13], adaptive resilient architectures are applied to ensure that the stochastic MASs under sensor–actuator attacks achieve uniform ultimate bounded consensus. In addition to adaptive control approach, a secure measurement preselector along with a neural network (NN) secure state observer are introduced in [14] for secure state estimating, and then bipartite tracking is achieved in the presence of adversarial sensor attacks. However, the adversarial attacks on actuator and sensor in previous literatures are modeled as disturbances, and most of the considered agent system is assumed to be general linear and deterministic. Usually, in addition to the disturbance, the attacks affect on sensor and actuator take a variety of forms, such as faults [15,16], measurement noise [17,18], sensor uncertainties [19,20] and unknown control directions [21,22] which fluctuate around the true values of sensor and actuator. For cyber–physical systems, the adversarial attacks are also modeled as time-varying and state-dependent which are more complicated than previously mentioned attack forms [23,24]. Thus, it is interesting and meaningful to consider a class of state-dependent sensor and actuator attacks with time-varying attack gain in the consensus of MASs.

In practical, systems do not merely suffer from sensor–actuator attacks but also commonly subject to uncertain dynamics, some of which probably do not match with the control input in the same channel and might bring significant adverse effects on systems performances [25,26]. It is thus of practical importance to investigate resilient consensus protocols for MASs to suppress sensor–actuator attacks and mismatched uncertainties, simultaneously. To address these issues, backstepping technique integrated with adaptive control method is employed in [27] to deal with sensor–actuator attacks and system uncertainties, and hence the asymptotic consensus of agents' output is achieved. A distributed robust adaptive control architecture is developed in [28] for addressing networked MASs subject to model uncertainty, exogenous stochastic disturbances, and compromised sensor and actuators. However, these aforementioned methods require full-state feedback, i.e., all the system states have to be measurable and full relative state information need to be transmitted. It increases the measurement and communication burdens of sensor and network for actual system, especially for the high order system having multiple states [29]. Therefore, output feedback control protocols are essential and practical significant for the resilient consensus of high-order MASs.

Motivated by the above mentioned limitations, a new compensator based dynamic output feedback controller is developed in this paper for the resilient leader-following consensus of high-order uncertain MASs under time-varying state dependent sensor–actuator attacks. The main contributions are summarized as follows: (i) A more realistic situation for the MASs is considered, in which high-order dynamics, mismatched uncertainties and time-varying state dependent sensor–actuator attacks are simultaneously taken into account. It covers the general linear or low-order system cases in [7,28], the deterministic system cases in [11,12] and the faults/sensor-uncertainty type attack cases in [15,20] as special case. (ii) Through introducing two online tuned dynamic gains, a new compensator driven only by the relative output measurements of neighboring agents is designed to mitigate and attenuate the effects of mismatched uncertainties and time-varying state dependent sensor–actuator attacks. To the best of authors' knowledge, it is the first attempt in simultaneous offsetting the influences of sensor–actuator attacks and mismatched uncertainties only using the neighbor agents' output information. (iii) Removing the impractical requirement that the compensators embedded in agents have to share information with their neighbors, a fully distributed dynamic output feedback resilient consensus controller is proposed. Another desirable feature of the proposed controller is that it can guarantee the asymptotic consensus rather than the commonly bound consensus results under sensor–actuator attacks.

**Notation:** In the following, if not explicitly stated, matrices are assumed to have compatible dimensions. $\mathbb{R}^n$ denotes the $n$-dimensional Euclidean space. $\mathbb{R}^{m\times n}$ denotes the set of $m \times n$ real matrices. $\boldsymbol{I}_n$ is the identity matrix with dimension $n$.

## 2. Problem formulation and preliminaries

In this paper, we consider a leader–follower networked MASs composed of $N+1$ agents, having $n$th order uncertain dynamics of agent $i \in \{0, 1, \dots, N\}$ given by

$$
\begin{aligned}
&\dot{x}_{i,m} = x_{i,m+1} + \boldsymbol{\Delta}_m^T(t)\overline{\boldsymbol{x}}_{i,m} \quad m = 1, \dots, n-1 \\
&\dot{x}_{i,n} = u_i + \boldsymbol{\Delta}_n^T(t)\boldsymbol{x}_i, \quad y_i = x_{i,1}
\end{aligned} \tag{1}
$$

where $\boldsymbol{x}_i = (x_{i,1}, \dots, x_{i,n})^T \in \mathbb{R}^n$, $u_i \in \mathbb{R}$ and $y_i \in \mathbb{R}$ are the local state, control input and output, respectively; $\overline{\boldsymbol{x}}_{i,m} = (x_{i,1}, \dots, x_{i,m})^T \in \mathbb{R}^m$; $u_0 = 0$; $\boldsymbol{\Delta}_m(t) \in \mathbb{R}^m$, $m = 1, \dots, n$ characterize the uncertainty of $i$th agent dynamics, satisfying $\boldsymbol{\Delta}_m(t) = \rho \boldsymbol{E}_m \boldsymbol{\Upsilon}_m(t)$, where $\rho > 0$ is a known constant, $\boldsymbol{E}_m \in \mathbb{R}^{m\times m}$ is a known matrix and $\boldsymbol{\Upsilon}_m(t) : \mathbb{R}^+ \to \mathbb{R}^m$ represents a time-varying matrix term satisfying $\boldsymbol{\Upsilon}_m(t)\boldsymbol{\Upsilon}_m^T(t) \le \boldsymbol{I}_m$. Observe that the term $\boldsymbol{\Upsilon}_m(t)$ can even be allowed to be state-dependent, i.e., $\boldsymbol{\Upsilon}_m(t) = \boldsymbol{\Upsilon}_m\left(t, \overline{\boldsymbol{x}}_{i,m}\right)$, as long as $\boldsymbol{\Upsilon}_m\left(t, \overline{\boldsymbol{x}}_{i,m}\right)\boldsymbol{\Upsilon}_m^T\left(t, \overline{\boldsymbol{x}}_{i,m}\right) \le \boldsymbol{I}_m$ is satisfied.

In our formulation, the agent indexed by 0 is referred as leader and the agents indexed by $1, \dots, N$ are called followers. We use a directed graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, \dots, N\}$ denotes the set of indexes corresponding to follower agent and $\mathcal{E}$ denotes the set of edges representing neighboring relationships, to describe the information exchanging among follower agents. $\mathcal{N}_i = \{j \in \mathcal{V} | (j, i) \in \mathcal{E}\}$ is defined as the neighboring set of agent $i$. Define the adjacency matrix associated with communication graph as $\boldsymbol{A} = [a_{ij}]$ with $a_{ij} > 0$ for $(i, j) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. Note that $a_{ii} = 0$ since agents are not self-connected. Define the in-degree matrix as a diagonal matrix $\boldsymbol{D} = \mathrm{diag}\left\{\sum_{j\in\mathcal{N}_1} a_{1j}, \dots, \sum_{j\in\mathcal{N}_N} a_{Nj}\right\}$. Then, the graph Laplacian matrix is defined as $\boldsymbol{L} = \boldsymbol{D} - \boldsymbol{A}$, which has all row sums equal to zero. Define pinning matrix of graph $\mathcal{G}$ as $\boldsymbol{B} = \mathrm{diag}\{b_1, \dots, b_N\}$ with $b_i > 0$ if and only if there exists an edge from the leader to the $i$th follower agent and $b_i > 0$ for at least one $i$. Since the leader is represented by vertex 0, an augmented graph $\bar{\mathcal{G}}$, which consists of graph $\mathcal{G}$, vertex 0 and edges between the leader and its neighbors, is obtained. Then, we have the following assumption:

**Assumption 1.** $\mathcal{G}$ is fixed, directed and the augmented graph $\bar{\mathcal{G}}$ contains a spanning tree with leader node 0 as root.

In this paper, the sensor and actuator attacks under consideration are a class of time-varying state-dependent cyber attacks, which maliciously modify the sensor measurement and control input signals to prevent the agents from achieving their consensus goal. Thus, the disrupted sensor measurements of $i$th agent is given as

$$y_i^c = x_{i,1} + \omega_i(t)x_{i,1} \tag{2}$$

where $y_i = x_{i,1}$ is the true output and $\omega_i(t)$ is an unknown time-varying weight. Note that $\omega_i(t)x_{i,1}$ is injected into the sensors maliciously and thus the output of the sensor $y_i^c = (1 + \omega_i(t))x_{i,1}$ is different from the ideal output $y_i = x_{i,1}$ Furthermore, the compromised control input is given by

$$u_i^c = \Omega_i(t)u_i \tag{3}$$

where $u_i$ is the ideal control input and $\Omega_i(t)$ represents a time-varying multiplicative actuator attack for agent $i$.

