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Multi-attribute group decision-making method based on multi-granulation weights and three-way decisions

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ABSTRACT

With the increasing complexity of decision-making problems and environment, the integration and fusion of three-way decisions, rough set and multi-attribute group decision-making (MAGDM) have become a major trend in the field of decision analysis. Although many researchers have presented various MAGDM methods under different environments, there are still some imperfections, such as the weight information is not comprehensive or flexible enough, the decision results lack interpretability, and the impact of risk attitude is not fully taken into account. In order to overcome the above shortcomings and improve the scientificity and rationality of decision-making, a novel data-driven MAGDM method under interval-valued intuitionistic uncertain linguistic environment is established based on the idea of multi-granulation and three-way decisions. Our contributions can be identified as follows: (1) The multi-granulation weight mining and fusion methods for experts and attributes are proposed, respectively; (2) The coarse-granulation grading information based on three-way decisions is developed to enhance the interpretability and reference value of decision results; (3) The expected value with risk attitude factor is defined to compare interval-valued intuitionistic uncertain linguistic variables (IVIULVs) and then is used to grade and rank alternatives under different risk attitudes. To illustrate the feasibility and practicality of the proposed method, a case of logistics supplier selection in e-commerce enterprises is demonstrated. Furthermore, the advantages and characteristics of the proposed method are highlighted via detailed comparison and thorough analysis.

1. Introduction

Research on the theories and methodologies of decision-making for real-world problems under uncertainty is important and necessary. Granular computing, as a rapidly growing information-processing paradigm in the domain of computational intelligence and human-centric system, has been employed as an effective and useful tool to deal with decision-making problems under uncertainty and attracted many researchers and practitioners [1]. Granular computing is regarded as an umbrella term to cover any theories, methodologies, techniques, and tools that make use of information granules in complex

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problem solving. As one of the classical models of granular computing, rough set theory, originated by Pawlak [2] in 1982, has developed rapidly in the past 30 years and continues growing healthy. Several extensions of the Pawlak rough set model have been established in line with the requirements of various decision-making problems. These extensions include dominance-based rough set [3], Bayesian rough set [4], decision-theoretic rough set [5,6], game-theoretic rough set [7], multi-granulation rough set [8], etc. Yao has insisted on monitoring and reporting the trends and development of rough set research by using scientometrics approach since 2013 [9–11]. He analyzed productivity and impact of rough set research domain and pointed out that decision-theoretic rough sets and decision-way decisions are new research trends.

Three-way decisions are newly emerging approach to decision-making under uncertainty, which provide insights into deeper understanding of rough sets and their applications in granular computing. Yao [12] firstly proposed an original model of three-way decisions with decision-theoretic rough sets based on Bayesian decision procedure. Furthermore, Yao [13] researched the interplay of three-way decisions and cognitive computing, and discussed general applicability of three-way decisions. Recently, based on the philosophy of three-way decisions as thinking in threes, Yao [14] discussed a wide sense of three-way decisions and proposed a trisecting-acting-outcome (TAO) model of three-way decisions. The basic idea of three-way decisions is to divide a whole into three parts, and to devise the most effective strategies to act upon the three parts, which reduces the complexity of the whole and is consistent with human's cognitions to solve the problem in the real world. Therefore, the methodology of three-way decisions is widely applied in many theoretic fields, such as attribute reduction [6], knowledge granulation [14,15], feature fusion [16], and formal concept analysis [17]. After a fast development in the past a few years, three-way decisions have achieved several successful results in many applications, such as multi-class decision [18], information filtering and classification [7,19], multi-label classification [20], cluster analysis [21], recommendation [22], decision support systems [23], etc.

With the increasing complexity of decision-making problems and environments, the combination of rough set, three-way decisions and decision-making methods becomes tighter and tighter in the field of decision analysis [24]. Zhang and Yao [7] applied game-theoretic rough set model to determine the multi-criteria three-way classifications. Liang [26] introduced group decision-making (GDM) into three-way decisions with decision-theoretic rough sets and proposed GDM-based three-way decisions. Liang [25] further constructed a novel three-way decisions based on decision-theoretic rough set under linguistic assessment with the aid of MAGDM. In recent years, the theory and methodology of rough set and three-way decisions have been applied in practice to solve various decision-making problems under uncertainty, such as the selection of new product ideas [25], strategic supplier selection [26], identification of distinctive streets [27], emergency decision-making [28], project investment [29], and medical diagnosis [31]. Liang [27] introduced prospect theory into three-way decisions and constructed a nonadditivity fusion of heterogeneous multi-attribute behavioral three-way decisions to deal with multi-attribute decision-making (MADM) problem under interval type-2 fuzzy environment. Sun [28] established a decision model using three-way decisions based on linguistic information-based decision-theoretic rough fuzzy set and applied it to solve MAGDM problem. Zhang [30] proposed a mechanism to make three-way group decisions with interval-valued decision-theoretic rough sets by aggregating inclusion measures. Sun [31] introduced multigranulation rough set theory and three-way decision principle into MAGDM and presented a new approach to MAGDM problems.

As one of the most important and flourishing research directions for decision-making under uncertainty, MAGDM methods are used to evaluate differences of opinion expressed by individual experts over multiple attributes in order to find an alternative that is most acceptable to the group of experts as a whole [28,32,33]. In this paper, we will focus on the objective determination methods of weights and the grading representation of decision results in MAGDM problems under interval-valued intuitionistic uncertain linguistic environment. IVIULVs are an important generalization of fuzzy sets, which can make the decision-making process more appropriate and effective [34–36]. Uncertain linguistic variables, interval-valued intuitionistic fuzzy sets and intuitionistic linguistic variables can be regarded as special cases of IVIULVs. Therefore, study of MAGDM methods based on IVIULVs is more universal and has important theoretical and application value.

It is well known that the weights of experts and attributes play an important role in the aggregating process. From the perspective of granular computing, the weight information can be depicted at different levels and divided into the following categories: (1) Coarse-granulation, that is, the weights of experts or attributes are one-dimensional vectors if each expert has the same weight for all the attributes, or each attribute has the same weight under all the experts; (2) Finer-granulation, that is, the weights of experts or attributes are two-dimensional vectors if each expert has different weights for different attributes or each attribute has different weights under different experts; (3) Finest-granulation, that is, the weights of experts are three-dimensional vectors if each expert has different weights for different attributes w.r.t. different alternatives. Similarly, the weights can also be mined and characterized from different perspectives. By fusing these weights the synthetic weights can be obtained, which is therefore considered to be multi-granulation weight information. To better illustrate the existing weight determination methods in detail, a comparative analysis is made and summarized in Tables 1 and 2.

Through Tables 1 and 2, it is not difficult to find out that the existing weight determination methods have the following limitations. (1) Most of the experts' objective weights are based on coarse-granulation and single-granulation, which are not comprehensive and flexible enough, and can not be directly used in incomplete cases; (2) The attribute weight determination methods are mainly carried out from the aspect of the size of value, and most of them are also based on single-granulation. In order to overcome these shortcomings, we take the evaluation value of each alternative under each attribute provided by each expert as the basic granulation, mining and analyzing the experts' finest-granulation weights from two aspects of uncertainty and closeness. Then by integrating the two kinds of weights, the multi-granularity weight of experts with

Table 1
Comparison of expert weight determination methods for MAGDM in different situations.

Methods	Form of evaluation information	Situations of expert weight	Expert weight determination methods	Granulation of expert weight
Gupta et al. [32]	IVIFNs	Completely unknown	The method based on advantage score and disadvantage score	coarse
Pang et al. [33]	ULVs	Completely unknown	The adjustment method based on the group consensus degree	coarse
Liu et al. [37]	MGLAI	Completely unknown	The extended TOPSIS based method	coarse
Wan et al. [38]	IFNs	Completely unknown	Group consensus based optimization model	coarse
Wan et al. [39]	TIFNs	Completely unknown	The credibility degree based method combining the evidence theory with Bayes approximation	coarse
Yue and Jia [40]	IVIFNs	Completely unknown	Combined weighting method based on extended TOPSIS	coarse
Meng et al. [35]	IVIULVs	Partly known	Shapley value based optimization method considering the interaction between experts	finer
Ye [41]	IVIFNs	Completely unknown	An exact model based on entropy measure	finest
Mohagheghi et al. [42]	PFSs	Completely unknown	Combined weighting method based on closeness coefficient of the individual judgement	multiple, finest
Zhang and Xu [43]	IFNs	Completely unknown	Bi-objective linear programming model considering both ranking and magnitude of the decision information	multiple, coarse
Wang et al. [44]	ULVs	Completely unknown	The method considering both uncertainty degree and deviation degree	multiple, coarse

Coarse-granulation means that each expert has the same weight for all the attributes. Finer-granulation means that each expert has different weights for different attributes. Finest-granulation means that each expert has different weights for different attributes w.r.t. different alternatives. TOPSIS: technique for order preference by similarity to ideal solution. IVIFNs: interval-valued intuitionistic fuzzy numbers. ULVs: uncertain linguistic variables. MGLAI: multi-granularity linguistic assessment information. IFNs: intuitionistic fuzzy numbers. TIFNs: triangular intuitionistic fuzzy numbers. PFSs: pythagorean fuzzy sets.

Table 2
Comparison of different attribute weight determination methods for MAGDM.

Methods	Form of evaluation information	Situations of attribute weight	Attribute weight determination methods	Granulation of attribute weight
Gupta et al. [32]	IVIFNs	Partly known	Linear programming optimization model	coarse
Meng et al. [35,36]	IVIULVs	Partly known	Shapley value based optimization method	coarse
Liu et al. [37]	MGLAI	Completely unknown	Standard and mean deviation based method	coarse
Mohagheghi et al. [42]	PFSs	Completely unknown	Entropy based method	coarse
Liu et al. [45]	LVs	Completely unknown	Combined weighting method based on statistical variance	multiple, coarse
Wan et al. [38]	IFNs	Completely unknown	Preference degree based optimization method	finer
Wan et al. [39]	TIFNs	Completely unknown	Shannon entropy based method	finer
Lourenzutti [46]	Heterogeneous information	Completely unknown	Given by experts	finer

Coarse-granulation means that each attribute has the same weight under all the experts. Finer-granulation means that each attribute has different weights under different experts. LVs: linguistic variables.

higher accuracy and stronger comprehensiveness can be obtained. For the weights of attributes, we establish the dominance granular structure by using the dominance relation between alternatives under different attributes, based on which the consistency degree is defined to characterize and measure the attribute importance from the perspective of the ranking of alternatives. Finally, we calculate and measure the weights of attributes from the aspects of maximizing deviation and consistency degree, respectively. By building an optimization model to fuse the two kinds of weights, the multi-granularity weight of attributes can be obtained.

Another key issue in MAGDM is the representation and interpretability of the decision results. The decision results obtained by the existing decision-making methods are basically given in the form of overall evaluation value or ranking of alternatives, where the first alternative in the ranking is usually regarded as the optimal one. This kind of decision results lacks the corresponding semantic interpretation and can only provide limited feedback information at the fine-granulation level for decision makers. Moreover, in the actual decision-making process, even the first alternative in the ranking may not be ideal. Decision makers need to be informed of the grading information of the alternative and its suitability to decision-making problems, so as to make a correct and reasonable choice. Therefore, it is necessary to depict and describe the decision results from multi-level and multi-view to form more perfect feedback information. We know that interval-valued intuitionistic uncertain linguistic information is a subjective reflection of the expert's preference, which can be naturally divided into three levels of high, medium and low. Based on the idea of three-way decisions, we divide the alternatives from two aspects, i.e., the comprehensive performance under all attributes and the overall evaluation value, so as to obtain two types of three-way grading information. By fusing these three-way grading information, the coarse-granulation grading information of the alternative can be obtained, which is an effective supplement to the fine-granulation ranking information and can enhance the interpretability of decision results.

In addition, in order to meet the actual needs of decision makers with different risk attitudes, we introduce risk attitude into the whole decision-making process. By defining the expected value with risk attitude factor and developing a comparison method considering risk attitude for IVIULVs, the influence of risk attitude on attribute weights and final decision-making results are analyzed and discussed.

Based on the above analysis, in order to improve the scientificity and rationality of decision-making, we establish a novel data-driven MAGDM method based on IVIULVs by integrating the idea of multi-granulation and three-way decisions into the decision-making process. Comparing with the existing MAGDM methods, the main contributions of our work can be identified as follows: (1) The multi-granulation weight mining and fusion methods for experts and attributes are proposed, respectively; (2) The coarse-granulation grading information based on three-way decisions is developed to improve the interpretability and reference value of decision results; (3) The risk attitude is introduced into the decision-making process, and its influence on attribute weight, alternative grading and ranking is considered in the proposed method.

The rest of the paper is organized as follows. Three-way decisions models, and basic concepts, operational laws, comparison method of IVIULVs are briefly reviewed in Section 2. Section 3 introduces the risk attitude into the comparison method of IVIULVs, and gives the definitions and measurement formulas of uncertainty and distance. In Section 4, the mining and fusion methods of multi-granulation weight are developed for experts and attributes, respectively. Section 5 introduces the detailed calculation process of the proposed method, including the acquisition process of coarse-granulation grading information. Section 6 illustrates the practicality and effectiveness of the proposed method through deeply analyzing a typical example of MAGDM problem. Section 7 analyses and compares the relationship and differences between the proposed method and other relevant MAGDM methods in detail. Section 8 draws our conclusions and points out future research directions.

2. Preliminaries

We start by recalling some well-known concepts that will be useful for subsequent developments throughout the paper. In this section, three-way decisions models, and basic concepts, aggregation operator, comparison method related to IVIULVs are briefly described in the following.

2.1. Three-way decisions models

The original three-way decisions are based on a trisecting-and-acting model defined as follows.

