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ORIGINAL ARTICLE

Hesitant fuzzy linguistic rough set over two universes model and its applications

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Abstract In practical decision making situations, decision makers usually express preferences by evaluating qualitative linguistic alternatives using the hesitant fuzzy linguistic term set. To analyze the hesitant fuzzy linguistic information effectively, we aim to apply the rough set over two universes model. Thus, it is necessary to study the fusion of the hesitant fuzzy linguistic term set and rough set over two universes. This paper proposes a general framework for the study of the hesitant fuzzy linguistic rough set over two universes. First, both the definitions and some fundamental properties will be developed, followed by construction of a general decision making rule based on the hesitant fuzzy linguistic information. Finally, we illustrate the newly proposed approach according to the basis of person-job fit, and discuss its applications compared to classical methods.

Keywords Hesitant fuzzy linguistic term set · Rough set over two universes · Hesitant fuzzy linguistic rough set over two universes · Decision making · Person-job fit

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1 Introduction

In real-world decision making activities, due to the inherent uncertainty of preference expression, and the management, storage and extraction of various useful information is not always presented as crisp numbers, it is believed that fuzzy numbers are advantageous for handling various complicated information systems. Fuzzy set theory [38], established by Zadeh in 1965, provides robust solutions in many application domains such as knowledge discovery, information processing, uncertainty mining, and machine learning [1, 8, 12, 18, 30, 31]. In fuzzy sets, the membership degree of an element is a single crisp value within $[0, 1]$. However, the classical fuzzy set experiences limitations when working with incomplete and uncertain information. Thus, many additional generalizations of fuzzy sets were developed [2, 19, 20, 35, 39].

During the decision making process, decision makers might hesitate among several possible membership values when determining the membership of an element belonging to a given set. To address this issue, Torra and Narukawa [29] and Torra [28] established the concept of the hesitant fuzzy set (HFS). Hesitant fuzzy sets are widely-used in modeling quantitative expressions when decision makers are likely to hesitate among several numbers in evaluating an alternative. However, when confronted with problems that are too complex or ill-defined to be addressed by utilizing quantitative expressions, it may be suitable to evaluate the membership degrees of alternatives by using qualitative instead of quantitative expressions. The fuzzy linguistic approach is generally regarded as an effective method to deal with these difficulties.

Motivated by the superiorities of the hesitant fuzzy set and fuzzy linguistic approach, Rodriguez et al. [25] introduced the concept of the hesitant fuzzy linguistic term set

(HFLTS). Compared to other generalizations of fuzzy sets, hesitant fuzzy linguistic term set potentially deals with hesitant fuzzy linguistic information perfectly. Suppose there is a human resource manager who intends to estimate the English writing level for a job seeker: the manager may consider the membership degree as between modest and competent, while the same individual may deem good as another justifiable answer. In this situation, the assessment can be expressed as modest and competent, or good. Following the introduction of the hesitant fuzzy linguistic term set, numerous scholars enriched the theory in different facets [10, 17, 26, 32, 34, 43]. In light of the above, decision making utilizing hesitant fuzzy linguistic information could better handle uncertain situations and qualitative information, and provide experts with more exemplary and flexible access to convey knowledge base understanding.

To analyze the hesitant fuzzy linguistic information, we aim to introduce the rough set theory to decision making problems based on hesitant fuzzy linguistic information. Rough set theory, due to Pawlak [21], is a widely-used mathematical tool to cope with various uncertainties in real-life applications [11, 14, 16, 41]. Since the equivalence relation in a classical rough set is a relatively restrictive condition that may hinder application domains, multiple generalizations of rough sets were developed [5–7, 9, 24, 37]. Among the various extension forms of the classical rough set, considering that rough sets and fuzzy sets are the two main tools used for processing uncertain information for information systems and are generally accepted as related, but distinct and complementary. Therefore, how to generalize the rough set model to a fuzzy case is significant for the development of rough set theory. Thus, Dubois and Prade [5] developed the concept of fuzzy rough sets. Since hesitant fuzzy sets hold many advantages over classical fuzzy sets, research into the integration of hesitant fuzzy sets with rough sets is a popular area of study. For instance, Yang et al. [36] constructed a hesitant fuzzy rough set that includes both constructive and axiomatic approaches. Deepak and John [4] studied the relationships between hesitant fuzzy rough approximation operators. Zhang et al. [42] further established an interval-valued hesitant fuzzy rough set model and utilized it in an illustrative medical diagnosis case. Liang and Liu [15] proposed the decision-theoretic rough set under hesitant fuzzy information, and researched its related ranking and resource allocation methods.

Since consideration of two universes expresses practical decision making information better than a single universe, many recent studies put emphasis on rough sets over two universes [22, 27, 33, 40]. By extending to two universes for a rough set, it is convenient to depict various intrinsic relationships composed of two different objections related to real-life decision making. In a single universe, there are some limitations for a rough set to analyze these relationships. For example, the research of the relationship between a disease set and a symptom set exerts a positive influence on modern clinical disease diagnosis problems. The rough set over two universes enables medical experts to obtain lower and upper rough approximations, which act as two important index sets for medical experts to determine the patient's disease. Therefore, a rough set over two universes could not only analyze many kinds of significant relationships in multi-attribute decision making activities, but also provide lower and upper rough approximations as two types of decision making basis according to the advantages of rough set theory. Conclusively, compared to a rough set on single universe and other multi-attribute decision making tools, the model of rough set over two universes could be regarded as a relatively ideal data analysis strategy.

In this paper, in order to deal with the problems of hesitant fuzzy linguistic term set data analysis, it is necessary to introduce a new rough set model called hesitant fuzzy linguistic (HFL) rough set over two universes. Since there are few studies on the combination of these two theories, we will explore both the definitions and basic properties of the HFL rough set over two universes model. Moreover, by utilizing the proposed model, we aim to present a general approach and show basic steps of decision making through a case study of person-job fit. In a real-life job market, person-job fit (P-J fit) tends to exert an ever-growing influence on various areas of business activities that aims to match an employee's skills and abilities to job demands. Moreover, it is generally acknowledged that the person-job fit increases job satisfaction and reduces turnover intention [3, 13, 23].

The presentation of the article is organized below: in the next section, we review basic knowledge about the fuzzy linguistic approach, hesitant fuzzy set and hesitant fuzzy linguistic term set. In Sect. 3, we introduce the hesitant fuzzy linguistic rough set over two universes and its related properties. Section 4 presents decision making rules and algorithm by utilizing the proposed model. Section 5 illustrates the steps of the proposed decision making method through a person-job fit example. In Sect. 6, we conclude this paper with some remarks.

2 Preliminaries

In this section, we briefly review the concepts of the fuzzy linguistic approach, hesitant fuzzy set and hesitant fuzzy linguistic term set.

2.1 Fuzzy linguistic approach

Since linguistic terms are close to human's expression of opinions, they are widely used by many decision makers in the real-world. To let fuzzy tools suit problems that are defined as qualitative situations in nature, the fuzzy linguistic approach [39] is an approximation method that represents qualitative aspects as linguistic values, by means of linguistic variables. The linguistic values are not numbers, but words or sentences used in a natural language.