Note that the considered sensor and actuator attacks in (2) and (3) are assumed to satisfy following assumption:

**Assumption 2.** For $\omega_i(t)$ and $\Omega_i(t)$, one has $\omega_i(t) \neq -1$ and $\Omega_i(t) \neq 0$. Furthermore, there exist known constants $\theta_1 \geq 1$, $\theta_2 > 0$ and unknown constant $\theta_3 \geq 1$ such that $|\omega_i(t)| \leq \theta_1$, $|\dot{\omega}_i(t)| \leq \theta_2$ and $|\Omega_i(t)| \leq \theta_3$, respectively. Obviously, there also exists a positive constant $\theta_4$ such that $\left|\frac{1}{1+\omega_i(t)}\right| \leq \theta_4$.

To this end, an useful Lemma which is important to derive the main results of this paper is presented as follows:

**Lemma 1.** *Let $\mathcal{B}_1, \mathcal{B}_2 \in \mathbb{R}^n$ and $\mathcal{A} \in \mathbb{R}^{n\times n}$ be the vectors and matrices, respectively, defined as $\mathcal{B}_1 = (0, \dots, 0, 1)^T$, $\mathcal{B}_2 = (1, 0, \dots, 0)^T$ and $\mathcal{A} = \begin{pmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ 0 & 0 & \cdots & 0 \end{pmatrix}$. Then, there exists column vectors $\boldsymbol{k}_g = \left(g_1, \dots, g_n\right)^T$, $\boldsymbol{\kappa} = \left(\kappa_1, \dots, \kappa_n\right)^T$, positive definite matrices $\boldsymbol{P}$, $\boldsymbol{Q}$ and constant $q_2 \in (0, 1)$ such that*

$$\boldsymbol{M}^T\boldsymbol{P} + \boldsymbol{P}\boldsymbol{M} \leq -\iota_1\boldsymbol{P}, \quad q_2\left(\boldsymbol{R}^T\boldsymbol{Q} + \boldsymbol{Q}\boldsymbol{R}\right) \leq -\iota_2\boldsymbol{Q}$$
$$\boldsymbol{S} = \begin{pmatrix} -3\boldsymbol{P} & -\boldsymbol{Q} \\ -\boldsymbol{Q} & \left(1 - q_2\right)\left(\boldsymbol{R}^T\boldsymbol{Q} + \boldsymbol{Q}\boldsymbol{R}\right) \end{pmatrix} \leq 0$$

*where $\iota_1 > 0$, $\iota_2 > 0$ are two positive constants and matrices $\boldsymbol{M} = \mathcal{A} + \mathcal{B}_1\boldsymbol{\kappa}^T$, $\boldsymbol{R} = \mathcal{A} - \boldsymbol{k}_g\mathcal{B}_2^T$. Moreover, defining $\boldsymbol{\Xi} = \mathrm{diag}\,(0, 1, \dots, n-1)$ and $\hat{\boldsymbol{E}} = \sum_{s=1}^n \widetilde{\boldsymbol{E}}_s\widetilde{\boldsymbol{E}}_s^T$ with $\widetilde{\boldsymbol{E}}_s = \mathrm{diag}\{\boldsymbol{E}_s, \boldsymbol{0}\} \in \mathbb{R}^{n\times n}$, there exists positive constants $\underline{\lambda}_1$, $\underline{\lambda}_2$, $h$, $\nu_1$, $l_i (i = 1, 2, 3, 4)$ such that $\underline{\lambda}_1\boldsymbol{I}_n \leq \boldsymbol{P}$, $\underline{\lambda}_2\boldsymbol{I}_n \leq \boldsymbol{Q}$, $-h\boldsymbol{P} \leq \boldsymbol{\Xi}^T\boldsymbol{P} + \boldsymbol{P}\boldsymbol{\Xi} \leq h\boldsymbol{P}$, $-h\boldsymbol{Q} \leq \boldsymbol{\Xi}^T\boldsymbol{Q} + \boldsymbol{Q}\boldsymbol{\Xi} \leq h\boldsymbol{Q}$, $\boldsymbol{Q}\boldsymbol{Q} \leq \nu_1\boldsymbol{Q}$, $\boldsymbol{P}\boldsymbol{k}_g\boldsymbol{k}_g^T\boldsymbol{P} \leq l_1\boldsymbol{P}$, $\hat{\boldsymbol{E}} \leq l_2\boldsymbol{P}$, $\hat{\boldsymbol{E}} \leq l_3\boldsymbol{Q}$, $\boldsymbol{Q}\mathcal{B}_1\boldsymbol{\kappa}^T\boldsymbol{\kappa}\mathcal{B}_1^T\boldsymbol{Q} \leq l_4\boldsymbol{Q}$.*

**Proof.** It is obvious that $\left(\mathcal{A}, \mathcal{B}_1\right)$ and $\left(\mathcal{A}, \mathcal{B}_2^T\right)$ are controllable and observable, respectively. Thus, column vectors $\boldsymbol{k}_g = \left(g_1, \dots, g_n\right)^T$ and $\boldsymbol{\kappa} = \left(\kappa_1, \dots, \kappa_n\right)^T$ can be found such that matrices $\boldsymbol{M}$ and $\boldsymbol{R}$ are Hurwitz. Then, the matrix inequalities in Lemma 1 are easily obtained and the proofs are omitted here for brevity.

**Remark 1.** In this paper, the norm-bounded system parameter uncertainty $\boldsymbol{\Delta}_m(t)$, which is assumed to be of the form $\boldsymbol{\Delta}_m(t) = \rho\boldsymbol{E}_m\boldsymbol{Y}_m(t)$ and $\boldsymbol{Y}_m(t)\boldsymbol{Y}_m^T(t) \leq \boldsymbol{I}_m$, is actually the result of model linearization and unmodeled dynamics. It has been widely used in the problem of robust control for uncertain systems, see for example, [28,30,31] and the references therein. Therefore, we consider the norm-bounded parameter uncertainty given as $\boldsymbol{\Delta}(t)$ for the agent dynamics in this paper. Besides, observe that the term $\boldsymbol{Y}_m(t)$ can even be allowed to be state-dependent, i.e., $\boldsymbol{Y}_m(t) = \boldsymbol{Y}_m\left(t, \overline{\boldsymbol{x}}_{i,m}\right)$, as long as $\boldsymbol{Y}_m\left(t, \overline{\boldsymbol{x}}_{i,m}\right)\boldsymbol{Y}_m^T\left(t, \overline{\boldsymbol{x}}_{i,m}\right) \leq \boldsymbol{I}_m$ is satisfied. Under these conditions, the considered model (1) can describe many practical systems, such as robot system [32], inverted pendulum system [33], circuit system [34] and so on. Moreover, many practical systems possessing parameter uncertainties which can be either exactly modeled or its unknown part over bounded by $\boldsymbol{Y}_m\left(t, \overline{\boldsymbol{x}}_{i,m}\right)\boldsymbol{Y}_m^T\left(t, \overline{\boldsymbol{x}}_{i,m}\right) \leq \boldsymbol{I}_m$, can also represented by model (1).

**Remark 2.** Note that $y_i^c$ in (2) is the only obtainable measurement under sensor attack. It is broadcasted to its neighbors and is used by the consensus protocol design for $i$th agent. Likewise, under actuator attack, the uncorrupted control input $u_i$ cannot be applied to the system and only the corrupted control input $u_i^c$ enters the model dynamics (1). Furthermore, we assume that attack gains $\omega_i(t) \neq -1$ and $\Omega_i(t) \neq 0$ to construct a feasible control protocol $u_i^c$ which based on the attacked output measurement $y_i^c$. Since $\omega_i(t) = -1$ and $\Omega_i(t) = 0$ result in $y_i^c \equiv 0$ and $u_i^c \equiv 0$, it is not possible to construct $u_i^c$ to guarantee the resilient leader-following consensus of MASs. Note that under the condition $\omega_i(t) = -1$ and $\Omega_i(t) = 0$ in Assumption 2, the case with "$\omega_i(t) = 1$ and $\Omega_i(t) = 1$" is allowed to hold at certain times.