Definition 1. [13] Let $(L, \succeq)$ denote a totally ordered set, (δ, ξ) denotes a pair of thresholds which satisfies $\delta \succ \xi$ (i.e., $\delta \succeq \xi \wedge \neg(\xi \succeq \delta)$). Suppose U be a set of objects, and $v : U \longrightarrow L$ be an evaluation function. For an object $x \in U$, $v(x)$ is its evaluation status value (ESV). Based on such an evaluation-based model, three pair-wise disjoint regions are defined as follows:

$$\text{Region } I(v) = \{x \in U \,|\, v(x) \succeq \delta\},$$
$$\text{Region } II(v) = \{x \in U \,|\, \xi \prec v(x) \prec \delta\},$$
$$\text{Region } III(v) = \{x \in U \,|\, v(x) \preceq \xi\},$$

where Region $I(v) \bigcup$ Region $II(v) \bigcup$ Region $III(v) = U$.

By adding a third component to the above model, a trisecting-acting-outcome (TAO) model was further proposed by Yao [14]. The function of trisecting is to divide a whole into three related and relatively independent parts. The resulting three parts are called a trisection of the whole. The function of acting is to apply a set of strategies to process the three parts. By trisecting a whole and acting on the resulting trisection, it would produce an expected outcome. The function of outcome evaluation is to measure the effectiveness of the results from a combined effort of trisecting and acting. The TAO model provides an architectural framework of three-way decisions. When applying the model to a particular application, one needs to use semantically and physically meaningful trisections, profitable actions, and informative measures of effectiveness.

2.2. Interval-valued intuitionistic uncertain linguistic variable

To deal with the qualitative fuzzy preferences, the experts usually use linguistic variables rather than numerical ones. The linguistic approach is an approximate technique, which represents qualitative aspects by means of linguistic variables. Let $S = \{s_\alpha | \alpha = 0, 1, \cdots, l\}$ be a finite and totally ordered discrete linguistic term set, where l is the even value, s_α represents a possible value for a linguistic variable, called linguistic term. S must have the following characteristics [47]:

(1) The set is ordered: $s_\alpha \geqslant s_\beta$ if $\alpha \geqslant \beta$;
(2) There is the negation operator: $neg(s_\alpha) = s_\beta$ such that $\beta = l - \alpha$;
(3) Max operator: $\max(s_\alpha, s_\beta) = s_\alpha$ if $s_\alpha \geqslant s_\beta$;
(4) Min operator: $\min(s_\alpha, s_\beta) = s_\alpha$ if $s_\alpha \leqslant s_\beta$.

For example, a set of seven terms S $(l = 6)$ could be

$$S = \{s_0 = \text{extremely poor}, s_1 = \text{very poor}, s_2 = \text{poor}, s_3 = \text{fair}, s_4 = \text{good},\ s_5 = \text{very good}, s_6 = \text{extremely good}\}.$$

In the integrating process of decision information, the integration results often do not match the elements in S. In order to facilitate computation and avoid the loss of information, the discrete term set S can be extended to continuous term set $\overline{S} = \{s_\alpha | s_0 \leqslant s_\alpha \leqslant s_l, \alpha \in [0, l]\}$ [48], whose elements also meet all the characteristics above. For arbitrary linguistic term s_α, if $s_\alpha \in S$, we call s_α the original term and α the original term index; otherwise, we call s_α the virtual term and α the virtual term index. In general, experts use the original linguistic terms to evaluate alternatives, and the virtual linguistic terms only appear during the operation. Let $I(s_\alpha)$ denote the term index of s_α in $\overline{S}$, i.e., $I(s_\alpha) = \alpha$.

In real situation, the input linguistic information may not match any of the original linguistic terms, and they may be located between two of them [48].

Definition 2. Let $\tilde{s} = [s_L, s_R]$, where $s_L, s_R \in \overline{S}$, s_L and s_R are the lower and upper limits, respectively, we call $\tilde{s}$ the uncertain linguistic variable.

Let $\tilde{S}$ be the set of all uncertain linguistic variables. Considering any three uncertain linguistic variables $\tilde{s} = [s_L, s_R]$, $\tilde{s}_1 = [s_{1L}, s_{1R}]$, and $\tilde{s}_2 = [s_{2L}, s_{2R}]$, $\tilde{s}, \tilde{s}_1, \tilde{s}_2 \in \tilde{S}$, $\rho \in [0, 1]$, the operational laws could be defined as:

(1) $\tilde{s}_1 \oplus \tilde{s}_2 = [s_{1L}, s_{1R}] \oplus [s_{2L}, s_{2R}] = [s_{1L} \oplus s_{2L}, s_{1R} \oplus s_{2R}] = [s_{1L+2L}, s_{1R+2R}]$;
(2) $\rho\tilde{s} = \rho[s_L, s_R] = [\rho s_L, \rho s_R] = [s_{\rho L}, s_{\rho R}]$.

By combining uncertain linguistic variables and interval-valued intuitionistic fuzzy set, Liu [34] proposed interval-valued intuitionistic uncertain linguistic set in 2013, based on which the corresponding operational laws, comparison method and aggregation operators were proposed [34,35]. These researches have laid an important theoretical foundation for the analysis of MAGDM based on IVIULVs.

Definition 3. [34] An interval-valued intuitionistic uncertain linguistic set (IVIULS) $\tilde{A}$ in $X = \{x_1, x_2, \cdots, x_n\}$ is expressed by

$$\tilde{A} = \left\{ \left\langle x([s_{\theta(x)}, s_{\tau(x)}], [u^L(x), u^U(x)], [v^L(x), v^U(x)]) \right\rangle \middle| x \in X \right\}$$

where $s_{\theta(x)}, s_{\tau(x)} \in \overline{S}$, the numbers $[u^L(x), u^U(x)]$ and $[v^L(x), v^U(x)]$, respectively represent the membership degree and non-membership degree of the element x to the uncertain linguistic variable $[s_{\theta(x)}, s_{\tau(x)}]$ with $[u^L(x), u^U(x)] \subseteq [0, 1]$, $[v^L(x), v^U(x)] \subseteq [0, 1]$, $u^U(x) + v^U(x) \leqslant 1$, $u^L(x) \geqslant 0$ and $v^L(x) \geqslant 0$ for each $x \in X$.

For brevity, Liu [34] further gave the definition of interval-valued intuitionistic uncertain linguistic variable (IVIULV). An IVIULV $\tilde{a}$ is defined by $\tilde{a} = \langle [s_{\theta(\tilde{a})}, s_{\tau(\tilde{a})}], [u^L(\tilde{a}), u^U(\tilde{a})], [v^L(\tilde{a}), v^U(\tilde{a})] \rangle$, where $[u^L(\tilde{a}), u^U(\tilde{a})]$ and $[v^L(\tilde{a}), v^U(\tilde{a})]$ respectively represent the membership degree and non-membership degree to the uncertain linguistic variable $[s_{\theta(\tilde{a})}, s_{\tau(\tilde{a})}]$ with $[u^L(\tilde{a}), u^U(\tilde{a})] \subseteq [0, 1]$, $[v^L(\tilde{a}), v^U(\tilde{a})] \subseteq [0, 1]$ and $u^U(\tilde{a}) + v^U(\tilde{a}) \leqslant 1$.

2.3. Aggregation operator of IVIULVs

In order to effectively aggregate IVIULVs, Liu [34] also proposed the following aggregation operator.

Definition 4. [34] Let $\tilde{a}_i = \langle [s_{\theta(\tilde{a}_i)}, s_{\tau(\tilde{a}_i)}], [u^L(\tilde{a}_i), u^U(\tilde{a}_i)], [v^L(\tilde{a}_i), v^U(\tilde{a}_i)] \rangle$ $(i = 1, 2, \cdots, n)$ be a set of the IVIULVs, and IVIULWAA: $\Omega^n \longrightarrow \Omega$, if

$$\begin{aligned} IVIULWAA_\omega(\tilde{a}_1, \tilde{a}_2, \cdots, \tilde{a}_n) &= \Sigma_{i=1}^n \omega_i \tilde{a}_i \\ &= \langle [s_{\Sigma_{i=1}^n \omega_i \theta(\tilde{a}_i)}, s_{\Sigma_{i=1}^n \omega_i \tau(\tilde{a}_i)}], \\ &\quad [1 - \prod_{i=1}^n (1 - u^L(\tilde{a}_i))^{\omega_i}, 1 - \prod_{i=1}^n (1 - u^U(\tilde{a}_i))^{\omega_i}, [\prod_{i=1}^n (v^L(\tilde{a}_i))^{\omega_i}, \prod_{i=1}^n (v^U(\tilde{a}_i))^{\omega_i}] \rangle \end{aligned} \tag{1}$$

where Ω is the set of all IVIULVs, and $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$ is the weight vector of $\tilde{a}_i (i = 1, 2, \cdots, n)$, $\omega_i \in [0, 1]$, $\Sigma_{i=1}^n \omega_i = 1$, then IVIULWAA is called the interval-valued intuitionistic uncertain linguistic weighted arithmetic average operator (IVIULWAA).

2.4. Comparison method of IVIULVs

Considering the application of IVIULVs in practical decision-making, Liu [34] gave a method to compare and rank IVIULVs by defining expected value and accuracy function.

Definition 5. [34] Let $\tilde{a}_1 = \langle [s_{\theta(\tilde{a}_1)}, s_{\tau(\tilde{a}_1)}], [u^L(\tilde{a}_1), u^U(\tilde{a}_1)], [v^L(\tilde{a}_1), v^U(\tilde{a}_1)] \rangle$ be an IVIULV, an expected value $E(\tilde{a}_1)$ of $\tilde{a}_1$ can be represented as follows

$$\begin{aligned} E(\tilde{a}_1) &= \frac{1}{2}(\frac{u^L(\tilde{a}_1) + u^U(\tilde{a}_1)}{2} + 1 - \frac{v^L(\tilde{a}_1) + v^U(\tilde{a}_1)}{2}) \times s_{(\theta(\tilde{a}_1) + \tau(\tilde{a}_1))/2} \\ &= s_{(\theta(\tilde{a}_1) + \tau(\tilde{a}_1)) \times (u^L(\tilde{a}_1) + u^U(\tilde{a}_1) + 2 - v^L(\tilde{a}_1) - v^U(\tilde{a}_1))/8} \end{aligned} \tag{2}$$

Definition 6. [34] Let $\tilde{a}_1 = \langle [s_{\theta(\tilde{a}_1)}, s_{\tau(\tilde{a}_1)}], [u^L(\tilde{a}_1), u^U(\tilde{a}_1)], [v^L(\tilde{a}_1), v^U(\tilde{a}_1)] \rangle$ be an IVIULV, an accuracy function $H(\tilde{a}_1)$ of $\tilde{a}_1$ can be represented as follows

$$\begin{aligned} H(\tilde{a}_1) &= (\frac{u^L(\tilde{a}_1) + u^U(\tilde{a}_1)}{2} + \frac{v^L(\tilde{a}_1) + v^U(\tilde{a}_1)}{2}) \times s_{(\theta(\tilde{a}_1) + \tau(\tilde{a}_1))/2} \\ &= s_{(\theta(\tilde{a}_1) + \tau(\tilde{a}_1)) \times (u^L(\tilde{a}_1) + u^U(\tilde{a}_1) + v^L(\tilde{a}_1) + v^U(\tilde{a}_1))/4} \end{aligned} \tag{3}$$

The comparison method of any two IVIULVs $\tilde{a}_1 = \langle [s_{\theta(\tilde{a}_1)}, s_{\tau(\tilde{a}_1)}], [u^L(\tilde{a}_1), u^U(\tilde{a}_1)], [v^L(\tilde{a}_1), v^U(\tilde{a}_1)] \rangle$ and $\tilde{a}_2 = \langle [s_{\theta(\tilde{a}_2)}, s_{\tau(\tilde{a}_2)}], [u^L(\tilde{a}_2), u^U(\tilde{a}_2)], [v^L(\tilde{a}_2), v^U(\tilde{a}_2)] \rangle$ is as follows [34]:

(1) If $E(\tilde{a}_1) > E(\tilde{a}_2)$, then $\tilde{a}_1 \succ \tilde{a}_2$;
(2) If $E(\tilde{a}_1) = E(\tilde{a}_2)$, then

$$\text{If } H(\tilde{a}_1) > H(\tilde{a}_2), \text{ then } \tilde{a}_1 \succ \tilde{a}_2;$$
$$\text{If } H(\tilde{a}_1) = H(\tilde{a}_2), \text{ then } \tilde{a}_1 = \tilde{a}_2.$$

3. Extended comparison method and proposed measure formulas

In this section, the comparison method is extended to the situation in which risk attitude is taken into account, and the corresponding measurement formulas are designed to make calculation and representation convenient.

3.1. Comparison method considering risk attitude

In the real decision-making process, different decision makers may have different attitudes towards risk. In order to facilitate decision makers of different risk types to compare and rank IVIULVs, we introduce risk attitude factor γ into Definitions 5 and 6, and propose the following definitions of expected value and accuracy function with risk attitude factor γ.

Definition 7. Let $\tilde{a}_1 = \langle [s_{\theta(\tilde{a}_1)}, s_{\tau(\tilde{a}_1)}], [u^L(\tilde{a}_1), u^U(\tilde{a}_1)], [v^L(\tilde{a}_1), v^U(\tilde{a}_1)] \rangle$ be an IVIULV, the expected value $E^\gamma(\tilde{a}_1)$ of $\tilde{a}_1$ when risk attitude factor is γ can be represented as follows

$$E^\gamma(\tilde{a}_1) = s_{((1-\gamma)\theta(\tilde{a}_1) + \gamma\tau(\tilde{a}_1)) \times ((1-\gamma)u^L(\tilde{a}_1) + \gamma u^U(\tilde{a}_1) + 1 - \gamma v^L(\tilde{a}_1) - (1-\gamma)v^U(\tilde{a}_1))/2} \tag{4}$$

where the value of γ depends on the decision maker's risk attitude. If the decision maker is risk-preference, then $\gamma > 0.5$; if the decision maker is risk-neutral, then $\gamma = 0.5$; if the decision maker is risk-averse, then $\gamma < 0.5$.