The fuzzy linguistic approach encompasses different linguistic computational models such as a 2-tuple linguistic model, a linguistic model based on type-2 fuzzy sets, a linguistic model based on membership functions, and a linguistic model based on ordinal scales. Usually, these linguistic models can be classified as either a function-based model or a symbolic linguistic model [25]. In this paper, we primarily utilize the ordered structure approach based on the symbolic linguistic model.

We consider $S = \{s_0, s_1, \ldots, s_g\}$ as a finite and totally ordered linguistic term set. The cardinality of the aforementioned linguistic term set is an odd number. The mid-term in a linguistic term set records an assessment of approximately 0.5 and the remaining linguistic terms are arranged symmetrically around it. Theoretically, the cardinality of S might be a sufficiently large positive integer. However, in management science and real-life decision making procedures, in order to combat problems more efficiently, it is generally acknowledged that the limit of cardinality is 11, or at most 13 [32]. In this paper, we define the limit of cardinality for S as 11, or at most 13.

For example, a set of seven terms S can be represented as follows:

$$S = \{s_0 : \textit{very poor}, s_1 : \textit{poor}, s_2 : \textit{medium poor}, \\ s_3 : \textit{fair}, s_4 : \textit{medium good}, s_5 : \textit{good}, s_6 : \textit{very good}\}.$$

Usually, it is required that the linguistic term S should satisfy the following additional characteristics:

1. The set is ordered: $s_i \le s_j \Leftrightarrow i \le j$. Thus, there exists a maximization operator: $\max(s_i, s_j) = s_i$, if $s_i \ge s_j$, and a minimization operator: $\min(s_i, s_j) = s_i$, if $s_i \le s_j$;
2. There is a negation operator: $neg(s_i) = s_{g-i}$, where the cardinality of the linguistic term set S is $g + 1$.

2.2 Hesitant fuzzy set

Torra and Narukawa [29] and Torra [28] presented the concept of hesitant fuzzy sets, which permits the membership degree of an element to a reference set expressed by several possible values.

Definition 2.1 [28] Let a set U be fixed. A hesitant fuzzy set (HFS) on U is in terms of a function h that when applied to U returns a subset of $[0, 1]$, which can be represented in terms of the following mathematical symbol:

$$F = \{\langle x, h_F(x)\rangle | x \in U\}, \tag{1}$$

where $h_F(x)$ is a set of some different finite values in $[0, 1]$, denoting the possible membership degrees of the element $x \in U$ to the set F. For convenience, $h_F(x)$ is the hesitant fuzzy element (HFE) and the set of all hesitant fuzzy elements is called HFEs.

2.3 Hesitant fuzzy linguistic term set

For HFS, an expert may hesitate among several possible values as the membership degree when evaluating an alternative. In a qualitative setting, an expert may also hesitate among several linguistic terms. To address such cases, based on the fuzzy linguistic approach and hesitant fuzzy set, Rodriguez et al. [25] presented the concept of hesitant fuzzy linguistic term sets.

Definition 2.2 [25] Suppose that $S = \{s_0, s_1, \ldots, s_g\}$ is a linguistic term set, and a hesitant fuzzy linguistic term set (HFLTS), H_S, is an ordered finite subset of the consecutive linguistic terms of S.

Since Definition 2.2 does not give specific mathematical form of HFLTS, Liao and Xu [17] mathematically refined the definition of HFLTS to be much easier understood.

Definition 2.3 [17] Let a set U be fixed and $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set. A hesitant fuzzy linguistic term set (HFLTS) on U is in terms of a function h that when applied to U returns a subset of S, which can be represented in terms of the following mathematical symbol:

$$A = \{\langle x, h_A(x)\rangle | x \in U\}, \tag{2}$$

where $h_A(x)$ is a set of some different ordered finite values in the linguistic term set S, denoting the possible membership degrees of the element $x \in U$ to the set A. For convenience, $h_A(x)$ is the hesitant fuzzy linguistic element (HFLE) and the set of all hesitant fuzzy linguistic elements is called HFLEs.

Suppose that U is the universe of discourse, then the set of all hesitant fuzzy linguistic term sets on U is denoted by $HFL(U)$.

Example 2.1 Suppose that $S = \{s_0 : \textit{very poor}, s_1 : \textit{poor}, s_2 : \textit{medium poor}, s_3 : \textit{fair}, s_4 : \textit{medium good}, s_5 : \textit{good}, s_6 : \textit{very good}\}$ is a linguistic term set. Let $U = \{x_1, x_2\}$ be a universe set, $h_A(x_1) = \{s_4 : \textit{medium good}, s_5 : \textit{good}\} = \{s_4, s_5\}$ and $h_A(x_2) = \{s_0 : \textit{very poor}, s_1 : \textit{poor},$

$s_2: medium\ poor\} = \{s_0, s_1, s_2\}$ be the HFLEs of $x_i (i = 1, 2)$ to a set A, respectively. Then, A can be expressed as:

$A = \{\langle x_1, \{s_4, s_5\}\rangle, \langle x_2, \{s_0, s_1, s_2\}\rangle\}$.

Here, we define two special hesitant fuzzy linguistic term sets as follows:

1. For all $x \in U$, we call A an empty hesitant fuzzy linguistic term set if and only if $h_A(x) = \{s_0\}$. In this case, the empty hesitant fuzzy linguistic term set is represented by $\emptyset$.
2. For all $x \in U$, we call A a full hesitant fuzzy linguistic term set if and only if $h_A(x) = \{s_g\}$. In this case, the full hesitant fuzzy linguistic term set is represented by U.

Wei et al. [32] defined the negation, max-union and min-intersection operations on hesitant fuzzy linguistic term sets based on the idea proposed by Torra [28].

Definition 2.4 [32] Suppose that $S = \{s_0, s_1, \ldots, s_g\}$ is a linguistic term set. Let U be the universe of discourse, $\forall A, B \in HFL(U)$, then

1. The negation of A, denoted by A^c, is defined as:

$$h_{A^c}(x) = \sim h_A(x) = \{s_{g-i} | i \in Ind(h_A(x))\}; \tag{3}$$

where $Ind(s_i)$ denotes the index i of a linguistic term s_i in a linguistic term set S, and $Ind(h_A(x))$ denotes the set of indexes of the linguistic terms in an HFLE $h_A(x)$.

2. The max-union of A and B, denoted by $A \cup B$, is defined as:

$$\begin{aligned} h_{A\cup B}(x) &= h_A(x) \vee h_B(x) \\ &= \{\max\{s_i, s_j\} | s_i \in h_A(x), s_j \in h_B(x)\}; \end{aligned} \tag{4}$$

3. The min-intersection of A and B, denoted by $A \cap B$, is defined as:

$$\begin{aligned} h_{A\cap B}(x) &= h_A(x) \wedge h_B(x) \\ &= \{\min\{s_i, s_j\} | s_i \in h_A(x), s_j \in h_B(x)\}. \end{aligned} \tag{5}$$

In above definition, the operations $^c, \cup, \cap$ are defined on HFLTSs, while operations $\sim, \vee, \wedge$ are defined on the corresponding HFLEs.