**Remark 3.** In Lemma 1, since $(\mathcal{A}, \mathcal{B}_1)$ is controllable and $(\mathcal{A}, \mathcal{B}_2^T)$ is observable, it is known that matrices $\boldsymbol{M}$ and $\boldsymbol{R}$ are Hurwitz by choosing vectors $\boldsymbol{k}_g$ and $\boldsymbol{\kappa}$ appropriately. Thus, there exists feasible solution matrices $\boldsymbol{P} > 0$ and $\boldsymbol{Q} > 0$ such that $\boldsymbol{M}^T\boldsymbol{P} + \boldsymbol{P}\boldsymbol{M} \le -\iota_1\boldsymbol{P}$ and $q_2(\boldsymbol{R}^T\boldsymbol{Q} + \boldsymbol{Q}\boldsymbol{R}) \le -\iota_2\boldsymbol{Q}$ with $\iota_1 > 0$, $\iota_2 > 0$ and $q_2 \in (0, 1)$. Simultaneously, the solution for $\boldsymbol{S} \le 0$ also exists since its diagonal elements $-3\boldsymbol{P}$ and $(1-q_2)(\boldsymbol{R}^T\boldsymbol{Q} + \boldsymbol{Q}\boldsymbol{R})$ are both negative definite. Besides, the solutions for positive constants $\underline{\lambda}_1$, $\underline{\lambda}_2$, $h$, $\nu_1$, $l_i (i = 1, 2, 3, 4)$ such that $\underline{\lambda}_1\boldsymbol{I}_n \le \boldsymbol{P}$, $\underline{\lambda}_2\boldsymbol{I}_n \le \boldsymbol{Q}$, $-h\boldsymbol{P} \le \boldsymbol{\Xi}^T\boldsymbol{P} + \boldsymbol{P}\boldsymbol{\Xi} \le h\boldsymbol{P}$, $-h\boldsymbol{Q} \le \boldsymbol{\Xi}^T\boldsymbol{Q} + \boldsymbol{Q}\boldsymbol{\Xi} \le h\boldsymbol{Q}$, $\boldsymbol{Q}\boldsymbol{Q} \le \nu_1\boldsymbol{Q}$, $\boldsymbol{P}\boldsymbol{k}_g\boldsymbol{k}_g^T\boldsymbol{P} \le l_1\boldsymbol{P}$, $\hat{\boldsymbol{E}} \le l_2\boldsymbol{P}$, $\hat{\boldsymbol{E}} \le l_3\boldsymbol{Q}$, $\boldsymbol{Q}\mathcal{B}_1\boldsymbol{\kappa}^T\boldsymbol{\kappa}\mathcal{B}_1^T\boldsymbol{Q} \le l_4\boldsymbol{Q}$ apparently exist

**Remark 4.** Here, we stress that it has been a difficult problem to handle sensor and actuator attacks by using output feedback control techniques. Output feedback control works based on the assumption that only output information is available. When sensor and actuator attacks are involved, the only available actual output and the control input are corrupted as shown in (2) and (3), respectively. Thus, we need to use only corrupted output for the design of controller which additionally corrupted by actuator attack to ensure full state consensus and mitigate the effects of joint sensor–actuator attacks simultaneously. To achieve this goal, we propose a compensator based output feedback consensus protocol that contains two online tuned dynamic gains.

## 3. Resilient consensus control under joint sensor–actuator attacks

In this subsection, we consider the resilient leader–follower consensus problem for system (1) under sensor attack (2) and actuator attack (3). A novel compensator-based dynamic output feedback consensus protocol is designed to mitigate the effects of joint sensor–actuator attack and guarantee the leader–follower consensus, i.e., $\lim_{t\to\infty} |x_{i,m} - x_{0,m}| = 0$ for $i = 1, \ldots, N$ and $m = 1, \ldots, n$.

Since each agent can only access the relative output measurements of its neighbors, the available information for $i$th agent is synthesized as $\sigma_{i,1} = \sum_{j\in\mathcal{N}_i} a_{ij}\left(y_i^c - y_j^c\right) + b_i\left(y_i^c - y_0^c\right) = (1+\omega_i(t))\sum_{j\in\mathcal{N}_i} a_{ij}\left(x_{i,1} - x_{j,1}\right) + (1+\omega_i(t))\, b_i\left(x_{i,1} - x_{0,1}\right)$. Based only on this, the distributed output feedback protocol together with a compensator is given as

$$
\begin{aligned}
\dot{z}_{i,m} &= z_{i,m+1} - g_m F_i^m\left(z_{i,1} - \sigma_{i,1}\right) - F_i\left(\gamma_i + \dot{\gamma}_i\right) z_{i,m} \\
\dot{z}_{i,n} &= \sum_{m=1}^{n} \kappa_m F_i^{n-m+1} z_{i,m} - g_n F_i^n\left(z_{i,1} - \sigma_{i,1}\right) \\
&\quad - F_i\left(\gamma_i + \dot{\gamma}_i\right) z_{i,n}, \qquad m = 1, \ldots, n-1
\end{aligned} \tag{4}
$$

$$
u_i^c = \Omega_i(t) u_i = \Omega_i(t) \sum_{m=1}^{n} \kappa_m F_i^{n-m+1} z_{i,m} \tag{5}
$$

where $\kappa_m, g_m, m = 1, \ldots, n$ are parameters provided in Lemma 1. $F_i (F_i(0) > 1)$ and $\gamma_i(\gamma_i(0) > 0)$ are two dynamic gains whose adaptive laws will be determined below. Then, we construct the following result:

**Theorem 1.** *Consider the high-order uncertain multi-agent system (1) under sensor attack (2) and actuator attack (3). Suppose that Assumptions 1–2 are satisfied. The compensator-based output feedback protocol is designed as (5) together with (4), in which $F_i$ and $\gamma_i$ are dynamically updated by*

$$
\begin{aligned}
\dot{F}_i &= \Theta_i\left(F_i, \zeta_{i,1}, \boldsymbol{\eta}_i\right) \\
&= -\frac{F_i}{h}\left(\frac{\tilde{\iota}(F_i - 1)}{3} - \beta - 1 - \varpi_2 - \Gamma_i\left(\zeta_{i,1}, \boldsymbol{\eta}_i\right)\right) \\
\dot{\gamma}_i &= \boldsymbol{\eta}_i^T \boldsymbol{P} \boldsymbol{\eta}_i
\end{aligned} \tag{6}
$$

*where matrix $\boldsymbol{P}$ and constant $h$ are given in Lemma 1, vector $\boldsymbol{\eta}_i = (\eta_{i,1}, \ldots, \eta_{i,n})^T$ with $\eta_{i,m} = \frac{z_{i,m}}{F_i^{m-1+h}}$ for $m = 1, \ldots, n$, $\zeta_{i,1} = \frac{z_{i,1} - \sigma_{i,1}}{F_i^h}$, $\beta$ is a positive adjustable constant, $\tilde{\iota} = \min\{\iota_1, \iota_2\}$ and $\varpi_2 = (1+\theta_1)(2\nu_1 + \rho^2 l_3) + \theta_2\theta_4(1+\nu_1) + 2l_4$ with $\iota_1$, $\iota_2$, $\nu_1$, $l_3$, $l_4$ are positive constants provided in Lemma 1, and $\Gamma_i(\zeta_{i,1}, \boldsymbol{\eta}_i)$ is a nonlinear function designed as*

$$
\Gamma_i\left(\zeta_{i,1}, \boldsymbol{\eta}_i\right) = \begin{cases} \dfrac{\zeta_{i,1}^2/2 + (\gamma_i + \dot{\gamma}_i)\zeta_{i,1}^2/\underline{\lambda}_2}{\zeta_{i,1}^2 + \boldsymbol{\eta}_i^T\boldsymbol{P}\boldsymbol{\eta}_i} & if \quad \zeta_{i,1}^2 + \boldsymbol{\eta}_i^T\boldsymbol{P}\boldsymbol{\eta}_i \neq 0 \\ 0 & if \quad \zeta_{i,1}^2 + \boldsymbol{\eta}_i^T\boldsymbol{P}\boldsymbol{\eta}_i = 0 \end{cases} \tag{7}
$$

*Then, if matrix inequalities in Lemma 1 hold, the full states of all follower agents reach consensus with that of leader.*

**Proof.** Define the synthesized single as

$$
\sigma_{i,m} = (1+\omega_i(t))\left(\sum_{j\in\mathcal{N}_i} a_{ij}\left(x_{i,m} - x_{j,m}\right) + b_i\left(x_{i,m} - x_{0,m}\right)\right)
$$

for $m = 1, \ldots, n$. Then, letting $r_{i,m} = z_{i,m} - \sigma_{i,m}$ and using (1) and (4), it can be derived that

$$
\dot{r}_{i,m} = r_{i,m+1} - g_m F_i^m r_{i,1} - \frac{\dot{\omega}_i(t)}{1+\omega_i(t)}\left(z_{i,m} - r_{i,m}\right)
$$

$$
\begin{aligned}
&-F_i(\gamma_i+\dot{\gamma}_i)z_{i,m}-\left(1+\omega_i(t)\right)\bar{\varphi}_{i,m}\\
\dot{r}_{i,n}&=\sum_{m=1}^{n}\kappa_m F_i^{n-m+1}z_{i,m}-\frac{\dot{\omega}_i(t)}{1+\omega_i(t)}\left(z_{i,n}-r_{i,n}\right)-U_i\\
&-g_nF_i^n r_{i,1}-F_i(\gamma_i+\dot{\gamma}_i)z_{i,n}-\left(1+\omega_i(t)\right)\bar{\varphi}_{i,n}
\end{aligned}
\tag{8}
$$

where $\bar{\varphi}_{i,m}=\boldsymbol{\Delta}_m^T(t)\sum_{j\in\mathcal{N}_i}a_{ij}\left(\bar{\boldsymbol{x}}_{i,m}-\bar{\boldsymbol{x}}_{j,m}\right)+\boldsymbol{\Delta}_m^T(t)b_i\left(\bar{\boldsymbol{x}}_{i,m}-\bar{\boldsymbol{x}}_{0,m}\right)$, $U_i=\left(1+\omega_i(t)\right)\left(\sum_{j\in\mathcal{N}_i}a_{ij}\left(u_i^c-u_j^c\right)+b_iu_i^c\right)=\left(1+\omega_i(t)\right)\Omega_i(t)\sum_{j\in\mathcal{N}_i}a_{ij}\left(u_i-u_j\right)+\left(1+\omega_i(t)\right)\Omega_i(t)+b_iu_i$.