Definition 8. Let $\tilde{a}_1 = \langle [s_{\theta(\tilde{a}_1)}, s_{\tau(\tilde{a}_1)}], [u^L(\tilde{a}_1), u^U(\tilde{a}_1)], [v^L(\tilde{a}_1), v^U(\tilde{a}_1)] \rangle$ be an IVIULV, the accuracy function $H^\gamma(\tilde{a}_1)$ of $\tilde{a}_1$ when risk attitude factor is γ can be represented as follows

$$H^\gamma(\tilde{a}_1) = s_{((1-\gamma)\theta(\tilde{a}_1)+\gamma\tau(\tilde{a}_1))\times((1-\gamma)u^L(\tilde{a}_1)+\gamma u^U(\tilde{a}_1)+\gamma v^L(\tilde{a}_1)+(1-\gamma)v^U(\tilde{a}_1))} \tag{5}$$

Different from the calculation formulas based on the average value of elements in Definitions 5 and 6, Definitions 7 and 8 use parameter γ to weight the elements and reflect the impact of risk attitude on the expected value and accuracy function. The higher the value of γ, the more optimistic the decision-maker is, the closer the assessment is to the maximum $\langle [s_{\tau(\tilde{a}_1)}, s_{\tau(\tilde{a}_1)}], [u^U(\tilde{a}_1), u^U(\tilde{a}_1)], [v^L(\tilde{a}_1), v^L(\tilde{a}_1)] \rangle$, and the larger the expected value and accuracy function correspondingly; Conversely, the smaller the value of γ, the more pessimistic the decision-maker is, the closer the assessment is to the minimum $\langle [s_{\theta(\tilde{a}_1)}, s_{\theta(\tilde{a}_1)}], [u^L(\tilde{a}_1), u^L(\tilde{a}_1)], [v^U(\tilde{a}_1), v^U(\tilde{a}_1)] \rangle$, and the smaller the expected value and accuracy function correspondingly.

In particular, when $\gamma = 0.5$, $E^{0.5}(\tilde{a}_1)$ and $H^{0.5}(\tilde{a}_1)$ degenerate to $E(\tilde{a}_1)$ and $H(\tilde{a}_1)$ defined in [34] (see Definitions 5 and 6), that is, $E(\tilde{a}_1)$ and $H(\tilde{a}_1)$ are special cases of $E^\gamma(\tilde{a}_1)$ and $H^\gamma(\tilde{a}_1)$, respectively.

Using Definitions 7 and 8, we can compare and rank two IVIULVs under different risk attitudes. When risk attitude factor is γ, the comparison method of any two IVIULVs $\tilde{a}_1 = \langle [s_{\theta(\tilde{a}_1)}, s_{\tau(\tilde{a}_1)}], [u^L(\tilde{a}_1), u^U(\tilde{a}_1)], [v^L(\tilde{a}_1), v^U(\tilde{a}_1)] \rangle$ and $\tilde{a}_2 = \langle [s_{\theta(\tilde{a}_2)}, s_{\tau(\tilde{a}_2)}], [u^L(\tilde{a}_2), u^U(\tilde{a}_2)], [v^L(\tilde{a}_2), v^U(\tilde{a}_2)] \rangle$ is as follows:

(1) If $E^\gamma(\tilde{a}_1) > E^\gamma(\tilde{a}_2)$, then $\tilde{a}_1 \succ \tilde{a}_2$;
(2) If $E^\gamma(\tilde{a}_1) = E^\gamma(\tilde{a}_2)$, then

$$\text{If } H^\gamma(\tilde{a}_1) > H^\gamma(\tilde{a}_2),\ \text{then } \tilde{a}_1 \succ \tilde{a}_2;$$
$$\text{If } H^\gamma(\tilde{a}_1) = H^\gamma(\tilde{a}_2),\ \text{then } \tilde{a}_1 = \tilde{a}_2.$$

3.2. Uncertainty and distance measurement

Next, the definition of uncertainty is given to measure the degree of uncertainty of an IVIULV. Let $\tilde{a}_1 = \langle [s_{\theta(\tilde{a}_1)}, s_{\tau(\tilde{a}_1)}], [u^L(\tilde{a}_1), u^U(\tilde{a}_1)], [v^L(\tilde{a}_1), v^U(\tilde{a}_1)] \rangle$ be an IVIULV, the uncertainty of $\tilde{a}_1$ includes two aspects: (1) the internal uncertainty of uncertain linguistic variable $[s_{\theta(\tilde{a}_1)}, s_{\tau(\tilde{a}_1)}]$; (2) the internal uncertainty of interval-valued intuitionistic fuzzy number $[u^L(\tilde{a}_1), u^U(\tilde{a}_1)], [v^L(\tilde{a}_1), v^U(\tilde{a}_1)]$. Therefore, the uncertainty of $\tilde{a}_1$ mainly depends on the distance between two subscripts $\theta(\tilde{a}_1)$ and $\tau(\tilde{a}_1)$, the distance between two membership degrees $u^L(\tilde{a}_1)$ and $u^U(\tilde{a}_1)$, the distance between two non-membership degrees $v^L(\tilde{a}_1)$ and $v^U(\tilde{a}_1)$, the distance between membership degrees and 1, and the distance between non-membership degrees and 0. The smaller the sum of these distances, the lower the degree of uncertainty of $\tilde{a}_1$, that is, the more accurate $\tilde{a}_1$ is; on the contrary, the larger the sum of these distances, the higher the degree of uncertainty of $\tilde{a}_1$, that is, the more vague $\tilde{a}_1$ is. Based on the comprehensive consideration of these distances, the following definition is given.

Definition 9. Let $\tilde{a}_1 = \langle [s_{\theta(\tilde{a}_1)}, s_{\tau(\tilde{a}_1)}], [u^L(\tilde{a}_1), u^U(\tilde{a}_1)], [v^L(\tilde{a}_1), v^U(\tilde{a}_1)] \rangle$ be an IVIULV, the uncertainty $\phi(\tilde{a}_1)$ of $\tilde{a}_1$ can be represented as follows

$$\begin{aligned}\phi(\tilde{a}_1) &= \frac{1}{l+1}(\tau(\tilde{a}_1) - \theta(\tilde{a}_1) + \frac{1}{2}(u^U(\tilde{a}_1) - u^L(\tilde{a}_1) + v^U(\tilde{a}_1) - v^L(\tilde{a}_1) \\ &\quad + \frac{1 - u^L(\tilde{a}_1) + 1 - u^U(\tilde{a}_1) + v^L(\tilde{a}_1) + v^U(\tilde{a}_1)}{2})) \\ &= \frac{1}{l+1}(\tau(\tilde{a}_1) - \theta(\tilde{a}_1) + \frac{2 + u^U(\tilde{a}_1) - 3u^L(\tilde{a}_1) + 3v^U(\tilde{a}_1) - v^L(\tilde{a}_1)}{4})\end{aligned} \tag{6}$$

where $l+1$ is the number of linguistic terms.

As can be seen from Definition 3, $u^L(\tilde{a}_1) \geqslant 0$, $v^L(\tilde{a}_1) \geqslant 0$, $u^U(\tilde{a}_1) + v^U(\tilde{a}_1) \leqslant 1$, $[u^L(\tilde{a}_1), u^U(\tilde{a}_1)] \subseteq [0,1]$ and $[v^L(\tilde{a}_1), v^U(\tilde{a}_1)] \subseteq [0,1]$. If $\tau(\tilde{a}_1) = \theta(\tilde{a}_1)$, $u^U(\tilde{a}_1) = u^L(\tilde{a}_1) = 1$ and $v^U(\tilde{a}_1) = v^L(\tilde{a}_1) = 0$, then $\phi(\tilde{a}_1) =$ reaches the minimum of 0, $\tilde{a}_1$ degenerates into an exact linguistic variable $\langle [s_{\tau(\tilde{a}_1)}, s_{\tau(\tilde{a}_1)}], [1,1], [0,0] \rangle$, namely $s_{\tau(\tilde{a}_1)}$; If $\theta(\tilde{a}_1) = 0$, $\tau(\tilde{a}_1) = l$, $u^U(\tilde{a}_1) = v^U(\tilde{a}_1) = 0.5$ and $u^L(\tilde{a}_1) = v^L(\tilde{a}_1) = 0$, then $\phi(\tilde{a}_1)$ reaches the maximum of 1, $\tilde{a}_1$ becomes the most uncertain variable $\langle [s_0, s_l], [0, 0.5], [0, 0.5] \rangle$. Thus the following property can be obtained.

Property 1. $0 \leqslant \phi(\tilde{a}_1) \leqslant 1$.

In order to calculate and measure the distance between two IVIULVs, we first use the expected value with risk attitude factor in Definition 7 to transform the IVIULVs into uncertain linguistic variables. The subscripts of the lower and upper limits of an IVIULV constitute an interval number. Then the distance between IVIULVs can be defined by using the normalized Euclidean distance of interval values.

For example, $\tilde{a}_1 = \langle [s_{\theta(\tilde{a}_1)}, s_{\tau(\tilde{a}_1)}], [u^L(\tilde{a}_1), u^U(\tilde{a}_1)], [v^L(\tilde{a}_1), v^U(\tilde{a}_1)] \rangle$ can be transformed into $[E^0(\tilde{a}_1), E^1(\tilde{a}_1)]$, where $E^0(\tilde{a}_1) = s_{\theta(\tilde{a}_1)\times(u^L(\tilde{a}_1)+1-v^U(\tilde{a}_1))/2}$, $E^1(\tilde{a}_1) = s_{\tau(\tilde{a}_1)\times(u^U(\tilde{a}_1)+1-v^L(\tilde{a}_1))/2}$. Then we can get the subscript of $E^0(\tilde{a}_1)$ and $E^1(\tilde{a}_1)$, i.e., $I(E^0(\tilde{a}_1)) = \theta(\tilde{a}_1) \times (u^L(\tilde{a}_1) + 1 - v^U(\tilde{a}_1))/2$, $I(E^1(\tilde{a}_1)) = \tau(\tilde{a}_1) \times (u^U(\tilde{a}_1) + 1 - v^L(\tilde{a}_1))/2$, the corresponding interval number is $[\theta(\tilde{a}_1) \times (u^L(\tilde{a}_1) + 1 - v^U(\tilde{a}_1))/2, \tau(\tilde{a}_1) \times (u^U(\tilde{a}_1) + 1 - v^L(\tilde{a}_1))/2]$.

Definition 10. Let $\tilde{a}_1 = \langle [s_{\theta(\tilde{a}_1)}, s_{\tau(\tilde{a}_1)}], [u^L(\tilde{a}_1), u^U(\tilde{a}_1)], [v^L(\tilde{a}_1), v^U(\tilde{a}_1)] \rangle$ and $\tilde{a}_2 = \langle [s_{\theta(\tilde{a}_2)}, s_{\tau(\tilde{a}_2)}], [u^L(\tilde{a}_2), u^U(\tilde{a}_2)], [v^L(\tilde{a}_2), v^U(\tilde{a}_2)] \rangle$ be two IVIULVs, the normalized Euclidean distance between them is defined as follows

$$d(\tilde{a}_1, \tilde{a}_2) = \frac{1}{2l}\Big(\frac{1}{2}((\theta(\tilde{a}_1)(u^L(\tilde{a}_1) + 1 - v^U(\tilde{a}_1)) - \theta(\tilde{a}_2)(u^L(\tilde{a}_2) + 1 - v^U(\tilde{a}_2)))^2 + (\tau(\tilde{a}_1)(u^U(\tilde{a}_1) + 1 - v^L(\tilde{a}_1)) - \tau(\tilde{a}_2)(u^U(\tilde{a}_2) + 1 - v^L(\tilde{a}_2)))^2)\Big)^{1/2} \tag{7}$$

where l is the maximum value that the subscripts may get, and l in the denominator ensures that the distance is between 0 and 1.

4. Mining and fusion of multi-granulation weight in MAGDM based on IVIULVs

Consider a MAGDM problem based on IVIULVs. Let $A = \{A_1, A_2, \cdots, A_m\}$ be a discrete set of alternatives, $C = \{C_1, C_2, \cdots, C_n\}$ be the set of attributes. Let $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$ be the weight vector of attributes, where $\omega_j \in [0, 1]$, $j = 1, 2, \cdots, n$, $\sum_{j=1}^{n} \omega_j = 1$. Let $E = \{e_1, e_2, \cdots, e_p\}$ be the set of experts, $\tilde{R} = \{\tilde{R}^1, \tilde{R}^2, \cdots, \tilde{R}^p\}$ be the set of decision matrices of p experts, where $\tilde{R}^k = (\tilde{R}^k_{ij})_{m\times n}$ is the decision matrix provided by e_k, and $\tilde{R}^k_{ij} = \langle [a^L_{ijk}, a^U_{ijk}], [u^L_{ijk}, u^U_{ijk}], [v^L_{ijk}, v^U_{ijk}] \rangle$ is a preference value in the form of IVIULV, given by expert $e_k \in E$ for alternative $A_i \in A$ w.r.t. attribute $C_j \in C$ $(1 \leqslant k \leqslant p, 1 \leqslant i \leqslant m, 1 \leqslant j \leqslant n)$.

In order to obtain more scientific and reasonable decision-making results, we introduce the idea of multi-granulation into the weight calculation process, and mine the objective synthetic weights of experts and attributes by analyzing the evaluation values and their potential correlations from finer granulation and more perspectives. The corresponding determination methods of multi-granulation weights are given below.

4.1. Calculation and integration of expert finest-granulation weight

To obtain the experts' weights with higher accuracy and stronger comprehensiveness, we take the evaluation value of each alternative on each attribute by each expert as the basic granulation, and mine the experts' weights from two aspects of uncertainty and closeness. Then, by calculating the uniformity of the two kinds of weights, the proportion factor is obtained to integrate the weights objectively.