Example 2.2 Suppose that $S = \{s_0: very\ poor, s_1: poor, s_2: medium\ poor, s_3: fair, s_4: medium\ good, s_5: good, s_6: very\ good\}$ is a linguistic term set. We let A, B be two hesitant fuzzy linguistic term sets. Suppose that $h_A(x) = \{s_1, s_2, s_3\}$ and $h_B(x) = \{s_3, s_4\}$ are two HFLEs of x to A and B. According to Definition 2.4, we have:

$h_{A^c}(x) = \{s_{6-3}, s_{6-2}, s_{6-1}\} = \{s_3, s_4, s_5\}$,
$h_A(x) \vee h_B(x) = \{\max\{s_1, s_3\}, \max\{s_1, s_4\}, \max\{s_2, s_3\}, \max\{s_2, s_4\}, \max\{s_3, s_3\}, \max\{s_3, s_4\}\} = \{s_3, s_4\}$,
$h_A(x) \wedge h_B(x) = \{\min\{s_1, s_3\}, \min\{s_1, s_4\}, \min\{s_2, s_3\}, \min\{s_2, s_4\}, \min\{s_3, s_3\}, \min\{s_3, s_4\}\} = \{s_1, s_2, s_3\}$.

Theorem 2.1 [32] Let U be the universe of discourse, Suppose that $S = \{s_0, s_1, \ldots, s_g\}$ is a linguistic term set. If we let A, B and C be three hesitant fuzzy linguistic term sets based on S, then the followings are true:

(1) Double negation law: $\sim(\sim h_A(x)) = h_A(x)$;
(2) De Morgan's laws: $\sim(h_A(x) \vee h_B(x)) = (\sim h_A(x)) \wedge (\sim h_B(x))$ and $\sim(h_A(x) \wedge h_B(x)) = (\sim h_A(x)) \vee (\sim h_B(x))$;
(3) Commutativity: $h_A(x) \vee h_B(x) = h_B(x) \vee h_A(x)$ and $h_A(x) \wedge h_B(x) = h_B(x) \wedge h_A(x)$;
(4) Associativity:$h_A(x) \vee (h_B(x) \vee h_C(x)) = (h_A(x) \vee h_B(x)) \vee h_C(x)$ and $h_A(x) \wedge (h_B(x) \wedge h_C(x)) = (h_A(x) \wedge h_B(x)) \wedge h_C(x)$;
(5) Distributivity:$h_A(x) \wedge (h_B(x) \vee h_C(x)) = (h_A(x) \wedge h_B(x)) \vee (h_A(x) \wedge h_C(x))$ and $h_A(x) \vee (h_B(x) \wedge h_C(x)) = (h_A(x) \vee h_B(x)) \wedge (h_A(x) \vee h_C(x))$.

Theorem 2.1 shows the basic properties of negation, max-union and min-intersection operations, which are defined on HFLEs.

Rodriguez et al. presented a context-free grammar G_H which aims to produce simple but rich linguistic expressions.

Definition 2.5 [26] Suppose that $S = \{s_0, s_1, \ldots, s_g\}$ is a linguistic term set, and E_{G_H} is a function that transforms the linguistic expression H, obtained by a context-free grammar G_H, into a hesitant fuzzy linguistic element $h_A(x)$, as follows: $E_{G_H}: H \rightarrow h_A(x)$.

By using the following, the linguistic expressions can be transformed into HFLEs.

(1) $E_{G_H}(s_i) = \{s_i | s_i \in S\}$;
(2) $E_{G_H}(at\ most\ s_i) = \{s_j | s_j \in S\ and\ s_j \leq s_i\}$;
(3) $E_{G_H}(lower\ than\ s_i) = \{s_j | s_j \in S\ and\ s_j < s_i\}$;
(4) $E_{G_H}(at\ least\ s_i) = \{s_j | s_j \in S\ and\ s_j \geq s_i\}$;
(5) $E_{G_H}(greater\ than\ s_i) = \{s_j | s_j \in S\ and\ s_j > s_i\}$;
(6) $E_{G_H}(between\ s_i\ and\ s_j) = \{s_k | s_k \in S\ and\ s_i \leq s_k \leq s_j\}$.

Although there are several methods that aim to compare the magnitude of HFLEs, we selected a comparison method for HFLEs based on pairwise comparisons of each linguistic term in the two HFLEs because of its advantages over other approaches when ranking alternatives to be obtained through utilizing intervals [10]. Moreover, considering that it is not suitable to compare discrete linguistic terms in HFLTSs through utilizing the comparison method for continuous numerical intervals. When comparing two hesitant fuzzy linguistic term sets, which may be of different lengths, the notion of pairwise comparisons of each linguistic term was constructed.

Prior to the introduction of the approach, we introduced the distance between two single linguistic terms. Let $s_i, s_j \in S$, and $d(s_i, s_j) = i - j$ be the distance between s_i and s_j.

Definition 2.6 [10] Suppose that $h_A(x_1)$ and $h_A(x_2)$ are two HFLEs on S, and the pairwise comparison matrix between $h_A(x_1)$ and $h_A(x_2)$ are defined as:

$$C(h_A(x_1), h_A(x_2)) = \left[d(s_i, s_j)\right]_{|h_A(x_1)|\times|h_A(x_2)|}, \tag{6}$$

where $s_i \in h_A(x_1)$ and $s_j \in h_A(x_2)$. Then, we let $C(h_A(x_1), h_A(x_2)) = [C_{mn}]$ be the pairwise comparison matrix between $h_A(x_1)$ and $h_A(x_2)$, where m and n denote row and column indexes in the pairwise comparison matrix. Then, the preference degree of $h_A(x_1)$ and $h_A(x_2)$ is defined as:

$$P(h_A(x_1) > h_A(x_2)) = \frac{\left|\sum_{C_{mn}>0} C_{mn}\right|}{\#\{C_{mn}=0\} + \sum |C_{mn}|}; \tag{7}$$

$$P(h_A(x_1) = h_A(x_2)) = \frac{\#\{C_{mn}=0\}}{\#\{C_{mn}=0\} + \sum |C_{mn}|}; \tag{8}$$

$$P(h_A(x_1) < h_A(x_2)) = \frac{\left|\sum_{C_{mn}<0} C_{mn}\right|}{\#\{C_{mn}=0\} + \sum |C_{mn}|}, \tag{9}$$

we say that $h_A(x_1)$ is superior to $h_A(x_2)$ with the degree of $P(h_A(x_1) > h_A(x_2))$, denoted by $P^A_{x_1>x_2}$; $h_A(x_1)$ is equal to $h_A(x_2)$ with the degree of $P(h_A(x_1) = h_A(x_2))$, denoted by $P^A_{x_1=x_2}$; $h_A(x_1)$ is inferior to $h_A(x_1)$ with the degree of $P(h_A(x_1) < h_A(x_2))$, denoted by $P^A_{x_1<x_2}$. Finally, $P^A_{x_1>x_2} + P^A_{x_1=x_2} + P^A_{x_1<x_2} = 1$.

In order to rank alternatives from the preference relation, there are several choice functions. We selected the concept of the non-dominance degree because it indicates the degree to which an alternative is not dominated by remaining ones. This is convenient for decision makers to rank alternatives from the above preference relation.