To go further, we first make sure that $F_i(t)$, $i=1,\dots,N$ updated by (6) are larger than 1. From (6), it can be seen $\Theta_i\left(1,\zeta_{i,1},\boldsymbol{\eta}_i\right)=\frac{F_i}{h}\left(\beta+1+\varpi_2\right)+\frac{F_i}{h}\Gamma_i\left(\zeta_{i,1},\boldsymbol{\eta}_i\right)>0$ for all $\zeta_{i,1}$, $\boldsymbol{\eta}_i$ and $\dot{F}_i=\Theta_i\left(1,\zeta_{i,1},\boldsymbol{\eta}_i\right)>0$ increase as $F_i$ increase. Thus, we can choose the initial condition $F_i(0)$ strictly larger than 1 to ensure $F_i(t)\geq 1$. Then, we introduce the following change of coordinates:

$$
\eta_{i,m}=\frac{z_{i,m}}{F_i^{m-1+h}},\quad \zeta_{i,m}=\frac{r_{i,m}}{F_i^{m-1+h}},\quad m=1,\dots,n
\tag{9}
$$

The novelty here is that $h$ is a strictly positive constant and it is chosen to satisfy inequalities $-h\boldsymbol{P}\leq\boldsymbol{\Xi}^T\boldsymbol{P}+\boldsymbol{P}\boldsymbol{\Xi}\leq h\boldsymbol{P}$ and $-h\boldsymbol{Q}\leq\boldsymbol{\Xi}^T\boldsymbol{Q}+\boldsymbol{Q}\boldsymbol{\Xi}\leq h\boldsymbol{Q}$ presented in Lemma 1. Further using (4) and (8), it follows that

$$
\begin{aligned}
\dot{\eta}_{i,m}&=F_i\eta_{i,m+1}-g_mF_i\zeta_{i,1}-F_i(\gamma_i+\dot{\gamma}_i)\eta_{i,m}\\
&\quad-(m-1+h)\frac{\dot{F}_i}{F_i}\eta_{i,m}\\
\dot{\eta}_{i,n}&=F_i\sum_{m=1}^{n}\kappa_m\eta_{i,m}-g_nF_i\zeta_{i,1}-F_i(\gamma_i+\dot{\gamma}_i)\eta_{i,n}\\
&\quad-(n-1+h)\frac{\dot{F}_i}{F_i}\eta_{i,n}\\
\dot{\zeta}_{i,m}&=F_i\zeta_{i,m+1}-F_i(\gamma_i+\dot{\gamma}_i)\eta_{i,m}-g_mF_i\zeta_{i,1}\\
&\quad-\frac{\dot{\omega}_i(t)}{1+\omega_i(t)}\left(\eta_{i,m}-\zeta_{i,m}\right)-(m-1+h)\frac{\dot{F}_i}{F_i}\zeta_{i,m}\\
&\quad-\left(1+\omega_i(t)\right)\varphi_{i,m}\\
\dot{\zeta}_{i,n}&=F_i\sum_{m=1}^{n}\kappa_m\eta_{i,m}+\frac{U_i}{F_i^{n-1+h}}-(n-1+h)\frac{\dot{F}_i}{F_i}\zeta_{i,n}\\
&\quad-\frac{\dot{\omega}_i(t)}{1+\omega_i(t)}\left(\eta_{i,n}-\zeta_{i,n}\right)-\left(1+\omega_i(t)\right)\varphi_{i,n}\\
&\quad-F_i(\gamma_i+\dot{\gamma}_i)\eta_{i,n}-g_nF_i\zeta_{i,1}
\end{aligned}
\tag{10}
$$

where $\varphi_{i,m}=\frac{\bar{\varphi}_{i,m}}{F_i^{m-1+h}}$ and parameters $g_m$, $\kappa_m$ for $m=1,\dots,n$ are given in Lemma 1. Letting $\boldsymbol{\eta}_i=\left(\eta_{i,1},\dots,\eta_{i,n}\right)^T$, $\boldsymbol{\zeta}_i=\left(\zeta_{i,1},\dots,\zeta_{i,n}\right)^T$ and $\boldsymbol{\varphi}_i=\left(\varphi_{i,1},\dots,\varphi_{i,n}\right)^T$, the global form of (10) can be written as

$$
\begin{aligned}
\dot{\boldsymbol{\eta}}_i&=F_i\boldsymbol{M}\boldsymbol{\eta}_i-F_i\boldsymbol{k}_g\zeta_{i,1}-F_i(\gamma_i+\dot{\gamma}_i)\boldsymbol{\eta}_i-\frac{\dot{F}_i}{F_i}\boldsymbol{\Pi}\boldsymbol{\eta}_i\\
\dot{\boldsymbol{\zeta}}_i&=F_i\boldsymbol{R}\boldsymbol{\zeta}_i-F_i\left(\gamma_i+\dot{\gamma}_i\right)\boldsymbol{\eta}_i-\left(1+\omega_i(t)\right)\boldsymbol{\varphi}_i-\boldsymbol{\Pi}\frac{\dot{F}_i}{F_i}\boldsymbol{\zeta}_i\\
&\quad-\frac{\dot{\omega}_i(t)}{1+\omega_i(t)}\left(\boldsymbol{\eta}_i-\boldsymbol{\zeta}_i\right)+F_i\boldsymbol{B}_1\boldsymbol{\kappa}^T\boldsymbol{\eta}_i+\frac{\boldsymbol{B}_1U_i}{F_i^{n-1+h}}
\end{aligned}
\tag{11}
$$

where $\boldsymbol{\Pi}=\operatorname{diag}(h,1+h,\dots,n-1+h)$, matrices $\boldsymbol{M}$, $\boldsymbol{R}$ and vectors $\boldsymbol{B}_1$, $\boldsymbol{B}_2$, $\boldsymbol{k}_g$, $\boldsymbol{\kappa}$ are provided in Lemma 1.

Based on the discussions above, consider the following Lyapunov candidate function as

$$
V=\sum_{i=1}^{N}\left(\left(\dot{\gamma}_i+2\gamma_i\right)\boldsymbol{\eta}_i^T\boldsymbol{P}\boldsymbol{\eta}_i+\left(\gamma_i-\gamma_i^*\right)^2+\boldsymbol{\zeta}_i^T\boldsymbol{Q}\boldsymbol{\zeta}_i\right)
\tag{12}
$$

where positive definite matrices $\boldsymbol{P}$ and $\boldsymbol{Q}$ are defined in Lemma 1 and $\gamma_i^*$ is a constant to be determined later. Using (11), the derivatives of $V$ is obtained as

$$
\begin{aligned}
\dot{V}&=\sum_{i=1}^{N}4F_i\left(\dot{\gamma}_i+\gamma_i\right)\boldsymbol{\eta}_i^T\boldsymbol{P}\left(\boldsymbol{M}\boldsymbol{\eta}_i-\boldsymbol{k}_g\zeta_{i,1}\right)\\
&\quad+\sum_{i=1}^{N}\left(2\left(\dot{\gamma}_i+\gamma_i-\gamma_i^*\right)-4F_i\left(\dot{\gamma}_i+\gamma_i\right)^2\right)\boldsymbol{\eta}_i^T\boldsymbol{P}\boldsymbol{\eta}_i\\
&\quad-\sum_{i=1}^{N}4\frac{\dot{F}_i}{F_i}\left(\dot{\gamma}_i+\gamma_i\right)\boldsymbol{\eta}_i^T\boldsymbol{P}\boldsymbol{\Pi}\boldsymbol{\eta}_i-\sum_{i=1}^{N}2\frac{\dot{F}_i}{F_i}\boldsymbol{\zeta}_i^T\boldsymbol{Q}\boldsymbol{\Pi}\boldsymbol{\zeta}_i
\end{aligned}
$$

$$
\begin{aligned}
&+\sum_{i=1}^{N}\left(2F_i\boldsymbol{\zeta}_i^T\boldsymbol{Q}\boldsymbol{R}\boldsymbol{\zeta}_i-2F_i\left(\dot{\gamma}_i+\gamma_i\right)\boldsymbol{\zeta}_i^T\boldsymbol{Q}\boldsymbol{\eta}_i\right)\\
&-\sum_{i=1}^{N}2\left(\left(1+\omega_i(t)\right)\boldsymbol{\zeta}_i^T\boldsymbol{Q}\boldsymbol{\varphi}_i+\frac{\dot{\omega}_i(t)}{1+\omega_i(t)}\boldsymbol{\zeta}_i^T\boldsymbol{Q}\left(\boldsymbol{\eta}_i-\boldsymbol{\zeta}_i\right)\right)\\
&+\sum_{i=1}^{N}2\boldsymbol{\zeta}_i^T\boldsymbol{Q}\boldsymbol{\mathcal{B}}_1\left(F_i\boldsymbol{\kappa}^T\boldsymbol{\eta}_i+\frac{U_i}{F_i^{n-1+h}}\right)
\end{aligned}\tag{13}
$$