(1) The expert finest-granulation weight based on uncertainty

The concept and Eq. (6) of uncertainty given in Definition 9 reflect from one side the reliability of expert evaluation information and the confidence of expert, which can be used to calculate finest-granulation weight of experts. The larger the uncertainty of evaluation value $\tilde{R}^k_{ij}$ is, the smaller the weight of expert e_k for alternative A_i w.r.t. attribute C_j is; conversely, the smaller the uncertainty of evaluation value $\tilde{R}^k_{ij}$ is, the larger the weight of expert e_k for alternative A_i w.r.t. attribute C_j is. Therefore, the finest-granulation weight of expert e_k for alternative A_i w.r.t. attribute C_j based on uncertainty can be calculated as follows

$$\lambda^{\phi}_{ijk} = \frac{1/\phi(\tilde{R}^k_{ij})}{\sum_{k=1}^{p} 1/\phi(\tilde{R}^k_{ij})} \tag{8}$$

(2) The expert finest-granulation weight based on closeness

Next, an extended TOPSIS method is presented to calculate the weights of experts from the closeness between experts' evaluation information. The traditional TOPSIS method, developed by Hwang and Yoon [49], is one of the major MADM techniques. It ranks the alternatives according to their distances from the positive and negative ideal solutions, i.e., the best alternative has simultaneously the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution. The positive ideal solution is identified with a "hypothetical alternative" which has the best values for all considered attributes whereas the negative ideal solution is identified with a "hypothetical alternative" which has the worst values for all considered attributes.

Inspired by the idea of Yue [40], we define a positive ideal decision (PID) and two negative ideal decisions (NID), which have the farthest distance from PID. The specific process is as follows.

Step 1. Determine the PID of the group for each alternative w.r.t. each attribute.

The PID should reflect the common decision aspirations and consistent judgements of group. So we define the Median of all individual decisions as the PID of group. Compared with the average used in [40], the Median is not affected by the extreme value, and is more suitable for describing the centralized trend of expert evaluation information.

Let $\{n_1, n_2, \cdots, n_q\}$ be a set of numbers, the Median of the numbers can be expressed as follows.

$$M_e\{n_1, n_2, \cdots, n_q\} = \begin{cases} n_{\sigma(\frac{q+1}{2})}, \text{if q is odd;} \\ (n_{\sigma(\frac{q}{2})} + n_{\sigma(\frac{q+1}{2})})/2, \text{if q is even,} \end{cases} \tag{9}$$

where $n_{\sigma(i)}$ represents the ith largest number in $\{n_1, n_2, \cdots, n_q\}$.

The PID of the group for alternative A_i w.r.t. attribute C_j can be denoted as $\tilde{R}^+_{ij} = \langle [a^{L+}_{ij}, a^{U+}_{ij}], [u^{L+}_{ij}, u^{U+}_{ij}], [v^{L+}_{ij}, v^{U+}_{ij}]\rangle$, where $a^{L+}_{ij} = M_e\{a^L_{ij1}, a^L_{ij2}, \cdots, a^L_{ijp}\}$, $a^{U+}_{ij} = M_e\{a^U_{ij1}, a^U_{ij2}, \cdots, a^U_{ijp}\}$, $u^{L+}_{ij} = M_e\{u^L_{ij1}, u^L_{ij2}, \cdots, u^L_{ijp}\}$, $u^{U+}_{ij} = M_e\{u^U_{ij1}, u^U_{ij2}, \cdots, u^U_{ijp}\}$, $v^{L+}_{ij} = M_e\{v^L_{ij1}, v^L_{ij2}, \cdots, v^L_{ijp}\}$, $v^{U+}_{ij} = M_e\{v^U_{ij1}, v^U_{ij2}, \cdots, v^U_{ijp}\}$.

Step 2. Divide the NID into two parts: L-NID and R-NID.

The minimum decision of all individual decisions is defined as left NID (L-NID), and the maximum decision of all individual decisions is defined as right NID (R-NID).

Step 3. Determine the L-NID of the group for each alternative w.r.t. each attribute.

The L-NID of the group for alternative A_i w.r.t. attribute C_j can be denoted as $\tilde{R}^-_{ijL} = \langle [a^{L-}_{ijL}, a^{U-}_{ijL}], [u^{L-}_{ijL}, u^{U-}_{ijL}], [v^{L-}_{ijL}, v^{U-}_{ijL}]\rangle$, where $a^{L-}_{ijL} = min_{1\leqslant k\leqslant p}\{a^L_{ijk}\}$, $a^{U-}_{ijL} = min_{1\leqslant k\leqslant p}\{a^U_{ijk}\}$, $u^{L-}_{ijL} = min_{1\leqslant k\leqslant p}\{u^L_{ijk}\}$, $u^{U-}_{ijL} = min_{1\leqslant k\leqslant p}\{u^U_{ijk}\}$, $v^{L-}_{ijL} = max_{1\leqslant k\leqslant p}\{v^L_{ijk}\}$, $v^{U-}_{ijL} = max_{1\leqslant k\leqslant p}\{v^U_{ijk}\}$.

Step 4. Determine the R-NID of the group for each alternative w.r.t. each attribute.

The R-NID of the group for alternative A_i w.r.t. attribute C_j can be denoted as $\tilde{R}^-_{ijR} = \langle [a^{L-}_{ijR}, a^{U-}_{ijR}], [u^{L-}_{ijR}, u^{U-}_{ijR}], [v^{L-}_{ijR}, v^{U-}_{ijR}]\rangle$, where $a^{L-}_{ijR} = max_{1\leqslant k\leqslant p}\{a^L_{ijk}\}$, $a^{U-}_{ijR} = max_{1\leqslant k\leqslant p}\{a^U_{ijk}\}$, $u^{L-}_{ijR} = max_{1\leqslant k\leqslant p}\{u^L_{ijk}\}$, $u^{U-}_{ijR} = max_{1\leqslant k\leqslant p}\{u^U_{ijk}\}$, $v^{L-}_{ijR} = min_{1\leqslant k\leqslant p}\{v^L_{ijk}\}$, $v^{U-}_{ijR} = min_{1\leqslant k\leqslant p}\{v^U_{ijk}\}$.

Step 5. Calculate the closeness coefficient between each expert and the group for each alternative w.r.t. each attribute.

Obviously, the smaller the distance $d(\tilde{R}^k_{ij}, \tilde{R}^+_{ij})$ is, the better the decision $\tilde{R}^k_{ij}$ is; the larger the distances $d(\tilde{R}^k_{ij}, \tilde{R}^-_{ijL})$ and $d(\tilde{R}^k_{ij}, \tilde{R}^-_{ijR})$ are, the better the decision $\tilde{R}^k_{ij}$ is. Therefore, the closeness coefficient between expert e_k and the group for alternative A_i w.r.t. attribute C_j can be defined as

$$c^k_{ij} = \frac{d(\tilde{R}^k_{ij}, \tilde{R}^-_{ijL}) + d(\tilde{R}^k_{ij}, \tilde{R}^-_{ijR})}{d(\tilde{R}^k_{ij}, \tilde{R}^+_{ij}) + d(\tilde{R}^k_{ij}, \tilde{R}^-_{ijL}) + d(\tilde{R}^k_{ij}, \tilde{R}^-_{ijR})} \tag{10}$$

This closeness coefficient depends on how far or how close the opinion of expert e_k departs from those of other experts. When an individual decision $\tilde{R}^k_{ij}$ is closer to group's unbiased opinion $\tilde{R}^+_{ij}$ and farther from $\tilde{R}^-_{ijL}$ and $\tilde{R}^-_{ijR}$, c^k_{ij} approaches to 1. Therefore, c^k_{ij} can be used to calculate the finest-granulation weights of experts.

Step 6. The finest-granulation weight of expert e_k for alternative A_i w.r.t. attribute C_j based on closeness can be obtained as follows

$$\lambda^c_{ijk} = \frac{c^k_{ij}}{\sum_{k=1}^{p} c^k_{ij}} \tag{11}$$

(3) The integration of two kinds of expert finest-granulation weight

In order to facilitate the comparison and analysis of the two kinds of expert weight, we define the concept of uniformity based on information entropy theory in this subsection. The entropy of a system as defined by Shannon [50] gives a measure of uncertainty about its actual structure. It has been a useful mechanism for characterizing the information content in various modes and applications in many diverse fields.

If $q = (q_1, q_2, \cdots, q_n)$ is a finite probability distribution, then its Shannon entropy is given by

$$S(q) = -\sum_{i=1}^{n} q_i \log_2 q_i \tag{12}$$

By replacing the probability distribution with the weight vector and normalizing the Eq. (12), we can give the following definition of uniformity.

Definition 11. The uniformity of weight vector $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_p)^T$ on distribution can be defined as

$$f(\lambda) = -\frac{1}{\log_2 p}\sum_{k=1}^{p}\lambda_k \log_2 \lambda_k \tag{13}$$

The larger $f(\lambda)$ is, the more uniform the distribution of the weight vector λ is. In particular, when the weight vector is a uniform distribution, $f(\lambda)$ reaches its maximum of 1. It is not difficult to obtain the following property.

Property 2. $0 < f(\lambda) \leqslant 1$.

The concept of uniformity can be used to determine the proportion of various weights in the process of weight integration. The more uniform the distribution of the weight vector is, the smaller the proportion of this kind of weight in the synthetic weight should be. Therefore, when the two kinds of experts' weights are synthesized, the kind of weight with more unbalanced distribution accounts for a larger proportion in the synthetic weights. According to the uniformity, the objective synthesis of two kinds of expert weight information can be realized. The specific integration process is as follows.

Step 1. Calculate the uniformity of two kinds of expert weight vector.

Let $\lambda_{ij}^{\phi} = (\lambda_{ij1}^{\phi}, \lambda_{ij2}^{\phi}, \cdots, \lambda_{ijp}^{\phi})^T$ be the finest-granulation weight vector of each expert for alternative A_i w.r.t. attribute C_j based on uncertainty, then its uniformity can be denoted by $f(\lambda_{ij}^{\phi})$.

Let $\lambda_{ij}^{c} = (\lambda_{ij1}^{c}, \lambda_{ij2}^{c}, \cdots, \lambda_{ijp}^{c})^T$ be the finest-granulation weight vector of each expert for alternative A_i w.r.t. attribute C_j based on closeness, then its uniformity can be denoted by $f(\lambda_{ij}^{c})$.

Step 2. Calculate the proportion η_{ij} of the weight λ_{ij}^{ϕ} in the synthetic weight.

$$\eta_{ij} = \frac{1 - f(\lambda_{ij}^{\phi})}{1 - f(\lambda_{ij}^{\phi}) + 1 - f(\lambda_{ij}^{c})} \tag{14}$$

Step 3. Calculate the objective synthetic weight λ_{ijk} of expert e_k for alternative A_i w.r.t. attribute C_j.

$$\lambda_{ijk} = \eta_{ij}\lambda_{ijk}^{\phi} + (1 - \eta_{ij})\lambda_{ijk}^{c} \tag{15}$$

4.2. Calculation and fusion of attribute multi-granulation weight

By using the expert weight λ_{ijk} and IVIULWAA operator (see Definition 4), the individual decision matrix can be aggregated into the group collective decision matrix, which is denoted as $\tilde{Z} = (\tilde{Z}_{ij})_{m\times n}$, where $\tilde{Z}_{ij} = \langle [a_{ij}^L, a_{ij}^U], [u_{ij}^L, u_{ij}^U], [v_{ij}^L, v_{ij}^U] \rangle$.

To obtain more objective and comprehensive attribute weight information, we measure the importance of attribute from the following two aspects: one is the size of value, we use the maximizing deviation formula to calculate the weights of attributes; the other is the ranking of alternatives, we introduce the dominance granular structure to measure the consistency degree between the rankings of alternatives under different attributes. And then, an optimization model is established to fuse the two kinds of weights effectively.

(1) The attribute weight based on maximizing deviation

The maximizing deviation method was proposed by Wang [51] to deal with MADM problems with numerical information. The theoretic foundation of this method is based on information theory, that is, the attribute providing more information should be evaluated a bigger weight [51]. So to the view of ranking the alternatives, if one attribute has similar attribute values across alternatives, it should be assigned a small weight; otherwise, the attribute which makes larger deviations should be evaluated a bigger weight. Therefore, once the group linguistic decision matrix is obtained, Eq. (16) can be used to get the attribute weights [52]:

$$\omega_j^d = \frac{\sum_{i=1}^{m}\sum_{h=1}^{m} d(\tilde{Z}_{ij}, \tilde{Z}_{hj})}{\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{h=1}^{m} d(\tilde{Z}_{ij}, \tilde{Z}_{hj})} \tag{16}$$

where $d(\tilde{Z}_{ij}, \tilde{Z}_{hj})$ represents the Euclidean distance between $\tilde{Z}_{ij}$ and $\tilde{Z}_{hj}$ (see Definition 10), ω_j^d represents the attribute weight based on maximizing deviation.

(2) The attribute weight based on consistency degree

On the basis of the group collective decision matrix $\tilde{Z}$, by using ω_j^d and IVIULWAA operator, the overall evaluation value of each alternative can be obtained and denoted as $\tilde{Z}_i = \langle [a_i^L, a_i^U], [u_i^L, u_i^U], [v_i^L, v_i^U] \rangle$ $(i = 1, 2, \cdots, m)$. Then by setting the risk attitude factor γ and using the comparison method considering risk attitude in section 3.1, the ranking of all alternatives under each attribute denoted by $R^{\gamma j}$ $(j = 1, 2, \cdots, n)$ and the ranking of all alternatives under all attributes denoted by $R^{\gamma *}$ can be obtained.

In order to depict the mutual relations between rankings effectively, we think of the similarity degree of dominance granular structures defined by Wang [53]. Wang had proved that similarity degree has the following good property, that is, when the positions of the two alternatives in front of the total ranking are exchanged, the similarity degree between the former and the latter is smaller, which means that the difference between the two total rankings is larger; conversely, when the positions of the two alternatives at the end of the total ranking are exchanged, the similarity degree is larger and the difference is smaller.

Inspired by Wang's similarity degree of two rankings [53], we measure the consistency degree between $R^{\gamma j}$ and $R^{\gamma *}$ by using the dominance relation [3] between alternatives under different attributes.