Definition 2.7 [26] Let $P_D = [P_{ij}]$ be a preference relation defined over a set of alternatives X. For the alternative x_i, the non-dominance degree is defined as:

$$NDD_i = \min\left\{1 - P^S_{ji}, j = 1, \ldots, n, j \neq i\right\}, \tag{10}$$

where $P^S_{ji} = \max\{P_{ji} - P_{ij}, 0\}$ denotes the degree to which x_i is strictly dominated by x_j. The non-dominated alternatives are defined as:

$$X^{ND} = \left\{x_i \middle| x_i \in X, NDD_i = \max_{x_j \in X}\{NDD_j\}\right\}. \tag{11}$$

Hierarchy plays a significant role in granular computing. In a classical set, hierarchy is characterized by set containment. Conversely, hierarchy is characterized by the comparisons of membership degrees in the background of a fuzzy set. Since the hesitant fuzzy linguistic term set is an extended form of fuzzy set, it is necessary to develop a new definition for comparing two hesitant fuzzy linguistic term sets. An HFL subset will be used to compare two hesitant fuzzy linguistic term sets. Let the *kth* largest value in $h_A(x)$ be denoted as $h_A^{\sigma(k)}(x)$, and the *kth* largest value in $h_B(x)$ be denoted as $h_B^{\sigma(k)}(x)$.

Definition 2.8 Let U be a non-empty and finite universe of discourse. For all $A, B \in HFL(U)$, if $h_A(x) \preceq h_B(x)$ holds for any $x \in U$ such that $h_A(x) \preceq h_B(x) \Leftrightarrow h_A^{\sigma(k)}(x) \leq h_B^{\sigma(k)}(x), \quad j = 1, 2 \ldots, l.$ We denote it by $A \subseteq B$.

This illustrates that the comparison of two hesitant fuzzy linguistic term sets is based on the comparisons of each value in corresponding HFLEs for all objects in the universe. Moreover, $\subseteq$ is reflexive, antisymmetric and transitive on $HFL(U)$.

3 Hesitant fuzzy linguistic rough set over two universes

In this section, we introduce the hesitant fuzzy linguistic (HFL) relation to classical model of fuzzy rough set and further extend the proposed rough set from a single universe to two universes. First, it is necessary to construct the concept of an HFL relation over two universes to form HFL rough approximation operators over two universes.

Definition 3.1 Suppose that $S = \{s_0, s_1, \ldots, s_g\}$ is a linguistic term set and U, V are two non-empty and finite universes of discourse. An HFL subset R of the universe $U \times V$ is called an HFL relation over $U \times V$. Then, R can be expressed as:

$$R = \{\langle (x, y), h_R(x, y)\rangle : (x, y) \in U \times V\}, \tag{12}$$

where $h_R(x,y)$ is a set of linguistic values in S for each $(x,y) \in U \times V$, denoting possible membership degrees of the relationships between x and y. Moreover, we denote the family of all HFL relations over two universes as $HFLR(U \times V)$. If $U = V$, the hesitant fuzzy linguistic relation $R \in HFLR(U \times V)$ reduces to the hesitant fuzzy linguistic relation on a single universe.

Next, we introduce a special HFL relation over two universes.

Definition 3.2 Suppose that $S = \{s_0, s_1, \ldots, s_g\}$ is a linguistic term set. Let $R \in HFLR(U \times V)$ for all $x \in U$, with R being serial if there exists a $y \in V$ such that $h_R(x,y) = \{s_g\}$. Then, R is referred to as a serial HFL relation over $U \times V$.

Since fuzzy rough sets can only deal with fuzzy information, it is common for real-life decision makers to express opinions by utilizing hesitant fuzzy linguistic information. To overcome this limitation, it is necessary to introduce the hesitant fuzzy linguistic relation to fuzzy rough set. Furthermore, considering the advantages of extending a single universe to two universes for the rough set, we develop the hesitant fuzzy linguistic rough set over two universes below.

Definition 3.3 Suppose that $S = \{s_0, s_1, \ldots, s_g\}$ is a linguistic term set and U, V are two non-empty and finite universes of discourse. $R \in HFLR(U \times V)$, the pair (U, V, R) is an HFL approximation space over two universes. For any $A \in HFL(V)$, the lower and upper approximations of A with respect to (U, V, R), denoted by $\underline{R}(A)$ and $\overline{R}(A)$, are two hesitant fuzzy linguistic term sets, defined as follows:

$$\underline{R}(A) = \{\langle x, h_{\underline{R}(A)}(x)\rangle | x \in U\}, \tag{13}$$

$$\overline{R}(A) = \left\{\left\langle x, h_{\overline{R}(A)}(x)\right\rangle \middle| x \in U\right\}, \tag{14}$$

where

$$h_{\underline{R}(A)}(x) = \wedge_{y\in V}\{h_{R^c}(x,y) \vee h_A(y)\},$$
$$h_{\overline{R}(A)}(x) = \vee_{y\in V}\{h_R(x,y) \wedge h_A(y)\}.$$

The pair $[\underline{R}(A), \overline{R}(A)]$ is referred to as a hesitant fuzzy linguistic rough set over two universes of A in terms of the hesitant fuzzy linguistic relation R, where both $\underline{R}(A)$ and $\overline{R}(A)$ are hesitant fuzzy linguistic term sets. $\underline{R}$ and $\overline{R}$ are the lower and upper HFL rough approximation operators over two universes, respectively.

If we let $U = V$, hesitant fuzzy linguistic rough approximations over two universes reduce to hesitant fuzzy linguistic rough approximations on single universe. Moreover, if the hesitant fuzzy linguistic relation R reduces to a binary relation, A reduces to a crisp set, and the hesitant fuzzy linguistic rough approximation operators reduce to the classical rough approximation operators. Similar to the practical meaning of rough set theory, the lower hesitant fuzzy linguistic rough approximation refers to all objects that are definitely contained in the set A, and the upper hesitant fuzzy linguistic rough approximation refers to all objects that are definitely contained and possibly contained in the set A. In real-life decision making activities, these two types of decision making ranking results denote the final decision making outcome along with minimum and maximum uncertainties, making the procedure more objective and logical. Thus, hesitant fuzzy linguistic rough approximation operators over two universes have advantages in modeling hesitant fuzzy linguistic information analysis and providing multiple ranking results for assessment by decision makers.

Example 3.1 Suppose that $S = \{s_0 : \textit{very poor}, s_1 : \textit{poor}, s_2 : \textit{medium poor}, s_3 : \textit{fair}, s_4 : \textit{medium good}, s_5 : \textit{good}, s_6 : \textit{very good}\}$ is a linguistic term set. HFLTS is denoted as A and an HFL relation over two universes is denoted as R, as seen below.