In the following, we will estimate each term on the right-hand side of (13). From inequalities $\boldsymbol{M}^T\boldsymbol{P}+\boldsymbol{P}\boldsymbol{M}\leq-\iota_1\boldsymbol{P}$ and $q_2\left(\boldsymbol{R}^T\boldsymbol{Q}+\boldsymbol{Q}\boldsymbol{R}\right)\leq-\iota_2\boldsymbol{Q}$ with $q_2\in(0,1)$ in Lemma 1, we have

$$
\begin{aligned}
&4F_i\left(\dot{\gamma}_i+\gamma_i\right)\boldsymbol{\eta}_i^T\boldsymbol{P}\boldsymbol{M}\boldsymbol{\eta}_i\leq-2F_i\iota_1\left(\dot{\gamma}_i+\gamma_i\right)\boldsymbol{\eta}_i^T\boldsymbol{P}\boldsymbol{\eta}_i\\
&2q_2F_i\boldsymbol{\zeta}_i^T\boldsymbol{Q}\boldsymbol{R}\boldsymbol{\zeta}_i\leq-F_i\iota_2\boldsymbol{\zeta}_i^T\boldsymbol{Q}\boldsymbol{\zeta}_i
\end{aligned}\tag{14}
$$

Utilizing $\boldsymbol{P}\boldsymbol{k}_g\boldsymbol{k}_g^T\boldsymbol{P}\leq l_1\boldsymbol{P}$ shown in Lemma 1, it is obtained that

$$
\begin{aligned}
&-4F_i\left(\dot{\gamma}_i+\gamma_i\right)\boldsymbol{\eta}_i^T\boldsymbol{P}\boldsymbol{k}_g\zeta_{i,1}\leq\left(\dot{\gamma}_i+\gamma_i\right)\left(4F_i^2l_1\boldsymbol{\eta}_i^T\boldsymbol{P}\boldsymbol{\eta}_i+\zeta_{i,1}^2\right)\\
&\leq16l_1^2F_i^3\boldsymbol{\eta}_i^T\boldsymbol{P}\boldsymbol{\eta}_i+F_i\left(\dot{\gamma}_i+\gamma_i\right)^2\boldsymbol{\eta}_i^T\boldsymbol{P}\boldsymbol{\eta}_i+\left(\dot{\gamma}_i+\gamma_i\right)\zeta_{i,1}^2
\end{aligned}\tag{15}
$$

According to (1) and (10), one has that $\left(1+\omega_i(t)\right)\varphi_{i,m}=\frac{(1+\omega_i(t))\bar{\varphi}_{i,m}}{F_i^{m-1+h}}=\frac{\boldsymbol{\Delta}_m^T(t)(\bar{\boldsymbol{z}}_{i,m}-\bar{\boldsymbol{r}}_{i,m})}{F_i^{m-1+h}}$ with $\bar{\boldsymbol{z}}_{i,m}=(z_{i,1},\ldots,z_{i,m})^T,\bar{\boldsymbol{r}}_{i,m}=(r_{i,1},\ldots,r_{i,m})^T\in\mathbb{R}^m$ and $\boldsymbol{\Delta}_m(t)\boldsymbol{\Delta}_m^T(t)=\rho^2\boldsymbol{E}_m\Upsilon_m(t)\Upsilon_m^T(t)\boldsymbol{E}_m^T\leq\rho^2\boldsymbol{E}_m\boldsymbol{E}_m^T$. Thus, since $F_i\geq1$ and utilizing inequalities $\boldsymbol{Q}\boldsymbol{Q}\leq\nu_1\boldsymbol{Q}$, $\hat{\boldsymbol{E}}\leq l_2\boldsymbol{P}$, $\hat{\boldsymbol{E}}\leq l_3\boldsymbol{Q}$ with $\widetilde{\boldsymbol{E}}_s=\mathrm{diag}\{\boldsymbol{E}_s,\boldsymbol{0}\}\in\mathbb{R}^{n\times n}$ and $\hat{\boldsymbol{E}}=\sum_{s=1}^n\widetilde{\boldsymbol{E}}_s\widetilde{\boldsymbol{E}}_s^T$ given in Lemma 1, it is obtained that

$$
\begin{aligned}
&-2\left(1+\omega_i(t)\right)\boldsymbol{\zeta}_i^T\boldsymbol{Q}\boldsymbol{\varphi}_i\\
&\leq\left(1+\theta_1\right)\left(2\boldsymbol{\zeta}_i^T\boldsymbol{Q}\boldsymbol{Q}\boldsymbol{\zeta}_i+\sum_{s=1}^{n}\frac{\bar{\boldsymbol{z}}_{i,s}^T\boldsymbol{\Delta}_s(t)\boldsymbol{\Delta}_s^T(t)\bar{\boldsymbol{z}}_{i,s}}{\left(F_i^{s-1+h}\right)^2}\right)\\
&\quad+\left(1+\theta_1\right)\sum_{s=1}^{n}\frac{\bar{\boldsymbol{r}}_{i,s}^T\boldsymbol{\Delta}_s(t)\boldsymbol{\Delta}_s^T(t)\bar{\boldsymbol{r}}_{i,s}}{\left(F_i^{s-1+h}\right)^2}\\
&\leq2\left(1+\theta_1\right)\nu_1\boldsymbol{\zeta}_i^T\boldsymbol{Q}\boldsymbol{\zeta}_i+\left(1+\theta_1\right)\boldsymbol{\eta}_i^T\hat{\boldsymbol{E}}\boldsymbol{\eta}_i+\left(1+\theta_1\right)\boldsymbol{\zeta}_i^T\hat{\boldsymbol{E}}\boldsymbol{\zeta}_i\\
&\leq\left(1+\theta_1\right)\left(\left(2\nu_1+\rho^2l_3\right)\boldsymbol{\zeta}_i^T\boldsymbol{Q}\boldsymbol{\zeta}_i+\rho^2l_2\boldsymbol{\eta}_i^T\boldsymbol{P}\boldsymbol{\eta}_i\right)
\end{aligned}\tag{16}
$$

Under Assumption 2, utilizing inequalities $\boldsymbol{Q}\boldsymbol{Q}\leq\nu_1\boldsymbol{Q}$, and $\underline{\lambda}_1\boldsymbol{I}_n\leq\boldsymbol{P}$, we easily have

$$
\begin{aligned}
&-2\frac{\dot{\omega}_i(t)}{1+\omega_i(t)}\boldsymbol{\zeta}_i^T\boldsymbol{Q}\left(\boldsymbol{\eta}_i-\boldsymbol{\zeta}_i\right)\\
&\leq\theta_2\theta_4\left(\nu_1+1\right)\boldsymbol{\zeta}_i^T\boldsymbol{Q}\boldsymbol{\zeta}_i+\frac{\theta_2\theta_4}{\underline{\lambda}_1}\boldsymbol{\eta}_i^T\boldsymbol{P}\boldsymbol{\eta}_i
\end{aligned}\tag{17}
$$

Let $\boldsymbol{\Xi}=\mathrm{diag}\left(h,1+h,\ldots,n-1+h\right)$. It follows that $\boldsymbol{\Pi}=\boldsymbol{\Xi}+h\boldsymbol{I}_n$ with $\boldsymbol{\Xi}$ shown in Lemma 1. This together with inequalities $-h\boldsymbol{P}\leq\boldsymbol{\Xi}^T\boldsymbol{P}+\boldsymbol{P}\boldsymbol{\Xi}\leq h\boldsymbol{P}$ and $-h\boldsymbol{Q}\leq\boldsymbol{\Xi}^T\boldsymbol{Q}+\boldsymbol{Q}\boldsymbol{\Xi}\leq h\boldsymbol{Q}$ in Lemma 1 implies that

$$
\begin{aligned}
&-4\left(\dot{\gamma}_i+\gamma_i\right)\frac{\dot{F}_i}{F_i}\boldsymbol{\eta}_i^T\boldsymbol{P}\boldsymbol{\Pi}\boldsymbol{\eta}_i-2\frac{\dot{F}_i}{F_i}\boldsymbol{\zeta}_i^T\boldsymbol{Q}\boldsymbol{\Pi}\boldsymbol{\zeta}_i\\
&\leq-h\left(2\frac{\dot{F}_i}{F_i}-\frac{\left|\dot{F}_i\right|}{F_i}\right)\left(2\left(\dot{\gamma}_i+\gamma_i\right)\boldsymbol{\eta}_i^T\boldsymbol{P}\boldsymbol{\eta}_i+\boldsymbol{\zeta}_i^T\boldsymbol{Q}\boldsymbol{\zeta}_i\right)
\end{aligned}\tag{18}
$$