For aforementioned MAGDM problem, if $E^\gamma(\tilde{Z}_{hj}) > E^\gamma(\tilde{Z}_{ij})$, or if $E^\gamma(\tilde{Z}_{hj}) = E^\gamma(\tilde{Z}_{ij})$ and $H^\gamma(\tilde{Z}_{hj}) \geqslant H^\gamma(\tilde{Z}_{ij})$, we have $\tilde{Z}_{hj} \succeq \tilde{Z}_{ij}$. It means that the alternative A_h dominates A_i under the attribute C_j when risk attitude factor is γ, denoted by $A_h(R^{\gamma j})^{\geqslant} A_i$, and the dominance relation $(R^{\gamma j})^{\geqslant}$ can be expressed as:

$$(R^{\gamma j})^{\geqslant} = \{(A_h, A_i) \in A \times A \mid \tilde{Z}_{hj} \succeq \tilde{Z}_{ij} \text{ when risk attitude factor is } \gamma\} \tag{17}$$

Therefore, the dominance class of the alternative A_i w.r.t. the attribute C_j when risk attitude factor is γ can be defined as follows:

$$([A_i]^{\gamma j})^{\geqslant} = \{A_h \in A \mid \tilde{Z}_{hj} \succeq \tilde{Z}_{ij} \text{ when risk attitude factor is } \gamma\} \tag{18}$$

Then the family set of the dominance classes of the alternative set A w.r.t. the attribute C_j can be further defined as:

$$A/(R^{\gamma j})^{\geqslant} = \{([A_1]^{\gamma j})^{\geqslant}, ([A_2]^{\gamma j})^{\geqslant}, \cdots, ([A_m]^{\gamma j})^{\geqslant}\} \tag{19}$$

$A/(R^{\gamma j})^{\geqslant}$ is called a dominance granular structure of A induced by $(R^{\gamma j})^{\geqslant}$.

Similarly, if $E^\gamma(\tilde{Z}_h) > E^\gamma(\tilde{Z}_i)$, or if $E^\gamma(\tilde{Z}_h) = E^\gamma(\tilde{Z}_i)$ and $H^\gamma(\tilde{Z}_h) \geqslant H^\gamma(\tilde{Z}_i)$, we have $\tilde{Z}_h \succeq \tilde{Z}_i$. It means that the alternative A_h dominates A_i under all attributes when risk attitude factor is γ, denoted by $A_h(R^{\gamma *})^{\geqslant} A_i$, and the dominance relation $(R^{\gamma *})^{\geqslant}$ can be expressed as:

$$(R^{\gamma *})^{\geqslant} = \{(A_h, A_i) \in A \times A \mid \tilde{Z}_h \succeq \tilde{Z}_i \text{ when risk attitude factor is } \gamma\} \tag{20}$$

Therefore, the dominance class of the alternative A_i under all the attributes when risk attitude factor is γ can be defined as follows:

$$([A_i]^{\gamma *})^{\geqslant} = \{A_h \in A \mid \tilde{Z}_h \succeq \tilde{Z}_i \text{ when risk attitude factor is } \gamma\} \tag{21}$$

Then the family set of the dominance classes of the alternative set A under all the attributes can be further defined as:

$$A/(R^{\gamma *})^{\geqslant} = \{([A_1]^{\gamma *})^{\geqslant}, ([A_2]^{\gamma *})^{\geqslant}, \cdots, ([A_m]^{\gamma *})^{\geqslant}\} \tag{22}$$

$A/(R^{\gamma *})^{\geqslant}$ is called a dominance granular structure of A induced by $(R^{\gamma *})^{\geqslant}$.

On the basis of the above representations, the concept of consistency degree is given below.

Definition 12. For a MAGDM problem, let $A = \{A_1, A_2, \cdots, A_m\}$ be a discrete set of alternatives, $C = \{C_1, C_2, \cdots, C_n\}$ be the set of attributes, $1 \leqslant i \leqslant m, 1 \leqslant j \leqslant n$. $R^{\gamma j}$ and $R^{\gamma *}$ represent, respectively, the ranking of all alternatives under attribute C_j and the ranking of all alternatives under all attributes when risk attitude factor is γ. The consistency degree between $R^{\gamma j}$ and $R^{\gamma *}$ is defined as

$$CON(R^{\gamma j}, R^{\gamma *}) = \frac{\sum_{i=1}^m \log_2 \dfrac{m}{|([A_i]^{\gamma j})^{\geqslant} \bigcup ([A_i]^{\gamma *})^{\geqslant}|}}{\sum_{i=1}^m \log_2 \dfrac{m}{|([A_i]^{\gamma j})^{\geqslant} \bigcap ([A_i]^{\gamma *})^{\geqslant}|}} \tag{23}$$

The larger $CON(R^{\gamma j}, R^{\gamma *})$ is, the closer the ranking $R^{\gamma j}$ is to the overall ranking $R^{\gamma *}$ when risk attitude factor is γ, which indicates that attribute C_j plays a greater role in the ranking and should be given a larger weight; otherwise, C_j should be given a smaller weight. Based on the consistency degree, the objective weight of attribute can be obtained as follows:

$$\omega_j^{\gamma c} = \frac{CON(R^{\gamma j}, R^{\gamma *})}{\sum_{j=1}^{n} CON(R^{\gamma j}, R^{\gamma *})} \tag{24}$$

(3) The fusion of two kinds of attribute objective weight

According to the attribute weight based on consistency $\omega_j^{\gamma c}$, the ranking information of attribute weights can be obtained and denoted as Ω^{γ}. For example, if $\omega_1^{0.5c} = 0.5$, $\omega_2^{0.5c} = 0.2$, $\omega_3^{0.5c} = 0.3$, then $\Omega^{0.5} = \{\omega_1^{0.5} > \omega_3^{0.5} > \omega_2^{0.5}\}$. Ω^{γ} can be used as a constraint for ranking the importance of attributes.

To determine the synthetic weight ω_j^{γ} of attribute under different risk attitudes, the maximizing deviation method is selected here. For attribute $C_j \in C$, when risk attitude factor is γ the deviation of alternative A_i to all the other alternatives can be defined as follows:

$$D_{ij}(\omega^{\gamma}) = \sum_{h=1}^{m} |E^{\gamma}(\tilde{Z}_{ij}) - E^{\gamma}(\tilde{Z}_{hj})|\omega_j^{\gamma} \tag{25}$$

Let $D_j(\omega^{\gamma}) = \sum_{i=1}^{m} D_{ij}(\omega^{\gamma}) = \sum_{i=1}^{m}\sum_{h=1}^{m} |E^{\gamma}(\tilde{Z}_{ij}) - E^{\gamma}(\tilde{Z}_{hj})|\omega_j^{\gamma}$. Then $D_j(\omega^{\gamma})$ represents the deviation value of all alternatives to other alternatives for attribute C_j.

In order to fuse the two kinds of attribute weights effectively and obtain the synthetic weight of attribute, a non-linear optimization model (M-1) is constructed by maximizing all deviation values for all attributes as follows:

$$maxD(\omega^{\gamma}) = \sum_{j=1}^{n} D_j(\omega^{\gamma}) = \sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{h=1}^{m} |E^{\gamma}(\tilde{Z}_{ij}) - E^{\gamma}(\tilde{Z}_{hj})|\omega_j^{\gamma}$$

$$\text{s.t.} \begin{cases} \omega_j^{\gamma} \geqslant min\{\omega_j^{d}, \omega_j^{\gamma c}\}, j = 1, \cdots, n \\ \omega_j^{\gamma} \in \Omega^{\gamma}, j = 1, \cdots, n \\ \sum_{j=1}^{n} \omega_j^{\gamma} = 1, j = 1, \cdots, n \end{cases} \tag{M-1}$$

According to the two kinds of attribute weights, the corresponding constraints are also given in model (M-1). The first constraint $\omega_j^{\gamma} \geqslant min\{\omega_j^{d}, \omega_j^{\gamma c}\}$ restricts the lower limit of attribute weight from the perspective of value, and the second constraint $\omega_j^{\gamma} \in \Omega^{\gamma}$ restricts the relationship between attribute weights from the perspective of ranking.

By solving model (M-1) using the Lingo software package, the current synthetic weight of attribute can be obtained. It is worth noting that current weight is not necessarily the final one. We need to use the current weight to recalculate the total ranking of all alternatives under all attributes. If it is the same with the previous ranking result, the current weight is just the final one. Otherwise, we need to update the attribute weight based on the consistency $\omega_j^{\gamma c}$ until the two total ranking results are the same. The synthetic weight ω_j^{γ} obtained at this time is the final weight.

By fusing the importance information of experts/attributes from multi-view, the process of determining the weights of experts/attributes is more comprehensive and objective, and the accuracy and comprehensiveness of calculation results can be effectively improved.

5. A data-driven MAGDM method based on multi-granulation weights and three-way decisions

Based on the idea of multi-granulation weights and three-way decisions, a novel data-driven MAGDM method is proposed to solve the MAGDM problems with completely unknown weights under interval-valued intuitionistic uncertain linguistic environment. In this method, the multi-granulation synthetic expert weights are obtained by integrating two kinds of finest-granulation expert weights based on uncertainty and closeness. By using these weights, the group collective decision matrix is determined. According to the group collective evaluation value of each alternative under all attributes, the set of alternatives is divided into three regions, i.e., acceptance, uncertainty and rejection. Then, based on maximizing deviation and consistency degree, the constraints on attribute weights are established, and the multi-granulation attribute weights are further obtained by solving the optimization model aiming at maximizing all deviation values for all attributes. Finally, the overall evaluation value of each alternative is determined by using the multi-granulation attribute weights and the final ranking result of all alternatives is obtained. According to the overall evaluation value of each alternative, the set of alternatives is divided into three grades, i.e., the set with higher evaluation, the set with medium evaluation and the set with lower evaluation. By fusing the above two types of three-way decision information, the coarse-granulation grading results can be obtained.

The detailed steps of the three-way MAGDM method based on multi-granulation weights are given as follows:

Step 1. Calculate the uncertainty $\phi(\tilde{R}_{ij}^k)$ by using Eq. (6), then by using Eq. (8) the expert finest-granulation weight based on uncertainty λ_{ijk}^{ϕ} is obtained;

Step 2. Determine the PID, L-NID and R-NID of the group for each alternative w.r.t. each attribute. By using Eqs. (7), (9) and (10), the closeness c_{ij}^k between each expert and the group for each alternative w.r.t. each attribute is calculated. Then by using Eq. (11), the expert finest-granulation weight based on closeness λ_{ijk}^c is obtained;

Step 3. Determine the proportion factor η_{ij} by using Eqs. (13) and (14). Then by using Eq. (15), the multi-granulation weight λ_{ijk} of expert is obtained.

Step 4. Aggregate the experts' individual decision matrices $\tilde{R}^k = (\tilde{R}_{ij}^k)_{m\times n}$ $(k = 1, 2, \ldots, p)$ into the group collective decision matrix $\tilde{Z} = (\tilde{Z}_{ij})_{m\times n}$ by using the expert weight λ_{ijk} and IVIULWAA operator (see Definition 4).

Step 5. Determine the three-way grading information of each alternative according to its performance under all attributes.

First, by using Eq. (4), the expected value $E^{\gamma}(\tilde{Z}_{ij})$ can be obtained, based on which the alternatives under each attribute are divided into the following three pair-wise disjoint regions by using three-way decisions idea:

$$
\begin{aligned}
&Region_{+}^{\gamma}(C_j) = \{A_i \in A | I(E^{\gamma}(\tilde{Z}_{ij})) > \tfrac{l}{2} + \delta\},\\
&Region_{o}^{\gamma}(C_j) = \{A_i \in A | \tfrac{l}{2} - \xi \leqslant I(E^{\gamma}(\tilde{Z}_{ij})) \leqslant \tfrac{l}{2} + \delta\},\\
&Region_{-}^{\gamma}(C_j) = \{A_i \in A | I(E^{\gamma}(\tilde{Z}_{ij})) < \tfrac{l}{2} - \xi\},
\end{aligned}
$$

where l is the maximum value of term index in S, $0 \leqslant \delta < 1$, $0 \leqslant \xi < 1$.

Then, the alternatives with higher evaluation value under all attributes are included in the acceptance region DES_{acc}^{γ}; the alternatives with lower evaluation value under all attributes are included in the rejection region DES_{rej}^{γ}; other alternatives are included in the uncertainty region DES_{unc}^{γ}. That is,

$$
\begin{aligned}
&DES_{acc}^{\gamma} = \{A_i \in A | A_i \in Region_{+}^{\gamma}(C_j),\ \text{for } \forall C_j \in C\},\\
&DES_{rej}^{\gamma} = \{A_i \in A | A_i \in Region_{-}^{\gamma}(C_j),\ \text{for } \forall C_j \in C\},\\
&DES_{unc}^{\gamma} = A - DES_{acc}^{\gamma} - DES_{rej}^{\gamma}.
\end{aligned}
$$

Therefore, the preliminary grading information of each alternative under different risk attitudes can be represented by DES_{acc}^{γ}, DES_{rej}^{γ} and DES_{unc}^{γ}.

Step 6. Calculate the attribute weight based on maximizing deviation ω_j^d by using Eq. (16).

Step 7. Based on the group collective decision matrix $\tilde{Z}$, by using the attribute weight ω_j^d and IVIULWAA operator, the overall evaluation value $\tilde{Z}_i$ of each alternative is obtained.

Step 8. Set the risk attitude factor γ.

Step 9. By using Eq. (4), the expected value $E^{\gamma}(\tilde{Z}_i)$ can also be obtained. Based on $E^{\gamma}(\tilde{Z}_{ij})$ and $E^{\gamma}(\tilde{Z}_i)$, the ranking result of all alternatives under each attribute $R^{\gamma j}$ and the one under all attributes $R^{\gamma *}$ are determined accordingly.

Step 10. Calculate the consistency degree $CON(R^{\gamma j}, R^{\gamma *})$ by using Eqs. (18), (21) and (23). Then by using Eq. (24), the attribute weight based on consistency degree $\omega_j^{\gamma c}$ is obtained, and the ranking information of attribute weight Ω^{γ} is determined accordingly.

Step 11. Determine the multi-granulation weight ω_j^{γ} of each attribute by solving the optimization model (M-1).

Step 12. Calculate the overall evaluation value of each alternative $\tilde{Z}_i^{\gamma}$ by using ω_j^{γ} and IVIULWAA operator.