$A = \{\langle x_1, \{\textit{between medium poor and fair}\}\rangle,$
$\langle x_2, \{\textit{between good and very good}\}\rangle, \langle x_3, \{\textit{fair}\}\rangle\},$
$R = \{\langle (x_1, y_1), \{\textit{very good}\}\rangle,$
$\langle (x_1, y_2), \{\textit{between fair and medium good}\}\rangle,$
$\langle (x_1, y_3), \{\textit{poor}\}\rangle,$
$\langle (x_2, y_1), \{\textit{between fair and medium good}\}\rangle,$
$\langle (x_2, y_2), \{\textit{very good}\}\rangle,$
$\langle (x_2, y_3), \{\textit{between good and very good}\}\rangle,$
$\langle (x_3, y_1), \{\textit{poor}\}\rangle,$
$\langle (x_3, y_2), \{\textit{between good and very good}\}\rangle,$
$\langle (x_3, y_3), \{\textit{very good}\}\rangle\}.$

In this example, R is serial. According to Definition 2.5, we transform the linguistic expressions into HFLEs.

$A = \{\langle x_1, \{s_2, s_3\}\rangle, \langle x_2, \{s_5, s_6\}\rangle, \langle x_3, \{s_3\}\rangle\},$
$R = \{\langle (x_1, y_1), \{s_6\}\rangle, \langle (x_1, y_2), \{s_3, s_4\}\rangle, \langle (x_1, y_3), \{s_1\}\rangle,$
$\langle (x_2, y_1), \{s_3, s_4\}\rangle, \langle (x_2, y_2), \{s_6\}\rangle, \langle (x_2, y_3), \{s_5, s_6\}\rangle,$
$\langle (x_3, y_1), \{s_1\}\rangle, \langle (x_3, y_2), \{s_5, s_6\}\rangle, \langle (x_3, y_3), \{s_6\}\rangle\}.$

Then, we can compute $\underline{R}(A)$ and $\overline{R}(A)$,

$$\begin{aligned} h_{\underline{R}(A)}(x_1) &= \wedge_{y\in V}\{h_{R^c}(x_1, y) \vee h_A(y)\} \\ &= (\{s_0\} \vee \{s_2, s_3\}) \wedge (\{s_2, s_3\} \vee \{s_5, s_6\}) \wedge (\{s_5\} \vee \{s_3\}) \\ &= \{s_2, s_3\} \wedge \{s_5, s_6\} \wedge \{s_5\} \\ &= \{s_2, s_3\}. \end{aligned}$$

Similarly, it is easy to acquire $h_{\underline{R}(A)}(x_2) = \{s_2, s_3\}$, $h_{\underline{R}(A)}(x_3) = \{s_3\}$, $h_{\overline{R}(A)}(x_1) = \{s_3, s_4\}$, $h_{\overline{R}(A)}(x_2) = \{s_5, s_6\}$, $h_{\overline{R}(A)}(x_3) = \{s_5, s_6\}$. Therefore, we have: $\underline{R}(A) = \{\langle x_1, \{s_2, s_3\}\rangle, \langle x_2, \{s_2, s_3\}\rangle, \langle x_3, \{s_3\}\rangle\}$, $\overline{R}(A) = \{\langle x_1, \{s_3, s_4\}\rangle, \langle x_2, \{s_5, s_6\}\rangle, \langle x_3, \{s_5, s_6\}\rangle\}$.

Theorem 3.1 Let U, V be two non-empty and finite universes of discourse. For any $A, B \in HFL(V)$, the following properties are true:

(1) $\underline{R}(A^c) = \left(\overline{R}(A)\right)^c$, $\overline{R}(A^c) = (\underline{R}(A))^c$;
(2) $A \subseteq B \Rightarrow \underline{R}(A) \subseteq \underline{R}(B)$, $A \subseteq B \Rightarrow \overline{R}(A) \subseteq \overline{R}(B)$;
(3) $\underline{R}(A \cap B) = \underline{R}(A) \cap \underline{R}(B)$, $\overline{R}(A \cup B) = \overline{R}(A) \cup \overline{R}(B)$;
(4) $\underline{R}(A \cup B) \supseteq \underline{R}(A) \cup \underline{R}(B)$, $\overline{R}(A \cap B) \subseteq \overline{R}(A) \cap \overline{R}(B)$;
(5) $\overline{R}(V) = \underline{R}(V) = U$, $\overline{R}(\emptyset) = \underline{R}(\emptyset) = \emptyset$.

Proof

(1) For all $x \in U$,

$$\begin{aligned} h_{\underline{R}(A^c)}(x) &= \wedge_{y\in V}\{h_{R^c}(x,y) \vee h_{A^c}(y)\} \\ &= \wedge_{y\in V}\{(\sim h_R(x,y)) \vee (\sim h_A(y))\} \\ &= \sim \left(\vee_{y\in V}\{h_R(x,y) \wedge h_A(y)\}\right) \\ &= h_{\left(\overline{R}(A)\right)^c}(x). \end{aligned}$$

Therefore, we have $\underline{R}(A^c) = \left(\overline{R}(A)\right)^c$. $\overline{R}(A^c) = (\underline{R}(A))^c$ is obtained similarly.

(2) Since $A \subseteq B$, by Definition 2.8, $h_A^{\sigma(k)}(y) \preceq h_B^{\sigma(k)}(y)$ for all $y \in V$. Therefore, it follows that $\wedge_{y\in V}\{h_{R^c}^{\sigma(k)}(x,y) \vee h_A^{\sigma(k)}(y)\} \leq \wedge_{y\in V}\{h_{R^c}^{\sigma(k)}(x,y) \vee h_B^{\sigma(k)}(y)\}$, and thus, for all $x \in U$, we have $h_{\underline{R}(A)}(x) \preceq h_{\underline{R}(B)}(x)$, which indicates $\underline{R}(A) \subseteq \underline{R}(B)$. $A \subseteq B \Rightarrow \overline{R}(A) \subseteq \overline{R}(B)$ is obtained in a similar manner.

(3) For all $x \in U$,

$$\begin{aligned} \underline{R}(A \cap B)(x) &= \wedge_{y\in V}\left\{h_{R^c}(x,y) \vee h_{(A\cap B)}(y)\right\} \\ &= \wedge_{y\in V}\{h_{R^c}(x,y) \vee (h_A(y) \wedge h_B(y))\} \\ &= \wedge_{y\in V}\{(h_{R^c}(x,y) \vee h_A(y)) \wedge (h_{R^c}(x,y) \vee h_B(y))\} \\ &= \left(\wedge_{y\in V}\{h_{R^c}(x,y) \vee h_A(y)\}\right) \wedge \\ &\quad \left(\wedge_{y\in V}\{h_{R^c}(x,y) \vee h_B(y)\}\right) \\ &= h_{\underline{R}(A)}(x) \wedge h_{\underline{R}(B)}(x) \\ &= h_{\underline{R}(A)\cap\underline{R}(B)}(x) \end{aligned}$$

Thus, $\underline{R}(A \cap B) = \underline{R}(A) \cap \underline{R}(B)$. Similarly, $\overline{R}(A \cup B) = \overline{R}(A) \cup \overline{R}(B)$ is obtained.

(4) Based on the above discussions, it is not difficult to prove $\underline{R}(A \cup B) \supseteq \underline{R}(A) \cup \underline{R}(B)$ and $\overline{R}(A \cap B) \subseteq \overline{R}(A) \cap \overline{R}(B)$.