It follows from the inequalities $\boldsymbol{Q}\boldsymbol{\mathcal{B}}_1\boldsymbol{\kappa}^T\boldsymbol{\kappa}\boldsymbol{\mathcal{B}}_1^T\boldsymbol{Q}\leq l_4\boldsymbol{Q}$ and $\underline{\lambda}_1\boldsymbol{I}_n\leq\boldsymbol{P}$ in Lemma 1 that

$$
\begin{aligned}
&2\boldsymbol{\zeta}_i^T\boldsymbol{Q}\boldsymbol{\mathcal{B}}_1\left(F_i\boldsymbol{\kappa}^T\boldsymbol{\eta}_i+\frac{U_i}{F_i^{n-1+h}}\right)\\
&=2F_i\left(1+c_i\left(1+\omega_i(t)\right)\right)\boldsymbol{\zeta}_i^T\boldsymbol{Q}\boldsymbol{\mathcal{B}}_1\boldsymbol{\kappa}^T\boldsymbol{\eta}_i\\
&\quad-2\left(1+\omega_i(t)\right)\boldsymbol{\zeta}_i^T\boldsymbol{Q}\boldsymbol{\mathcal{B}}_1\frac{\sum_{j\in\mathcal{N}_i}a_{ij}F_j\boldsymbol{\kappa}^T\boldsymbol{\eta}_j}{F_i^{n-1+h}}\\
&\leq2l_4\boldsymbol{\zeta}_i^T\boldsymbol{Q}\boldsymbol{\zeta}_i+N\left(1+\theta_1\right)^2\theta_3^2\frac{F_i^2}{\underline{\lambda}_1}\boldsymbol{\eta}_i^T\boldsymbol{P}\boldsymbol{\eta}_i\\
&\quad+\left(1+(1+N)\left(1+\theta_1\right)\theta_3\right)^2\frac{F_i^2}{\underline{\lambda}_1}\boldsymbol{\eta}_i^T\boldsymbol{P}\boldsymbol{\eta}_i
\end{aligned}\tag{19}
$$

where $\boldsymbol{A}$ is adjacency matrix associated with communication graph and $c_i=\sum_{j\in\mathcal{N}_i}a_{ij}+b_i$.

Substituting (14)–(19) into (13) yields

$$
\begin{aligned}
\dot{V} \leq & -\sum_{i=1}^{N} \chi_{i,1}\left(\dot{\gamma}_i+\gamma_i\right)\boldsymbol{\eta}_i^T \boldsymbol{P}\boldsymbol{\eta}_i-\sum_{i=1}^{N} \chi_{i,2}\boldsymbol{\zeta}_i^T \boldsymbol{Q}\boldsymbol{\zeta}_i \\
& +\sum_{i=1}^{N}\left(F_i \boldsymbol{\delta}_i^T \boldsymbol{S}\boldsymbol{\delta}_i+\varpi_{i,1}\boldsymbol{\eta}_i^T \boldsymbol{P}\boldsymbol{\eta}_i+\left(\dot{\gamma}_i+\gamma_i\right)\zeta_{i,1}^2\right) \\
& -\sum_{i=1}^{N} 2\gamma_i^* \boldsymbol{\eta}_i^T \boldsymbol{P}\boldsymbol{\eta}_i
\end{aligned} \tag{20}
$$

where $\boldsymbol{\delta}_i=\left(\left(\dot{\gamma}_i+\gamma_i\right)\boldsymbol{\eta}_i^T,\boldsymbol{\zeta}_i^T\right)$, $\chi_{i,1}=2F_i\tilde{\iota}+2h\left(2\frac{\dot{F}_i}{F_i}-\frac{|\dot{F}_i|}{F_i}\right)-2$ with $\tilde{\iota}=\min\{\iota_1,\iota_2\}$, $\chi_{i,2}=F_i\tilde{\iota}+h\left(2\frac{\dot{F}_i}{F_i}-\frac{|\dot{F}_i|}{F_i}\right)-\varpi_2$ with $\varpi_2$ given in (6), $\varpi_{i,1}=16l_1^2F_i^3+(1+\theta_1)\rho^2 l_2+\left(\left(1+(1+N)\left(1+\theta_1\right)\theta_3\right)^2+N\left(1+\theta_1\right)^2\theta_3^2\right)F_i^2/\underline{\lambda}_1+\theta_2\theta_4/\underline{\lambda}_1$ and matrix $\boldsymbol{S}$ is shown in Lemma 1. Note that in the following, we use abbreviated notation $\Gamma_i$ instead of $\Gamma_i\left(\zeta_{i,1},\boldsymbol{\eta}_i\right)$ for the neatness.

Then, using (6), if $\dot{F}_i\geq 0$ ($F_i\geq 1$), we have $\chi_{i,1}=\frac{4F_i\tilde{\iota}}{3}+\frac{2\varsigma\tilde{\iota}}{3}+2\beta+2\varpi_2+2\Gamma_i\geq 2\tilde{\iota}+2\beta+2\Gamma_i$, $\chi_{i,2}=\frac{2F_i\tilde{\iota}}{3}+\frac{\varsigma\tilde{\iota}}{3}+\beta+1+\Gamma_i\geq\tilde{\iota}+\beta+\Gamma_i$. Else, if $\dot{F}_i\leq 0$ ($F_i\geq 1$), we obtain $\chi_{i,1}=2\varsigma\tilde{\iota}+6\beta+4+6\varpi_2+6\Gamma_i\geq 2\tilde{\iota}+2\beta+2\Gamma_i$, $\chi_{i,2}=\varsigma\tilde{\iota}+3\beta+3+2\varpi_2+3\Gamma_i\geq\tilde{\iota}+\beta+\Gamma_i$. Utilizing (7) and inequality $\underline{\lambda}_2\boldsymbol{I}_n\leq\boldsymbol{Q}$ in Lemma 1, if $\zeta_{i,1}^2+\boldsymbol{\eta}_i^T\boldsymbol{P}\boldsymbol{\eta}_i\neq 0$, one has

$$
\begin{aligned}
& -2\Gamma_i\left(\dot{\gamma}_i+\gamma_i\right)\boldsymbol{\eta}_i^T\boldsymbol{P}\boldsymbol{\eta}_i-\Gamma_i\boldsymbol{\zeta}_i^T\boldsymbol{Q}\boldsymbol{\zeta}_i+\left(\dot{\gamma}_i+\gamma_i\right)\zeta_{i,1}^2 \\
= & -\left(\dot{\gamma}_i+\gamma_i\right)\zeta_{i,1}^2\left(1-\frac{\zeta_{i,1}^2}{\boldsymbol{\eta}_i^T\boldsymbol{P}\boldsymbol{\eta}_i+\zeta_{i,1}^2}\right) \\
& -\frac{2\left(\dot{\gamma}_i+\gamma_i\right)^2\boldsymbol{\eta}_i^T\boldsymbol{P}\boldsymbol{\eta}_i\zeta_{i,1}^2}{\underline{\lambda}_2\left(\boldsymbol{\eta}_i^T\boldsymbol{P}\boldsymbol{\eta}_i+\zeta_{i,1}^2\right)}-\frac{\boldsymbol{\zeta}_i^T\boldsymbol{Q}\boldsymbol{\zeta}_i\zeta_{i,1}^2}{2\left(\boldsymbol{\eta}_i^T\boldsymbol{P}\boldsymbol{\eta}_i+\zeta_{i,1}^2\right)} \\
& -\frac{\left(\dot{\gamma}_i+\gamma_i\right)\boldsymbol{\zeta}_i^T\boldsymbol{Q}\boldsymbol{\zeta}_i\zeta_{i,1}^2}{\underline{\lambda}_2\left(\boldsymbol{\eta}_i^T\boldsymbol{P}\boldsymbol{\eta}_i+\zeta_{i,1}^2\right)}+\left(\dot{\gamma}_i+\gamma_i\right)\zeta_{i,1}^2 \\
\leq & -\frac{2\left(\dot{\gamma}_i+\gamma_i\right)^2\boldsymbol{\eta}_i^T\boldsymbol{P}\boldsymbol{\eta}_i\zeta_{i,1}^2}{\underline{\lambda}_2\left(\boldsymbol{\eta}_i^T\boldsymbol{P}\boldsymbol{\eta}_i+\zeta_{i,1}^2\right)}-\frac{\boldsymbol{\zeta}_i^T\boldsymbol{Q}\boldsymbol{\zeta}_i\zeta_{i,1}^2}{2\left(\boldsymbol{\eta}_i^T\boldsymbol{P}\boldsymbol{\eta}_i+\zeta_{i,1}^2\right)}\leq 0
\end{aligned}
$$