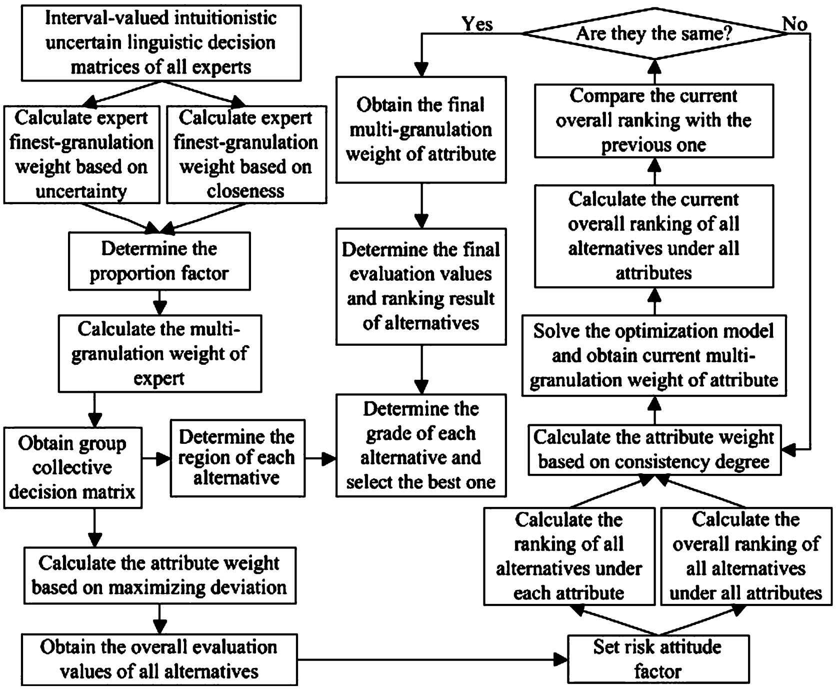

Fig. 1. The schematic illustration of the data-driven MAGDM method based on multi-granulation weights and three-way decisions.

Step 13. Recalculate the expected value $E^\gamma(\tilde{Z}_i^\gamma)$ by using Eq. (4). Determine the current overall ranking result under all attributes and compare it with the previous one. If they are the same, the current weight ω_j^γ is the final weight of attribute; else, update the ranking $R^{\gamma*}$ and go to Step 10.

Step 14. Under the current risk attitude γ, according to the final overall evaluation value of each alternative $\tilde{Z}_i^\gamma$ and its expected value $E^\gamma(\tilde{Z}_i^\gamma)$, the final ranking result of all alternatives is obtained. At the same time, the three-way grading information of the alternatives based on the expected value $E^\gamma(\tilde{Z}_i^\gamma)$ is further obtained as follows.

$$Region_+^\gamma(*) = \{A_i \in A | I(E^\gamma(\tilde{Z}_i^\gamma)) > \tfrac{l}{2} + \delta\},$$
$$Region_o^\gamma(*) = \{A_i \in A | \tfrac{l}{2} - \xi \leqslant I(E^\gamma(\tilde{Z}_i^\gamma)) \leqslant \tfrac{l}{2} + \delta\},$$
$$Region_-^\gamma(*) = \{A_i \in A | I(E^\gamma(\tilde{Z}_i^\gamma)) < \tfrac{l}{2} - \xi\}.$$

Step 15. By fusing the two types of three-way decision information obtained in Step 5 and Step 14, the alternatives can be divided from high to low into five grades, that is, DES_{acc}^γ, $DES_{unc}^\gamma \bigcap Region_+^\gamma(*)$, $DES_{unc}^\gamma \bigcap Region_o^\gamma(*)$, $DES_{unc}^\gamma \bigcap Region_-^\gamma(*)$, and DES_{rej}^γ.

These coarse-granulation grading results and the fine-granulation ranking result provide multi-level and multi-view feedback information for decision makers, which can help them to make flexible choices and reasonable decisions according to practical problems and application requirements.

Fig. 1 illustrates the various steps of the data-driven MAGDM method based on multi-granulation weights and three-way decisions. Fig. 2 shows the acquisition process of coarse-granulation grading information based on three-way decisions.

6. An illustrative example

With the rapid development of e-commerce, more and more people choose online shopping. Logistics transportation is an urgent problem for e-commerce enterprises to solve. Choosing suitable logistics suppliers can effectively improve the business level of enterprises. In this section, the proposed method is applied to the logistics supplier selection in e-commerce enterprises. Suppose there are five logistics distribution providers (A_1, A_2, A_3, A_4, A_5) to choose from, which are to be evaluated by four experts (e_1, e_2, e_3, e_4) under five attributes: C_1–the capability of logistics enterprise, C_2–the quality of logistics service, C_3–the degree of informatization, C_4–the development prospects of enterprises, C_5-the cost of logistics service. The linguistic term set used by the experts is $S = \{s_0 = \text{extrimly poor}, s_1 = \text{very poor}, s_2 = \text{poor}, s_3 = \text{fair}, s_4 = \text{good}, s_5 = \text{very good}, s_6 = \text{extrimly good}\}$. According to their own experience, preference and hesitation, each expert chooses original terms from the linguistic term set to give preference information in the form of IVIULVs, and forms

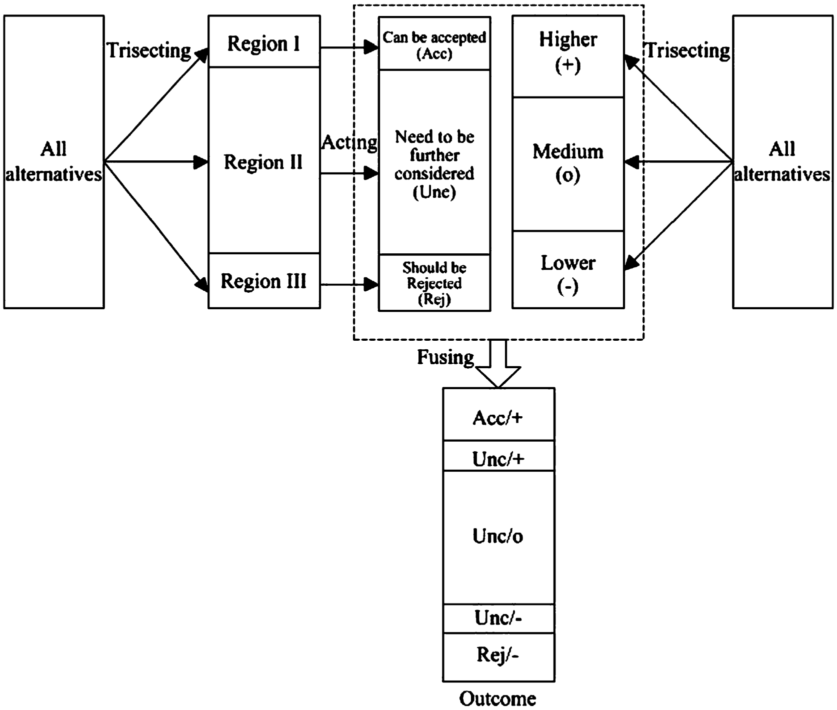

Fig. 2. The schematic illustration of the coarse-granulation grading decision-making process based on three-way decisions.

Table 3
Decision matrix $\tilde{R}^1$.

A	C				
	C_1	C_2	C_3	C_4	C_5
A_1	$\langle[s_4, s_5], [0.5, 0.6], [0.2, 0.3]\rangle$	$\langle[s_4, s_5], [0.5, 0.6], [0.3, 0.4]\rangle$	$\langle[s_5, s_6], [0.8, 0.8], [0.1, 0.1]\rangle$	$\langle[s_4, s_4], [0.8, 0.8], [0.1, 0.1]\rangle$	$\langle[s_5, s_5], [0.7, 0.8], [0.1, 0.2]\rangle$
A_2	$\langle[s_4, s_5], [0.8, 0.8], [0.1, 0.2]\rangle$	$\langle[s_4, s_6], [0.7, 0.8], [0.2, 0.2]\rangle$	$\langle[s_3, s_5], [0.6, 0.7], [0.2, 0.3]\rangle$	$\langle[s_3, s_5], [0.5, 0.6], [0.2, 0.4]\rangle$	$\langle[s_3, s_5], [0.6, 0.8], [0.1, 0.1]\rangle$
A_3	$\langle[s_2, s_4], [0.5, 0.7], [0.2, 0.2]\rangle$	$\langle[s_4, s_4], [0.6, 0.8], [0.1, 0.2]\rangle$	$\langle[s_5, s_5], [0.7, 0.8], [0.1, 0.2]\rangle$	$\langle[s_5, s_5], [0.7, 0.8], [0.1, 0.2]\rangle$	$\langle[s_3, s_4], [0.8, 0.8], [0.1, 0.2]\rangle$
A_4	$\langle[s_4, s_6], [0.6, 0.7], [0.2, 0.3]\rangle$	$\langle[s_2, s_4], [0.5, 0.6], [0.1, 0.2]\rangle$	$\langle[s_3, s_4], [0.5, 0.7], [0.2, 0.3]\rangle$	$\langle[s_3, s_5], [0.7, 0.7], [0.2, 0.3]\rangle$	$\langle[s_5, s_6], [0.6, 0.8], [0.1, 0.2]\rangle$
A_5	$\langle[s_1, s_3], [0.6, 0.7], [0.2, 0.2]\rangle$	$\langle[s_3, s_4], [0.7, 0.8], [0.1, 0.1]\rangle$	$\langle[s_5, s_6], [0.8, 0.9], [0, 0.1]\rangle$	$\langle[s_4, s_5], [0.7, 0.8], [0.1, 0.2]\rangle$	$\langle[s_3, s_4], [0.8, 0.8], [0.1, 0.1]\rangle$

Table 4
Decision matrix $\tilde{R}^2$.

A	C				
	C_1	C_2	C_3	C_4	C_5
A_1	$\langle[s_5, s_5], [0.6, 0.7], [0.1, 0.3]\rangle$	$\langle[s_3, s_4], [0.7, 0.7], [0.1, 0.2]\rangle$	$\langle[s_4, s_5], [0.7, 0.8], [0.1, 0.2]\rangle$	$\langle[s_6, s_6], [0.7, 0.8], [0.1, 0.2]\rangle$	$\langle[s_5, s_6], [0.5, 0.6], [0.3, 0.4]\rangle$
A_2	$\langle[s_4, s_6], [0.5, 0.7], [0.2, 0.3]\rangle$	$\langle[s_4, s_5], [0.6, 0.7], [0.2, 0.3]\rangle$	$\langle[s_5, s_5], [0.8, 0.8], [0.1, 0.2]\rangle$	$\langle[s_5, s_6], [0.6, 0.7], [0.1, 0.2]\rangle$	$\langle[s_4, s_5], [0.7, 0.9], [0.1, 0.1]\rangle$
A_3	$\langle[s_4, s_5], [0.6, 0.7], [0.1, 0.2]\rangle$	$\langle[s_3, s_5], [0.8, 0.9], [0.1, 0.1]\rangle$	$\langle[s_2, s_3], [0.6, 0.6], [0.2, 0.4]\rangle$	$\langle[s_4, s_4], [0.7, 0.7], [0.2, 0.3]\rangle$	$\langle[s_3, s_5], [0.5, 0.6], [0.1, 0.2]\rangle$
A_4	$\langle[s_4, s_5], [0.8, 0.8], [0.1, 0.2]\rangle$	$\langle[s_3, s_4], [0.6, 0.7], [0.2, 0.3]\rangle$	$\langle[s_2, s_3], [0.6, 0.7], [0.1, 0.3]\rangle$	$\langle[s_2, s_4], [0.8, 0.8], [0.1, 0.2]\rangle$	$\langle[s_4, s_4], [0.5, 0.7], [0.2, 0.3]\rangle$
A_5	$\langle[s_2, s_3], [0.7, 0.8], [0.1, 0.2]\rangle$	$\langle[s_3, s_5], [0.6, 0.8], [0.1, 0.2]\rangle$	$\langle[s_4, s_5], [0.7, 0.8], [0.1, 0.2]\rangle$	$\langle[s_4, s_6], [0.8, 0.9], [0.1, 0.1]\rangle$	$\langle[s_4, s_5], [0.7, 0.8], [0.1, 0.2]\rangle$

a decision matrix for each logistics distribution supplier w.r.t. different attributes. The decision matrices from all experts $\tilde{R}^k = (\tilde{R}^k_{ij})_{m\times n}$ $(k = 1, 2, 3, 4)$ are listed in Tables 3–6.

The specific decision-making process is as follows:

Table 5
Decision matrix $\tilde{R}^3$.

A	C				
	C_1	C_2	C_3	C_4	C_5
A_1	$\langle[s_5,s_5],[0.5,0.6],[0.3,0.4]\rangle$	$\langle[s_6,s_6],[0.8,0.9],[0.1,0.1]\rangle$	$\langle[s_5,s_5],[0.8,0.9],[0.1,0.1]\rangle$	$\langle[s_5,s_6],[0.7,0.8],[0.2,0.2]\rangle$	$\langle[s_4,s_4],[0.8,0.8],[0.1,0.2]\rangle$
A_2	$\langle[s_5,s_6],[0.6,0.7],[0.1,0.3]\rangle$	$\langle[s_4,s_6],[0.5,0.7],[0.1,0.3]\rangle$	$\langle[s_5,s_5],[0.6,0.7],[0.2,0.3]\rangle$	$\langle[s_5,s_5],[0.8,0.9],[0.1,0.1]\rangle$	$\langle[s_5,s_5],[0.5,0.7],[0.2,0.3]\rangle$
A_3	$\langle[s_4,s_4],[0.8,0.9],[0.1,0.1]\rangle$	$\langle[s_5,s_5],[0.7,0.8],[0.1,0.2]\rangle$	$\langle[s_2,s_3],[0.7,0.8],[0.1,0.2]\rangle$	$\langle[s_4,s_4],[0.7,0.8],[0.1,0.2]\rangle$	$\langle[s_4,s_4],[0.7,0.8],[0.1,0.2]\rangle$
A_4	$\langle[s_2,s_4],[0.8,0.9],[0.1,0.1]\rangle$	$\langle[s_3,s_4],[0.6,0.8],[0.1,0.2]\rangle$	$\langle[s_4,s_5],[0.6,0.8],[0.1,0.2]\rangle$	$\langle[s_4,s_5],[0.6,0.7],[0.1,0.3]\rangle$	$\langle[s_3,s_4],[0.7,0.8],[0.1,0.2]\rangle$
A_5	$\langle[s_1,s_2],[0.6,0.6],[0.2,0.3]\rangle$	$\langle[s_3,s_3],[0.7,0.8],[0.1,0.2]\rangle$	$\langle[s_5,s_6],[0.8,0.9],[0,0.1]\rangle$	$\langle[s_4,s_5],[0.7,0.8],[0.1,0.2]\rangle$	$\langle[s_3,s_4],[0.8,0.8],[0.1,0.1]\rangle$

Table 6
Decision matrix $\tilde{R}^4$.