(5) This theorem holds when R is serial. For all $x \in U$, we have $h_\emptyset(x) = \{s_0\}$. Suppose that there exists $y' \in V$ such that $h_R(x, y') = \{s_g\}$. Then,

$$\begin{aligned} h_{\underline{R}(\emptyset)}(x) &= \wedge_{y\in V}\{h_{R^c}(x,y) \vee \{s_0\}\} \\ &= \wedge_{y\in V}\{h_{R^c}(x,y)\} \\ &= h_{R^c}(x,y') \wedge \left(\wedge_{y\neq y'}\{h_{R^c}(x,y)\}\right) \\ &= \{s_0\} \wedge \left(\wedge_{y\neq y'}\{h_{R^c}(x,y)\}\right) \\ &= \{s_0\} \end{aligned}$$

Thus, we obtain $\underline{R}(\emptyset) = \emptyset$. $\overline{R}(\emptyset) = \emptyset$ is determined in a similar manner. For all $x \in U$, we have $h_V(x) = \{s_g\}$, $h_{\underline{R}(V)}(x) = \wedge_{y\in V}\{h_{R^c}(x,y) \vee \{s_g\}\} = \{s_g\}$, and therefore obtain $\underline{R}(V) = U$. $\overline{R}(V) = U$ is obtained in an identical fashion.

□

Theorem 3.1 indicates the fundamental properties of hesitant fuzzy linguistic rough sets over two universes: (1) shows the complement of hesitant fuzzy linguistic rough approximations over two universes; (2) shows the monotone of hesitant fuzzy linguistic rough approximations over two universes; (3) and (4) show the multiplication and addition of hesitant fuzzy linguistic rough approximations over two universes; (5) illustrates the normality and conormality of hesitant fuzzy linguistic rough approximations over two universes.

Theorem 3.2 Let U, V be two non-empty and finite universes of discourse. Suppose that $R_1, R_2 \in HFLR(U \times V)$. If $R_1 \subseteq R_2$, for any $A \in HFL(V)$, then the following properties are true:

(1) $\underline{R_1}(A) \supseteq \underline{R_2}(A)$, for all $A \in HFL(V)$;
(2) $\overline{R_1}(A) \subseteq \overline{R_2}(A)$, for all $A \in HFL(V)$.

Proof Since $R_1 \subseteq R_2$, by Definition 2.8, we have $h_{R_1}^{\sigma(k)}(x,y) \leq h_{R_2}^{\sigma(k)}(x,y)$ for all $(x,y) \in (U \times V)$. It follows that

$$\begin{aligned} h_{\underline{R_1}(A)}(x) &= \wedge_{y\in V}\left\{h_{R_1^c}^{\sigma(k)}(x,y) \vee h_A^{\sigma(k)}(y)\right\} \\ &\geq \wedge_{y\in V}\left\{h_{R_2^c}^{\sigma(k)}(x,y) \vee h_A^{\sigma(k)}(y)\right\} \\ &= h_{\underline{R_2}(A)}(x) \end{aligned}$$

Hence, for each $x \in U$, we have $h_{\underline{R_1}(A)}(x) \succeq h_{\underline{R_2}(A)}(x)$, which means $\underline{R_1}(A) \supseteq \underline{R_2}(A)$. $\overline{R_1}(A) \subseteq \overline{R_2}(A)$ is similarly obtained. □

This theorem shows that the lower and upper approximations in HFL rough sets over two universes are monotonic due to the monotonic forms of the HFL relations over two universes.

4 Decision making approach based on HFL rough set over two universes

In this section, we introduce a new approach to decision making utilizing the HFL rough set over two universes. The main points of our model and decision making methods are summarized as follows.

4.1 Application model

Suppose that $U = \{x_1, x_2, \ldots, x_j\}$ is the alternatives set and $V = \{y_1, y_2, \ldots, y_k\}$ is the general characteristic factors set. $R \in HFLR(U \times V)$ is an HFL relation reflecting the relevancy degree between alternatives and general characteristic factors. We also let $A \in HFL(V)$ be an evaluation set that reflects the general characteristic factors of a new occurred alternative. Then, we obtain an HFL decision making information system (U, V, R, A).

Based on the decision making strategy developed in [27], we present decision rules by using an HFL rough set over two universes. First, we denote three decision making index sets:

$$T_1 = \left\{ i \middle| \max_{x_i \in U} \{\underline{R}(A)(x_i)\} \right\}; \tag{15}$$

$$T_2 = \left\{ j \middle| \max_{x_j \in U} \{\overline{R}(A)(x_j)\} \right\}; \tag{16}$$

$$T_3 = \left\{ k \middle| \max_{x_k \in U} \{\underline{R}(A)(x_k) \oplus \overline{R}(A)(x_k)\} \right\}. \tag{17}$$

From the viewpoint of the classical operational risk decision making principle, and according to the definitions of the above decision making index sets, we can obtain the explanation for decision making index sets T_1, T_2 and T_3. These sets are composed of subscripts of the largest hesitant fuzzy linguistic element in the corresponding hesitant fuzzy linguistic term set. Specifically, since the lower hesitant fuzzy linguistic rough approximation refers to all objects that are definitely contained in the set A, and the upper hesitant fuzzy linguistic rough approximation refers to all objects that are definitely contained and possibly contained in the set A. T_1 is the max-min decision criterion, and is the final decision making result with minimum uncertainties; T_2 is the max-max decision making criterion of risk decision making, and is the final decision making result with maximum uncertainties; T_3 is the weighted decision criterion of T_1 and T_2 with the weighted value 0.5, and is the final decision making result with medium uncertainties. When determining T_3, linguistic operational laws are used [17]: For any linguistic terms s_α, $s_\beta \in S$, we have $s_\alpha \oplus s_\beta = s_{\alpha+\beta}$ and $\lambda s_\alpha = s_{\lambda\alpha} (\lambda \in [0,1])$. To obtain a more objective decision making result, the decision rules are presented as follows, based on the above definitions:

(1) If $T_1 \cap T_2 \cap T_3 \neq \emptyset$, then $x_i (i \in T_1 \cap T_2 \cap T_3)$ is the optimal decision making result;
(2) If $T_1 \cap T_2 \cap T_3 = \emptyset$ and $T_1 \cap T_2 \neq \emptyset$, then $x_i (i \in T_1 \cap T_2)$ is the optimal decision making result;
(3) If $T_1 \cap T_2 \cap T_3 = \emptyset$ and $T_1 \cap T_2 = \emptyset$, then $x_i (i \in T_3)$ is the optimal decision making result.

By virtue of the decision making index sets T_1, T_2 and T_3, the proposed decision rules are a multi-faceted decision making scheme that considers multiple situations. Concretely speaking, situation (1) denotes little difference between the three decision making index sets; situation (2) denotes some difference between the three decision making index sets; situation (3) denotes many differences between the three decision making index sets. By utilizing the multi-faceted decision making scheme, decision makers obtain more accurate results than when using other approaches.

4.2 Algorithm for application model

We present an algorithm for the decision making model based on an HFL rough set over two universes as follows:

Algorithm 1 The decision making procedure by utilizing an HFL rough set over two universes.