and if $\zeta_{i,1}^2+\boldsymbol{\eta}_i^T\boldsymbol{P}\boldsymbol{\eta}_i=0$, one further obtains $-\Gamma_i\boldsymbol{\zeta}_i^T\boldsymbol{Q}\boldsymbol{\zeta}_i-2\Gamma_i\left(\dot{\gamma}_i+\gamma_i\right)\boldsymbol{\eta}_i^T\boldsymbol{P}\boldsymbol{\eta}_i+\left(\dot{\gamma}_i+\gamma_i\right)\zeta_{i,1}^2=0$ since $\Gamma_i=0$ from (7) and $\zeta_{i,1}^2+\boldsymbol{\eta}_i^T\boldsymbol{P}\boldsymbol{\eta}_i=0$ if and only if $\zeta_{i,1}^2=0$ and $\boldsymbol{\eta}_i^T\boldsymbol{P}\boldsymbol{\eta}_i=0$. Therefore, choosing $\gamma_i^*>\varpi_{i,1}/2$ and using $\boldsymbol{S}\leq 0$ from Lemma 1, all of these together with (20) lead to

$$
\dot{V}=-(\tilde{\iota}+\beta)\sum_{i=1}^{N}\left(2\left(\dot{\gamma}_i+\gamma_i\right)\boldsymbol{\eta}_i^T\boldsymbol{P}\boldsymbol{\eta}_i+\boldsymbol{\zeta}_i^T\boldsymbol{Q}\boldsymbol{\zeta}_i\right)\leq 0 \tag{21}
$$

Therefore, we can know from (21) that $V$ is bounded and so are $\boldsymbol{\eta}_i$, $\boldsymbol{\zeta}_i$, $\gamma_i$. By noting from (6) that $\gamma_i, i=1,\dots,n$ are monotonically increasing, it then follows that the adaptive gains $\gamma_i$ converge to some finite values. By (21), $\dot{V}\equiv 0$ implies that $\boldsymbol{\eta}_i=0$ and $\boldsymbol{\zeta}_i=0$. Thus, using LaSalle's Invariance principle, we can obtain $\lim_{t\to\infty}\boldsymbol{\eta}_i=0$ and $\lim_{t\to\infty}\boldsymbol{\zeta}_i=0$. According to (6), one knows that $\lim_{t\to\infty}F_i(t)$ are upper-bounded by a constant $1+3(\beta+1+\varpi_2)/\tilde{\iota}$ as $\lim_{t\to\infty}\boldsymbol{\eta}_i=0$ and $\lim_{t\to\infty}\zeta_{i,1}=0$. Thus, using $\boldsymbol{\eta}_i=\left(\eta_{i,1},\dots,\eta_{i,n}\right)^T$, $\boldsymbol{\zeta}_i=\left(\zeta_{i,1},\dots,\zeta_{i,n}\right)^T$ and based on definition (9), we further obtain $\lim_{t\to\infty}z_{i,m}=0$ and $\lim_{t\to\infty}r_{i,m}=0$. As $r_{i,m}=z_{i,m}-\sigma_{i,m}$, it is then concluded that $\lim_{t\to\infty}\sigma_{i,m}=0$. For $m=1,\dots,n$, letting $\boldsymbol{\sigma}_m=(\sigma_{1,m},\dots,\sigma_{N,m})$, $\boldsymbol{x}_m=(x_{1,m},\dots,x_{N,m})$ and $\underline{\boldsymbol{x}}_{0,m}=(x_{0,m},\dots,x_{0,m})\in\mathbb{R}^N$, we have $\boldsymbol{\sigma}_m=(\boldsymbol{L}+\boldsymbol{B})\left(\boldsymbol{x}_m-\underline{\boldsymbol{x}}_{0,m}\right)$. Since $(\boldsymbol{L}+\boldsymbol{B})$ is a positive matrix under Assumption 1, it is further concluded that $\lim_{t\to\infty}x_{i,m}-x_{0,m}=0$, that is, full states consensus is achieved. This completes the proof.

**Remark 5.** The compensator-based dynamic output feedback consensus protocol (5) depends only on the relative output measurements of neighboring agents and independent of any global information of the communication graph, and thereby are fully distributed. It is thus easier to implement than those output feedback control method presented in some existing literatures where the observers embedded at followers need to communicate with each other and the global graph matrix need to be known. Furthermore, compensator state $z_{i,m}$ in (4) are available to agent $i$ if there is an internal local information transmission loop between the compensator embedded at agent $i$ and the actuator of $i$th agent. Therefore, there is no need to measure the feedback signal $z_{i,m}$ by employing sensors, and thus $z_{i,m}$ but not $(1+\omega_i(t))z_{i,m}$ is utilized for the controller design in (5). Note that it is not hard to add such an information transmission loop between the compensator and the actuator in practice, and hence it is commonly assumed in existing results that each agent $i$ can use this local information for feedback [35].

**Remark 6.** Under adaptive law (6), the final value of $F_i(t)$ is upper bounded by constant $1+3(\beta+1+\varpi_2)/\tilde{\iota}$ due to $\lim_{t\to\infty}\Gamma_i\left(\zeta_{i,1},\boldsymbol{\eta}_i\right)=0$, and thus the constant $\gamma_i^*$ whose value depends on $F_i(t)$ is obtainable. It should be noticed that $\gamma_i^*$ is not directly used for the consensus controller design and it is vital to ensure that controller (5) with compensator (4) is applicable to sensor–actuator attacks case. Besides, following the proof process of Theorem 1, the determination of control gains and compensator parameters in (4)–(5) is

**Table 1**
Comparison of the considered problems.

| Multi-agent systems | [11] | [12] | [13] | [27] | [28] | [29] |
|---|---|---|---|---|---|---|
| High-order unmatched system | × | × | × | √ | × | √ |
| Uncertainty | × | × | × | √ | √ | √ |
| Output feedback | √ | √ | × | × | × | √ |
| Sensor–actuator attacks | √ | √ | √ | √ | √ | × |
| Attack model | × | × | × | √ | × | * |
| Whether the controller is suitable for the system in this paper | No | No | No | No | No | No |

independent of the upper bound of actuator attacks $\theta_3$. The constant $\theta_3$ only affect the convergent upper bound (given as $\gamma_i^*$) of dynamic parameters $\gamma_i$. Therefore, $\theta_3$ in Assumption 2 can assumed to be unknown.

**Remark 7.** In this paper, a new compensator based dynamic output feedback controller is developed, by which the effects of mismatched uncertainties and time-varying state dependent sensor–actuator attacks are mitigated simultaneously. The specific work including two aspects. First, a novel distributed compensator (4) is constructed for each agent based only on the attacked output measurement, which is a key component for the designed resilient consensus controller. A distinct feature of compensator (4) is that it includes two dynamic parameters $F_i(t)$ and $\gamma_i(t)$, in which $F_i(t)$ is introduced to counteract the effects of unmatched uncertainties and the time-varying state dependent sensor–actuator attacks and $\gamma_i(t)$ is introduced to play a key role in finding a function $V$ such that $\dot{V}$ is negative definite. Second, a specific term $\left(\gamma_i - \gamma_i^*\right)^2$ is introduced for the construction of Lyapunov function $V$. Note that the introduction of this term provide a flexible definite negative term which is vital to ensure that the derivative of $V$ along system (11) is negative definite.

**Remark 8.** In practice, the adversarial attacks are usually modeled as time-varying and state-dependent given as (2)–(3) [23,24]. Compared with the attacks in the other forms of disturbances [11–14], faults [15,16], measurement noise [17,18], sensor uncertainties [19,20] and unknown control directions [21,22], our problem is technically more challenging and thus is particularly worthy of investigating. Specific reasons are given as follows: First, compared with the disturbance form attacks in [11–14], in which the attacks can be separated from the measured state and estimated by introducing an adaptive control architecture, the time-varying state-dependent attacks obviously cannot been isolated from the actual measured state and thus, the proposed adaptive control method in [11–14] is inapplicable. Second, from a system theoretical point of view, faults in [15,16], measurement noise in [17,18] and disturbance formed attack are fundamentally same. Hence, the traditional adaptive fault-tolerant control schemes cannot be directly applied for the case with time-varying state-dependent attack. Third, the sensor uncertainties in [19,20] and unknown control directions in [21,22] are actually the same as time-varying state-dependent sensor attack and time-invariant state-dependent actuator attack, respectively. Thus, the proposed control methods cannot be applied to the case considering time-varying state-dependent sensor and actuator attacks simultaneously.

**Remark 9.** From a system theoretical point of view, faults and attacks are fundamentally same. However, the attacks may be undetectable because they are strategically optimized in a coordinated way by malicious adversaries while the faults cannot collude with each other. Therefore, the attacks are usually with some system information such that the traditional adaptive fault-tolerant control schemes cannot be directly applied. At the performance level, the attack-resilient control expects the controlled system to restore the nominal operation with simultaneously minimizing the performance loss. Therefore, the results on actuator faults cannot be applied to the case for actuator attacks, especially for the case considering time-varying state-dependent sensor and actuator attacks simultaneously.