A	C				
	C_1	C_2	C_3	C_4	C_5
A_1	$\langle[s_4,s_5],[0.7,0.8],[0.1,0.2]\rangle$	$\langle[s_5,s_5],[0.6,0.6],[0.1,0.3]\rangle$	$\langle[s_5,s_6],[0.8,0.8],[0.1,0.1]\rangle$	$\langle[s_4,s_4],[0.8,0.8],[0.1,0.1]\rangle$	$\langle[s_5,s_5],[0.7,0.8],[0.1,0.2]\rangle$
A_2	$\langle[s_5,s_6],[0.7,0.8],[0.1,0.2]\rangle$	$\langle[s_6,s_6],[0.7,0.8],[0.2,0.2]\rangle$	$\langle[s_4,s_6],[0.8,0.8],[0.2,0.2]\rangle$	$\langle[s_4,s_5],[0.5,0.6],[0.3,0.3]\rangle$	$\langle[s_4,s_5],[0.9,0.9],[0,0.1]\rangle$
A_3	$\langle[s_3,s_4],[0.7,0.7],[0.2,0.2]\rangle$	$\langle[s_4,s_4],[0.7,0.8],[0.1,0.2]\rangle$	$\langle[s_5,s_5],[0.7,0.7],[0.1,0.2]\rangle$	$\langle[s_5,s_5],[0.7,0.8],[0.1,0.2]\rangle$	$\langle[s_3,s_4],[0.8,0.8],[0.1,0.1]\rangle$
A_4	$\langle[s_6,s_6],[0.6,0.7],[0.2,0.3]\rangle$	$\langle[s_3,s_3],[0.5,0.6],[0.2,0.3]\rangle$	$\langle[s_4,s_4],[0.6,0.7],[0.2,0.3]\rangle$	$\langle[s_3,s_4],[0.7,0.7],[0.2,0.2]\rangle$	$\langle[s_5,s_6],[0.6,0.8],[0.2,0.2]\rangle$
A_5	$\langle[s_3,s_4],[0.5,0.7],[0.3,0.3]\rangle$	$\langle[s_4,s_4],[0.6,0.8],[0.1,0.2]\rangle$	$\langle[s_6,s_6],[0.9,0.9],[0,0.1]\rangle$	$\langle[s_5,s_5],[0.8,0.8],[0.1,0.1]\rangle$	$\langle[s_4,s_5],[0.8,0.9],[0.1,0.1]\rangle$

Table 7
The expert finest-granulation weight based on uncertainty λ_{ijk}^{ϕ}.

Experts	Alternatives	Attributes				
		C_1	C_2	C_3	C_4	C_5
e_1	A_1	0.119	0.069	0.106	0.382	0.283
	A_2	0.303	0.085	0.053	0.052	0.103
	A_3	0.054	0.272	0.395	0.255	0.150
	A_4	0.099	0.102	0.147	0.181	0.167
	A_5	0.164	0.113	0.080	0.089	0.259
e_2	A_1	0.405	0.081	0.094	0.191	0.057
	A_2	0.150	0.138	0.569	0.098	0.189
	A_3	0.097	0.047	0.082	0.235	0.077
	A_4	0.195	0.175	0.152	0.189	0.485
	A_5	0.294	0.058	0.074	0.053	0.229
e_3	A_1	0.344	0.591	0.695	0.045	0.377
	A_2	0.261	0.077	0.320	0.757	0.498
	A_3	0.745	0.340	0.091	0.255	0.613
	A_4	0.110	0.178	0.158	0.295	0.177
	A_5	0.278	0.461	0.080	0.089	0.259
e_4	A_1	0.132	0.259	0.106	0.382	0.283
	A_2	0.286	0.701	0.058	0.093	0.210
	A_3	0.104	0.340	0.431	0.255	0.160
	A_4	0.596	0.545	0.543	0.336	0.171
	A_5	0.264	0.368	0.767	0.769	0.253

Step 1. Calculate the expert finest-granulation weight based on uncertainty.

(1) By using Eq. (6), the results of uncertainty $\phi(\tilde{R}_{ij}^k)$ are obtained.

(2) By using Eq. (8), the results of expert finest-granulation weight based on uncertainty λ_{ijk}^{ϕ} are obtained and shown in Table 7.

Step 2. Calculate the expert finest-granulation weight based on closeness.

Table 8
The expert finest-granulation weight based on closeness λ^c_{ijk}.

Experts	Alternatives	Attributes				
		C_1	C_2	C_3	C_4	C_5
e_1	A_1	0.246	0.251	0.259	0.254	0.286
	A_2	0.255	0.289	0.215	0.242	0.238
	A_3	0.215	0.267	0.235	0.252	0.280
	A_4	0.273	0.225	0.265	0.269	0.258
	A_5	0.282	0.288	0.303	0.289	0.245
e_2	A_1	0.244	0.251	0.187	0.224	0.212
	A_2	0.229	0.229	0.251	0.249	0.285
	A_3	0.249	0.230	0.244	0.225	0.239
	A_4	0.272	0.313	0.220	0.235	0.235
	A_5	0.257	0.241	0.182	0.202	0.286
e_3	A_1	0.265	0.204	0.295	0.267	0.216
	A_2	0.267	0.286	0.298	0.238	0.233
	A_3	0.247	0.231	0.278	0.272	0.220
	A_4	0.221	0.246	0.216	0.229	0.241
	A_5	0.232	0.230	0.303	0.289	0.245
e_4	A_1	0.245	0.295	0.259	0.254	0.286
	A_2	0.248	0.196	0.236	0.271	0.243
	A_3	0.288	0.272	0.242	0.252	0.262
	A_4	0.235	0.216	0.298	0.267	0.266
	A_5	0.229	0.241	0.213	0.221	0.225

Table 9
The proportion factor η_{ij}.

Alternatives	Attributes				
	C_1	C_2	C_3	C_4	C_5
A_1	0.995	0.976	0.971	0.989	0.931
A_2	0.954	0.973	0.981	0.998	0.980
A_3	0.990	0.980	0.991	0.209	0.987
A_4	0.986	0.950	0.956	0.932	0.990
A_5	0.862	0.985	0.961	0.979	0.244

(1) Calculate the PID of the group for each alternative w.r.t. each attribute by using Eq. (9). Determine the L-NID and R-NID of the group for each alternative w.r.t. each attribute.
(2) By using Eq. (7), the Euclidean distance between each expert and the PID of the group, the Euclidean distance between each expert and the L-NID of the group, and the Euclidean distance between each expert and the R-NID of the group are obtained.
(3) By using Eq. (10), the results of closeness between each expert and the group are obtained.
(4) By using Eq. (11), the results of expert finest-granulation weight based on closeness λ^c_{ijk} are obtained and shown in Table 8.

Step 3. Determine the multi-granulation weights of experts.

(1) By using Eqs. (13) and (14), the proportion factors are obtained and shown in Table 9.
(2) By using Eq. (15), the multi-granulation weights of experts are obtained and listed in Table 10. Fig. 3 shows the multi-granulation weights of each expert under each attribute for different alternatives visually.

Step 4. By using the experts' multi-granulation weights and IVIULWAA operator, the collective decision matrix is obtained and shown in Table 11.

Step 5. By using Eq. (4), the expected values $E^\gamma(\tilde{Z}_{ij})$ under different risk attitude factors are obtained and listed in Table 12. Let $\delta = 0.5$, $\xi = 0.5$, due to $l = 6$, according to the expected values we can get the following three regions:

$$Region^\gamma_+(C_j) = \{A_i \in A | I(E^\gamma(\tilde{Z}_{ij})) > 3.5\},$$
$$Region^\gamma_o(C_j) = \{A_i \in A | 2.5 \leqslant I(E^\gamma(\tilde{Z}_{ij})) \leqslant 3.5\},$$
$$Region^\gamma_-(C_j) = \{A_i \in A | I(E^\gamma(\tilde{Z}_{ij})) < 2.5\}.$$

Table 10
The multi-granulation weights of experts λ_{ijk}.

Experts	Alternatives	Attributes				
		C_1	C_2	C_3	C_4	C_5
e_1	A_1	0.119	0.073	0.110	0.381	0.283
	A_2	0.301	0.090	0.056	0.053	0.106
	A_3	0.056	0.272	0.394	0.252	0.152
	A_4	0.102	0.108	0.152	0.187	0.168
	A_5	0.181	0.115	0.088	0.093	0.248
e_2	A_1	0.404	0.085	0.096	0.191	0.067
	A_2	0.154	0.140	0.563	0.098	0.191
	A_3	0.098	0.051	0.083	0.227	0.079
	A_4	0.196	0.182	0.155	0.192	0.482
	A_5	0.289	0.061	0.078	0.056	0.272
e_3	A_1	0.344	0.582	0.683	0.047	0.366
	A_2	0.261	0.083	0.319	0.755	0.492
	A_3	0.740	0.338	0.093	0.268	0.608
	A_4	0.111	0.182	0.160	0.290	0.178
	A_5	0.272	0.457	0.088	0.093	0.248
e_4	A_1	0.133	0.260	0.110	0.381	0.283
	A_2	0.284	0.687	0.062	0.093	0.211
	A_3	0.106	0.339	0.430	0.252	0.161
	A_4	0.591	0.528	0.532	0.331	0.172
	A_5	0.259	0.367	0.745	0.758	0.232

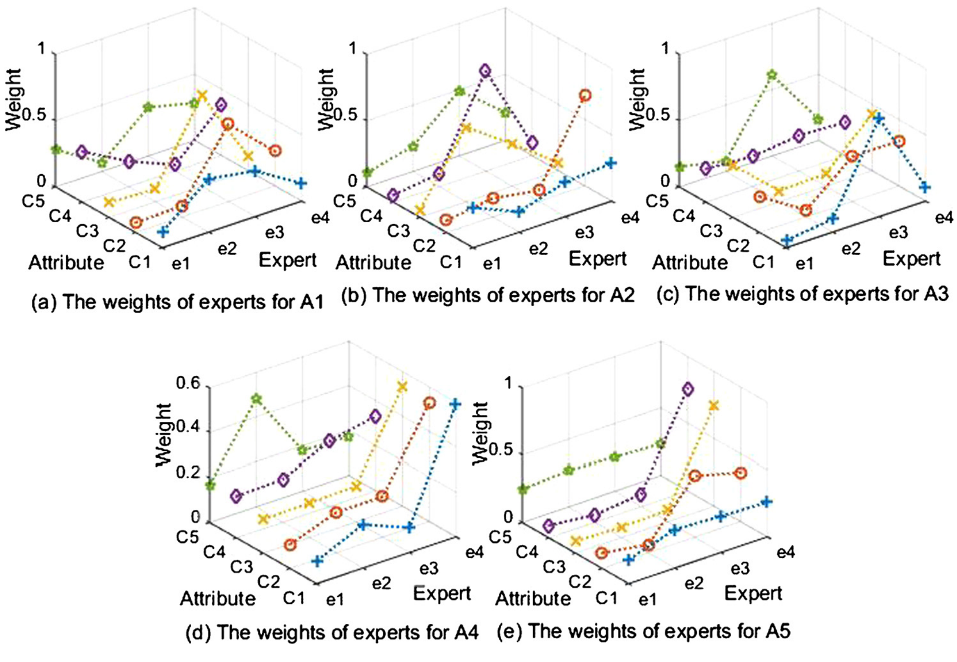

Fig. 3. The multi-granulation weights of each expert w.r.t. each attribute for different alternative.

Then the preliminary grading information DES^{γ}_{acc}, DES^{γ}_{rej} and DES^{γ}_{unc} of each alternative under different risk attitude factors are obtained and listed in Table 12.

Step 6. By using Eq. (16), the objective weights of attributes based on maximizing deviations are obtained, $\omega^d_1 = 0.229$, $\omega^d_2 = 0.258$, $\omega^d_3 = 0.259$, $\omega^d_4 = 0.149$, $\omega^d_5 = 0.105$.

Step 7. By using ω^d_j and IVIULWAA operator, the overall evaluation values of all alternatives are obtained, $\tilde{Z}_1 = \langle[s_{4.882}, s_{5.076}], [0.73, 0.808], [0.115, 0.164]\rangle$, $\tilde{Z}_2 = \langle[s_{4.854}, s_{5.413}], [0.71, 0.793], [0, 0.204]\rangle$, $\tilde{Z}_3 = \langle[s_{4.181}, s_{4.374}], [0.713, 0.804], [0.107, 0.179]\rangle$, $\tilde{Z}_4 = \langle[s_{3.696}, s_{4.369}], [0.615, 0.72], [0.159, 0.259]\rangle$, $\tilde{Z}_5 = \langle[s_{3.828}, s_{4.375}], [0.759, 0.82], [0, 0.152]\rangle$.

Table 11
The collective decision matrix.