Require: The relation R between the universe U and V provided by an expert and an evaluation set A.
Ensure: The optimal decision making result.
1: Transform the linguistic expressions into HFLEs;
2: Calculate the $\underline{R}(A)$ and $\overline{R}(A)$ with respect to (U, V, R);
3: Construct the pairwise comparison matrix over a set of alternatives in $\underline{R}(A)$ and $\overline{R}(A)$;
4: Compute the preference degrees of alternatives and construct the corresponding preference matrix;
5: Compute the non-dominance degree and determine the ranking results over a set of alternatives in $\underline{R}(A)$ and $\overline{R}(A)$;
6: Compute T_1, T_2, T_3, $T_1 \cap T_2 \cap T_3$ and $T_1 \cap T_2$, and determine the optimal decision making result.

5 An illustrative example

The basic steps of the proposed decision making approach are illustrated using an example of a person-job fit problem. Job requirements linguistic information of a typical OTT (over the top) player from a local recruitment website in China was extracted for use in this example. We modeled a person-job fit problem by utilizing the HFL rough set over two universes.

5.1 Case description

Suppose that Mr. X is a job seeker wanting to know which job suit for him. and hires an expert to conduct occupation evaluations. Let $U = \{x_1, x_2, x_3, x_4, x_5\}$ denotes five common departments in an OTT company. Where x_i stands for "administrative department", "marketing department", "research & development department", "financial department" and "sales department". The universe $V = \{y_1, y_2, y_3, y_4, y_5, y_6, y_7\}$ denotes seven professional abilities in an OTT player. Where y_i stands for "mathematical ability", "software application capability", "writing ability", "verbal ability", "organization and management skill", "detail awareness ability" and "social skills".

Let $R \in HFLR(U \times V)$ be a hesitant fuzzy linguistic relation extracted from a recruitment website's job requirements information. Mr. X needs to conduct the self-evaluation denoted as $A = \{\langle y_i, h_A(y_i)\rangle | y_i \in V\}$, where y_i contains seven linguistic values within S: $S = \{s_0 : \textit{Extremely Limited User}, s_1 : \textit{Limited User}, s_2 : \textit{Modest User}, s_3 : \textit{Competent User}, s_4 : \textit{Good User}, s_5 : \textit{Very Good User}, s_6 : \textit{Expert User}\}$.

The original information given by the expert is presented in Table 1.

To match Mr. X's professional abilities to requirements information, Mr. X conducts a self-evaluation denoted by A.

$$
\begin{aligned}
A = \{&\langle y_1, \{\textit{between Good User and Very Good User}\}\rangle,\\
&\langle y_2, \{\textit{between Very Good User and Expert User}\}\rangle,\\
&\langle y_3, \{\textit{Competent User}\}\rangle,\\
&\langle y_4, \{\textit{between Good User and Very Good User}\}\rangle,\\
&\langle y_5, \{\textit{between Good User and Very Good User}\}\rangle,\\
&\langle y_6, \{\textit{between Competent User and Good User}\}\rangle,\\
&\langle y_7, \{\textit{Modest User}\}\rangle\}.
\end{aligned}
$$

5.2 Decision making process

The steps for Algorithm 1 are followed to transform the linguistic expressions into HFLEs. We rewrite the relation R, shown in Table 2, and the self-evaluation set A as follows.

$$
\begin{aligned}
A = \{&\langle y_1, \{s_4, s_5\}\rangle, \langle y_2, \{s_5, s_6\}\rangle, \langle y_3, \{s_3\}\rangle,\\
&\langle y_4, \{s_4, s_5\}\rangle, \langle y_5, \{s_4, s_5\}\rangle, \langle y_6, \{s_3, s_4\}\rangle, \langle y_7, \{s_2\}\rangle\}.
\end{aligned}
$$

Then, we calculate the $\underline{R}(A)$ and $\overline{R}(A)$ in terms of the approximation space (U, V, R) as follows.

Table 1 The relation between departments and abilities presented by linguistic expressions

R	y_1	y_2	y_3	y_4	y_5	y_6	y_7
x_1	{*Competent User*}	{*Good User*}	{*Very Good User*}	{*Very Good User*}	{*Expert User*}	{*Very Good User*}	{*Good User*}
x_2	{*Competent User*}	{*Good User*}	{*Very Good User*}	{*Competent User*}	{*Good User*}	{*Competent User*}	{*Expert User*}
x_3	{*Very Good User*}	{*Expert User*}	{*Very Good User*}	{*Good User*}	{*Good User*}	{*Good User*}	{*Competent User*}
x_4	{*Very Good User*}	{*Very Good User*}	{*Competent User*}	{*Competent User*}	{*Very Good User*}	{*Expert User*}	{*Competent User*}
x_5	{*Competent User*}	{*Competent User*}	{*Good User*}	{*Expert User*}	{*Competent User*}	{*Very Good User*}	{*Expert User*}

Table 2 The relation between departments and abilities presented by HFLEs

R	y_1	y_2	y_3	y_4	y_5	y_6	y_7
x_1	$\{s_3\}$	$\{s_4\}$	$\{s_5\}$	$\{s_5\}$	$\{s_6\}$	$\{s_5\}$	$\{s_4\}$
x_2	$\{s_3\}$	$\{s_4\}$	$\{s_5\}$	$\{s_3\}$	$\{s_4\}$	$\{s_3\}$	$\{s_6\}$
x_3	$\{s_5\}$	$\{s_6\}$	$\{s_5\}$	$\{s_4\}$	$\{s_4\}$	$\{s_4\}$	$\{s_3\}$
x_4	$\{s_5\}$	$\{s_5\}$	$\{s_3\}$	$\{s_3\}$	$\{s_5\}$	$\{s_6\}$	$\{s_3\}$
x_5	$\{s_3\}$	$\{s_3\}$	$\{s_4\}$	$\{s_6\}$	$\{s_3\}$	$\{s_5\}$	$\{s_6\}$

$$\underline{R}(A) = \{\langle x_1, \{s_2\}\rangle, \langle x_2, \{s_2\}\rangle, \langle x_3, \{s_3\}\rangle, \langle x_4, \{s_3\}\rangle, \langle x_5, \{s_2\}\rangle\},$$
$$\overline{R}(A) = \{\langle x_1, \{s_4, s_5\}\rangle, \langle x_2, \{s_4\}\rangle, \langle x_3, \{s_5, s_6\}\rangle, \langle x_4, \{s_5\}\rangle, \langle x_5, \{s_4, s_5\}\rangle\}.$$

Since $s_2 \leq s_3$, the ranking order of linguistic values in $\underline{R}(A)$ is: $x_3 = x_4 > x_1 = x_2 = x_5$. This indicates the optimal choice for Mr. X is the research & development department, or the financial department. $T_1 = \{3, 4\}$ is easily obtained.

Next, the ranking order of the linguistic values in $\overline{R}(A)$ is determined. There are four different linguistic values in $\overline{R}(A)$, i.e., $\{s_4, s_5\}$, $\{s_4\}$, $\{s_5\}$ and $\{s_5, s_6\}$.