**Remark 10.** Compared with the linear models and state-independent bounded attacks considered in Ref. [13], the high-order unmatched uncertainty multi-agent system with time-varying state-independent sensor–actuator attacks and unmeasurement of full-state information is considered in this paper. And thus, the proposed consensus controllers in [13] cannot extended to the case in this paper. Besides, the asymptotically consensus result is obtained in this paper, which is better than the bounded consensus result given in literature [13]. In addition to [13], the output feedback consensus controllers are proposed in Refs. [11,12]. However, compared to the problem considered in this paper, the sensor–actuator attacks are assumed to be time-varying state-independent and bounded, and the high-order unmatched uncertainty still leave out of consideration. Similar to [13], only bounded consensus result but not asymptotically consensus result is obtained in literatures [11,12]. In Ref. [28], although the system uncertainty is further considered, the assumptions of state-independent bounded sensor–actuator attacks and measurement of full-state information are still necessary. These make the proposed controller in [28] cannot be applied to the system considered in this paper. Different from [28], both high-order unmatched uncertainty and state-dependent sensor–actuator attacks are considered in [27] and this paper. However, in Ref. [27], the uncertainty is considered with constant uncertain parameter and the full-state information need to be measured, and thus the designed controller in [27] cannot extended to the system considered in this paper. Besides, except for sensor–actuator attacks, the same dynamic output feedback consensus problem as this paper is considered in [29] for high-order unmatched nonlinear systems. For clarity, the detailed comparisons of considered problems between these literatures and this paper have been shown in Table 1. Note that in line 6 of Table 1, √ and × means that the considered attack models are same and different respectively. * indicates that no attack is considered.

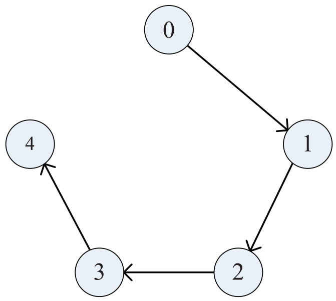


**Fig. 1.** Topology of augmented graph $\bar{\mathcal{G}}$.

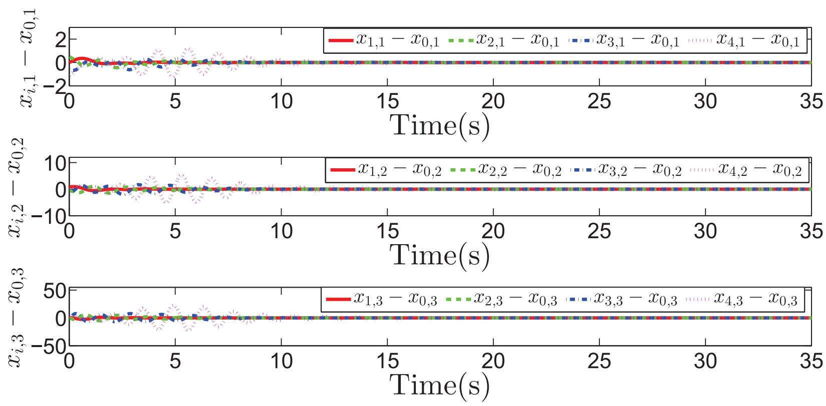


**Fig. 2.** Trajectories of consensus errors under joint sensor–actuator attacks.

## 4. Numerical examples

In this section, we consider a group of networked multiple systems consisting of 1 leader (labeled as 0) and 4 followers (labeled as 1,2,3,4) subject to sensor and actuator attacks to demonstrate the effectiveness of the proposed theoretical results. The communication topology among agents is given in Fig. 1, in which there is a spanning tree satisfying Assumption 1. The dynamics of agent system is descried by (1) with $n = 3$ and $\boldsymbol{\Delta}_1(t) = -(1 + 0.2\sin(t))$, $\boldsymbol{\Delta}_2(t) = (0.3\cos(t), 0.3\sin(t))^T$, $\boldsymbol{\Delta}_3(t) = (-0.3\cos(0.5t), -0.3\sin(0.5t), -1 + 0.3cos(0.5t))^T$, for $i = 0, 1, 2, 3, 4$. The time-varying state-dependent sensor and actuator attack are considered as (2) and (3), respectively, with $\omega_i(t) = 0.2 + 0.1\sin(0.5t)$ and $\Omega_i(t) = 1 + 0.4\sin(0.1t)$, which satisfies Assumption 2 with $\theta_1 = 0.3$, $\theta_2 = 0.05$, $\theta_3 = 1.4$, $\theta_4 = 1$.

According to the results in Theorem 1, we construct the dynamic output feedback consensus controller as (5) with compensator (4) and $F_i$, $\gamma_i$ updated by (6), in which $g_1 = 3$, $g_2 = 3$, $g_3 = 1$, $\kappa_1 = -1$, $\kappa_2 = -3$, $\kappa_3 = -3$, $h = 5$, $\tilde{\imath} = 1$, $\beta = 0.1$, $\varpi_2 = 13.186$ and matrix $\boldsymbol{P} = \begin{pmatrix} 2.38 & 3.5 & 1.37 \\ 3.5 & 6.8 & 2.7 \\ 1.3 & 2.7 & 2.1 \end{pmatrix}$. Then, for the initial condition $x_{0,1} = 0.1$, $x_{1,1} = 0.1$, $x_{2,1} = 0.6$, $x_{3,1} = -0.5$, $x_{4,1} = -1$, $x_{0,2} = 0.1$, $x_{1,2} = 1$, $x_{2,2} = -0.5$, $x_{3,2} = -1$, $x_{4,2} = 0.5$, and $x_{0,3} = 0.1$, $x_{1,3} = 0.5$, $x_{2,3} = -0.5$, $x_{3,3} = 0.2$, $x_{4,3} = 0.6$, $F_i(0) = 1 (i = 1, 2, 3, 4)$, $\gamma_1(0) = 2$, $\gamma_2(0) = 3$, $\gamma_3(0) = 3$, $\gamma_4(0) = 2$, Figs. 2–3 show the simulation results under joint sensor–actuator attacks. It can be observer form Fig. 2 that the consensus under both sensor attack and joint sensor–actuator attacks are indeed achieved. Fig. 3 shows that the dynamic gains $F_i$ and $\gamma_i$ are converge to some finite constant values.

## 5. Conclusion

In this paper, the resilient leader-following consensus problem for high-order multi-agent systems in the presence of unmatched uncertainties and time-varying sensor–actuator attacks has been considered. By introducing a novel compensator, in which two dynamic gains are suitably selected to dominate the uncertainties and sensor–actuator attacks, a novel resilient distributed controller has been designed to guarantee the leader-following consensus. A significant advantage of the proposed controller is that it requires no assumption that the compensator embedded in each follower have to share information with their neighbors and it is independent of global communication graph information, and hence is fully distributed. Note that the controller has been shown to be effective in the presence of joint sensor and actuator attacks that are time-varying and state dependent. It has theoretically proved that the

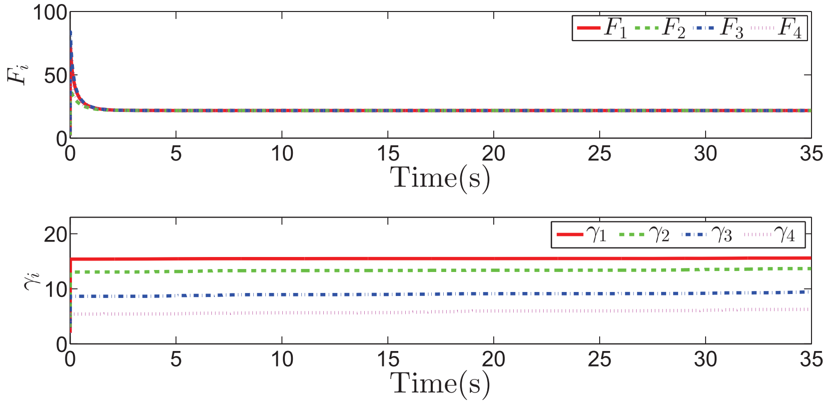


**Fig. 3.** Trajectories of dynamic parameters $F_i, \gamma_i, i = 1, 2, 3, 4$ under joint sensor–actuator attacks.

asymptotic leader-following consensus is achieved, and the resilient control gain and compensator parameters can be obtained by solving matrix inequalities. A simulation example bas been given to illustrate the effectiveness of the proposed theoretical results.

Up to now, the output feedback based resilient consensus problem for high-order uncertain multi-agent systems under unbounded malicious attacks is still an open problem need to be solved. The research on this problem requires to estimate the unknown bounds of attacks based only on the attacked output state information while deal with the full state dependent uncertainties. It is an interesting problem in our future work.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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