The preference degrees of $\{s_4, s_5\}$, $\{s_4\}$, $\{s_5\}$ and $\{s_5, s_6\}$ are computed:

$P(\{s_4, s_5\} > \{s_4\}) = 0.5;$
$P(\{s_4, s_5\} < \{s_4\}) = 0;$
$P(\{s_4, s_5\} > \{s_5\}) = 0;$
$P(\{s_4, s_5\} < \{s_5\}) = 0.5;$
$P(\{s_4, s_5\} > \{s_5, s_6\}) = 0;$
$P(\{s_4, s_5\} < \{s_5, s_6\}) = 0.8;$
$P(\{s_4\} > \{s_5\}) = 0;$
$P(\{s_4\} < \{s_5\}) = 1;$
$P(\{s_4\} > \{s_5, s_6\}) = 0;$
$P(\{s_4\} < \{s_5, s_6\}) = 1;$
$P(\{s_5\} > \{s_6\}) = 0;$
$P(\{s_5\} < \{s_6\}) = 0.5.$

We rank $\{s_4, s_5\}$, $\{s_4\}$, $\{s_5\}$ and $\{s_5, s_6\}$ by using the non-dominance degree method described in Definition 2.7. From this, the following is obtained:

$$P_D^S = \begin{pmatrix} - & 0.5 & 0 & 0 \\ 0 & - & 0 & 0 \\ 0.5 & 1 & - & 0 \\ 0.8 & 1 & 0.5 & - \end{pmatrix}.$$

Thus, we have:

$NDD_1 = \min\{(1-0)), (1-0.5), (1-0.8)\} = 0.2;$
$NDD_2 = \min\{(1-0.5), (1-1), (1-1)\} = 0;$
$NDD_3 = \min\{(1-0), (1-0), (1-0.5)\} = 0.5;$
$NDD_4 = \min\{(1-0), (1-0), (1-0)\} = 1.$

Finally, we obtain $\{s_5, s_6\} > \{s_5\} > \{s_4, s_5\} > \{s_4\}$. Therefore, the ranking order of linguistic values in $\overline{R}(A)$ is: $x_3 > x_4 > x_1 = x_5 > x_2$. According to the ranking result, the optimal choice for Mr. X is the research & development department, while the marketing department is the worst. We obtain $T_2 = \{3\}$. Then, according to Algorithm 1, we also obtain $T_3 = \{3\}$. Since $T_1 \cap T_2 \cap T_3 \neq \emptyset$, the optimal recruitment department is the research & development department.

5.3 Comparative analysis

Here, we compare the newly proposed decision making rules with the method of cosine similarity measure between HFLTSs [17]. The cosine similarity measure is defined as the inner product of two vectors divided by the product of their lengths. Prior to the introduction of cosine similarity measure between HFLTSs, all the distance and similarity measures are based on algebra distance measures, such as the Hamming distance, the Euclidean distance, and the Hausdorff distance. The cosine similarity measure between HFLTSs is based on the geometric point of view. Moreover, the cosine distance and similarity measures are more easily understood, as their geometric meanings are intuitive. Therefore, the method of cosine similarity measure between HFLTSs is a typical and effective approach for analysis.

5.3.1 Cosine similarity measure

As presented by Liao and Xu [17], the cosine similarity measure between the HFLTSs H_S^1 and H_S^2 is proposed as:

$$\rho(H_S^1, H_S^2) = \frac{\sum_{i=1}^{N}\left(\frac{1}{L_i}\sum_{l=1}^{L_i}\left(\frac{\left|\delta_l^1(x_i)\right|}{2\tau+1}\cdot\frac{\left|\delta_l^2(x_i)\right|}{2\tau+1}\right)\right)}{\sqrt{\sum_{i=1}^{N}\left(\frac{1}{L_i}\sum_{l=1}^{L_i}\left(\frac{\delta_l^1(x_i)}{2\tau+1}\right)^2\right)\cdot\sum_{i=1}^{N}\left(\frac{1}{L_i}\sum_{l=1}^{L_i}\left(\frac{\delta_l^2(x_i)}{2\tau+1}\right)^2\right)}}, \quad (18)$$

where $H_S^1 = \{\langle x_i, h_S^1(x_i)\rangle | x_i \in X\}$ and $H_S^2 = \{\langle x_i, h_S^2(x_i)\rangle | x_i \in X\}$ $(i = 1, 2, \ldots, N)$, with $h_S^k(x_i) = \left\{s_{\delta_l^k}(x_i) | s_{\delta_l^k}(x_i) \in S, l = 1, 2, \ldots L_i\right\}$, $k = 1, 2$, L_i is the maximum number of linguistic terms in $h_S^1(x_i)$ and $h_S^2(x_i)$ (the shorter one should be extended until they are equal in length), N is the cardinality of X, and τ denotes the index of a linguistic term.

5.3.2 Result analysis and discussions

Through utilizing the above cosine similarity measure formula, the following is obtained: $\rho(A, x_1) = 0.94$, $\rho(A, x_2) = 0.88$, $\rho(A, x_3) = 0.98$, $\rho(A, x_4) = 0.96$, $\rho(A, x_5) = 0.87$. The results reveal ranking order of x_i as

$x_3 > x_4 > x_1 > x_2 > x_5$. According to the ranking result, the ideal department for Mr. X is research & development. The ranking order is similar to that determined by Algorithm 1. Conclusively, both methods indicate the optimal department as research & development.

Moreover, there are extra steps for the cosine similarity measure between the hesitant fuzzy linguistic term sets which may reduce result precision. Since the aforementioned cosine similarity measure is based on an assumption that each HFLE are equal lengths, if HFLE lengths differ, the shorter one should be extended until the HFLEs are equal lengths. To some extent, it is inevitable that original HFLE information is changed. Thus, such an adjustment is less well justified in theory and practice. Consequently, the proposed approach is more flexible in dealing with the person-job fit problem.

Comparing to the above theoretical results to existing literature, the main contribution of the proposed decision making model is in providing different decision making criteria based on the advantages of rough set theory. Specifically, by introducing the notion of a rough set into hesitant fuzzy linguistic information analysis, decision makers can refer to two types of ranking results: lower and upper hesitant fuzzy linguistic rough approximations over two universes. These yield a final result with minimum and maximum uncertainties, respectively. Furthermore, based on the multi-type results, the multi-faceted recruitment decision rules provide more accurate recruitment outcomes than other methods. The hesitant fuzzy linguistic rough set over two universes model is superior in providing robust solutions for person-job fit problems.

Despite the contributions mentioned above, this study may also serve as directions for further research. First, although this study focuses on real-life decision making problems by utilizing HFL rough sets over two universes, it would be interesting to investigate several theoretical parts of the proposed rough set model, such as the axiomatic approach, attribute reduction, and uncertainty measures based on HFL rough sets over two universes. It is also worth investigating the relationships between HFL rough sets over two universes and multigranulation rough sets, and related applications in a group decision making background.

6 Conclusions

In this paper, we developed a general framework for the study of HFL rough sets over two universes. The lower and upper hesitant fuzzy linguistic rough approximation operators over two universes were defined, and related properties of HFL rough sets over two universes were proved. Lastly, a comprehensive algorithm for real-life decision making was established and illustrated by a numerical example under the background of person-job fit. This research discusses some theoretical and practical aspects of HFL rough sets over two universes, and further study may investigate how to measure uncertainties and construct efficient attribute reduction methods for HFL information systems. Finally, how to utilize the proposed approach to solve more decision making problems should also be discussed in future research.

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