A	C				
	C_1	C_2	C_3	C_4	C_5
A_1	$\langle[s_{4.748}, s_5]$, $[0.573, 0.675]$, $[0.158, 0.314]\rangle$	$\langle[s_{5.338}, s_{5.497}]$, $[0.735, 0.826]$, $[0.108, 0.156]\rangle$	$\langle[s_{4.904}, s_{5.22}]$, $[0.792, 0.875]$, $[0.1, 0.107]\rangle$	$\langle[s_{4.43}, s_{4.477}]$, $[0.78, 0.8]$, $[0.103, 0.118]\rangle$	$\langle[s_{4.634}, s_{4.701}]$, $[0.732, 0.79]$, $[0.108, 0.21]\rangle$
A_2	$\langle[s_{4.545}, s_{5.699}]$, $[0.69, 0.763]$, $[0.111, 0.237]\rangle$	$\langle[s_{5.374}, s_{5.86}]$, $[0.674, 0.781]$, $[0.189, 0.219]\rangle$	$\langle[s_{4.826}, s_{5.062}]$, $[0.74, 0.767]$, $[0.135, 0.233]\rangle$	$\langle[s_{4.801}, s_{5.098}]$, $[0.755, 0.864]$, $[0.115, 0.128]\rangle$	$\langle[s_{4.387}, s_5]$, $[0.685, 0.815]$, $[0, 0.172]\rangle$
A_3	$\langle[s_{3.782}, s_{4.098}]$, $[0.765, 0.867]$, $[0.112, 0.120]\rangle$	$\langle[s_{4.288}, s_{4.389}]$, $[0.682, 0.807]$, $[0.1, 0.193]\rangle$	$\langle[s_{4.471}, s_{4.647}]$, $[0.693, 0.748]$, $[0.106, 0.212]\rangle$	$\langle[s_{4.505}, s_{4.505}]$, $[0.7, 0.781]$, $[0.117, 0.219]\rangle$	$\langle[s_{3.608}, s_{4.079}]$, $[0.725, 0.789]$, $[0.1, 0.179]\rangle$
A_4	$\langle[s_{4.96}, s_{5.582}]$, $[0.677, 0.755]$, $[0.162, 0.245]\rangle$	$\langle[s_{2.892}, s_{3.472}]$, $[0.539, 0.665]$, $[0.164, 0.267]\rangle$	$\langle[s_{3.537}, s_{4.005}]$, $[0.586, 0.719]$, $[0.161, 0.281]\rangle$	$\langle[s_{3.098}, s_{4.477}]$, $[0.698, 0.722]$, $[0.143, 0.243]\rangle$	$\langle[s_{4.162}, s_{4.68}]$, $[0.577, 0.757]$, $[0.157, 0.243]\rangle$
A_5	$\langle[s_{1.807}, s_{2.987}]$, $[0.61, 0.711]$, $[0.182, 0.248]\rangle$	$\langle[s_{3.367}, s_{3.604}]$, $[0.661, 0.8]$, $[0.1, 0.185]\rangle$	$\langle[s_{5.667}, s_{5.922}]$, $[0.877, 0.894]$, $[0, 0.106]\rangle$	$\langle[s_{4.758}, s_{5.056}]$, $[0.784, 0.808]$, $[0.1, 0.114]\rangle$	$\langle[s_{3.504}, s_{4.504}]$, $[0.777, 0.83]$, $[0.1, 0.121]\rangle$

Step 8. By using Eq. (4), the expected value $E^\gamma(\tilde{Z}_i)$ under different risk attitude factors are obtained. Based on the expected values, the ranking results of all alternatives under different risk attitude factors are determined and shown in Table 13.

Step 9. By using Eqs. (18), (21), (23) and (24), the attribute weights based on the consistency degree under different risk attitude factors are obtained and shown in Table 14. And the ranking information of attribute weight is determined, e.g. $\Omega^{0.4} = \{\omega_4^{0.4} > \omega_5^{0.4} > \omega_2^{0.4} > \omega_3^{0.4} > \omega_1^{0.4}\}$.

Taking the case of $\gamma = 0.4$ in Table 13 for example, the dominance classes of alternatives w.r.t. attribute C_1 can be obtained by using Eq. (18):

$$([A_1]^{0.4C_1})^{\geqslant} = \{A_1, A_2, A_4\}, \qquad ([A_2]^{0.4C_1})^{\geqslant} = \{A_2, A_4\},$$
$$([A_3]^{0.4C_1})^{\geqslant} = \{A_1, A_2, A_3, A_4\}, \qquad ([A_4]^{0.4C_1})^{\geqslant} = \{A_4\},$$
$$([A_5]^{0.4C_1})^{\geqslant} = \{A_1, A_2, A_3, A_4, A_5\}.$$

The dominance classes of alternatives based on overall ranking can be obtained by using Eq. (21):

$([A_1]^{0.4*})^{\geqslant} = \{A_1, A_2\}$, $([A_2]^{0.4*})^{\geqslant} = \{A_2\}$, $([A_3]^{0.4*})^{\geqslant} = \{A_1, A_2, A_3, A_5\}$, $([A_4]^{0.4*})^{\geqslant} = \{A_1, A_2, A_3, A_4, A_5\}$, $([A_5]^{0.4*})^{\geqslant} = \{A_1, A_2, A_5\}$.

Then by using Eq. (23), the consistency degree between $R^{0.4C_1}$ and $R^{0.4*}$ can be obtained:

$$CON(R^{0.4C_1}, R^{0.4*}) = \frac{\log_2\frac{5}{3} + \log_2\frac{5}{2} + \log_2\frac{5}{5} + \log_2\frac{5}{5} + \log_2\frac{5}{5}}{\log_2\frac{5}{2} + \log_2\frac{5}{1} + \log_2\frac{5}{3} + \log_2\frac{5}{1} + \log_2\frac{5}{3}} = 0.277$$

Similarly, the consistency degree of all attributes can be obtained. Finally, by using Eq. (24), the attribute weight $\omega_j^{\gamma c}$ can be determined.

Step 10. The multi-granulation weights of attributes under different risk attitude factors are determined by solving the optimization model (M-1) and shown in Table 15. Fig. 4 shows the relationship between attribute weights and the impact of risk attitude factors on the weights visually.

Step 11. By using the multi-granulation weights of attributes listed in Table 15 and IVIULWAA operator, the current overall evaluation values of all alternatives under different risk attitude factors are calculated and shown in Table 16.

Step 12. Determine the final multi-granulation weights of attributes.

(1) Recalculate the expected values of current overall evaluation values by using Eq. (4), and obtain the collective ranking results of alternatives under different risk attitude factors, which are also listed in Table 16.
(2) Since the ranking results in Table 16 are the same as the overall rankings in Table 13, the calculation results in Table 15 are the final weights of attributes, and the expected values and ranking results in Table 16 are the final decision results.

Step 13. Based on the final expected values $E^\gamma(\tilde{Z}_i^\gamma)$, the grading information of the alternatives under all risk attitudes are obtained according to the following grading standards.

Table 12
The expected values and grading information under different risk attitude factors.

Factor	Alternatives	Attributes					Grade
		C_1	C_2	C_3	C_4	C_5	
0	A_1	$s_{2.989}$,-	$s_{4.214}$,+	$s_{4.132}$,+	$s_{3.681}$,+	$s_{3.528}$,+	Unc
	A_2	$s_{3.304}$,o	$s_{3.91}$,+	$s_{3.638}$,+	$s_{3.907}$,+	$s_{3.318}$,o	Unc
	A_3	$s_{3.111}$,o	$s_{3.192}$,o	$s_{3.311}$,o	$s_{3.335}$,o	$s_{2.789}$,o	Unc
	A_4	$s_{3.55}$,+	$s_{1.839}$,-	$s_{2.308}$,-	$s_{2.255}$,-	$s_{2.775}$,o	Unc
	A_5	$s_{1.23}$,-	$s_{2.485}$,-	$s_{5.019}$,+	$s_{3.974}$,+	$s_{2.901}$,o	Unc
0.1	A_1	$s_{3.067}$,o	$s_{4.263}$,+	$s_{4.181}$,+	$s_{3.692}$,+	$s_{3.57}$,+	Unc
	A_2	$s_{3.434}$,o	$s_{3.983}$,+	$s_{3.685}$,+	$s_{3.96}$,+	$s_{3.432}$,o	Unc
	A_3	$s_{3.158}$,o	$s_{3.247}$,o	$s_{3.36}$,o	$s_{3.376}$,o	$s_{2.852}$,o	Unc
	A_4	$s_{3.635}$,+	$s_{1.91}$,-	$s_{2.384}$,-	$s_{2.375}$,-	$s_{2.866}$,o	Unc
	A_5	$s_{1.327}$,-	$s_{2.54}$,o	$s_{5.077}$,+	$s_{4.008}$,+	$s_{2.977}$,o	Unc
0.2	A_1	$s_{3.145}$,o	$s_{4.313}$,+	$s_{4.23}$,+	$s_{3.704}$,+	$s_{3.613}$,+	Unc
	A_2	$s_{3.566}$,+	$s_{4.056}$,+	$s_{3.734}$,+	$s_{4.014}$,+	$s_{3.547}$,+	Acc
	A_3	$s_{3.205}$,o	$s_{3.301}$,o	$s_{3.409}$,o	$s_{3.417}$,o	$s_{2.915}$,o	Unc
	A_4	$s_{3.721}$,+	$s_{1.982}$,-	$s_{2.461}$,-	$s_{2.497}$,-	$s_{2.958}$,o	Unc
	A_5	$s_{1.425}$,-	$s_{2.596}$,o	$s_{5.135}$,+	$s_{4.042}$,+	$s_{3.094}$,o	Unc
0.3	A_1	$s_{3.223}$,o	$s_{4.363}$,+	$s_{4.28}$,+	$s_{3.716}$,+	$s_{3.655}$,+	Unc
	A_2	$s_{3.701}$,+	$s_{4.13}$,+	$s_{3.782}$,+	$s_{4.068}$,+	$s_{3.665}$,+	Acc
	A_3	$s_{3.253}$,o	$s_{3.356}$,o	$s_{3.459}$,o	$s_{3.459}$,o	$s_{2.979}$,o	Unc
	A_4	$s_{3.808}$,+	$s_{2.056}$,-	$s_{2.539}$,o	$s_{2.621}$,o	$s_{3.051}$,o	Unc
	A_5	$s_{1.526}$,-	$s_{2.653}$,o	$s_{5.193}$,+	$s_{4.076}$,+	$s_{3.191}$,o	Unc
0.4	A_1	$s_{3.303}$,o	$s_{4.414}$,+	$s_{4.329}$,+	$s_{3.728}$,+	$s_{3.698}$,+	Unc
	A_2	$s_{3.838}$,+	$s_{4.204}$,+	$s_{3.831}$,+	$s_{4.123}$,+	$s_{3.784}$,+	Acc
	A_3	$s_{3.301}$,o	$s_{3.411}$,o	$s_{3.509}$,+	$s_{3.5}$,o	$s_{3.043}$,o	Unc
	A_4	$s_{3.896}$,+	$s_{2.13}$,-	$s_{2.619}$,o	$s_{2.747}$,o	$s_{3.146}$,o	Unc
	A_5	$s_{1.628}$,-	$s_{2.71}$,o	$s_{5.252}$,+	$s_{4.11}$,+	$s_{3.29}$,o	Unc
0.5	A_1	$s_{3.383}$,o	$s_{4.464}$,+	$s_{4.379}$,+	$s_{3.739}$,+	$s_{3.74}$,+	Unc
	A_2	$s_{3.977}$,+	$s_{4.279}$,+	$s_{3.88}$,+	$s_{4.178}$,+	$s_{3.905}$,+	Acc
	A_3	$s_{3.349}$,o	$s_{3.466}$,o	$s_{3.559}$,+	$s_{3.541}$,+	$s_{3.108}$,o	Unc
	A_4	$s_{3.985}$,+	$s_{2.206}$,-	$s_{2.699}$,o	$s_{2.874}$,o	$s_{3.242}$,o	Unc
	A_5	$s_{1.733}$,-	$s_{2.767}$,o	$s_{5.311}$,+	$s_{4.144}$,+	$s_{3.389}$,o	Unc
0.6	A_1	$s_{3.463}$,o	$s_{4.515}$,+	$s_{4.43}$,+	$s_{3.751}$,+	$s_{3.783}$,+	Unc
	A_2	$s_{4.119}$,+	$s_{4.355}$,+	$s_{3.929}$,+	$s_{4.233}$,+	$s_{4.028}$,+	Acc
	A_3	$s_{3.398}$,o	$s_{3.522}$,+	$s_{3.61}$,+	$s_{3.582}$,+	$s_{3.174}$,o	Unc
	A_4	$s_{4.076}$,+	$s_{2.284}$,-	$s_{2.781}$,o	$s_{3.003}$,o	$s_{3.339}$,o	Unc
	A_5	$s_{1.839}$,-	$s_{2.825}$,o	$s_{5.37}$,+	$s_{4.179}$,+	$s_{3.489}$,o	Unc
0.7	A_1	$s_{3.544}$,+	$s_{4.566}$,+	$s_{4.481}$,+	$s_{3.763}$,+	$s_{3.826}$,+	Acc
	A_2	$s_{4.262}$,+	$s_{4.432}$,+	$s_{3.979}$,+	$s_{4.289}$,+	$s_{4.153}$,+	Acc
	A_3	$s_{3.447}$,o	$s_{3.577}$,+	$s_{3.661}$,+	$s_{3.624}$,+	$s_{3.241}$,o	Unc
	A_4	$s_{4.167}$,+	$s_{2.363}$,-	$s_{2.864}$,o	$s_{3.133}$,o	$s_{3.438}$,o	Unc
	A_5	$s_{1.948}$,-	$s_{2.884}$,o	$s_{5.429}$,+	$s_{4.213}$,+	$s_{3.589}$,+	Unc
0.8	A_1	$s_{3.626}$,+	$s_{4.617}$,+	$s_{4.531}$,+	$s_{3.775}$,+	$s_{3.869}$,+	Acc
	A_2	$s_{4.409}$,+	$s_{4.509}$,+	$s_{4.029}$,+	$s_{4.345}$,+	$s_{4.279}$,+	Acc
	A_3	$s_{3.496}$,o	$s_{3.633}$,+	$s_{3.712}$,+	$s_{3.665}$,+	$s_{3.308}$,o	Unc
	A_4	$s_{4.259}$,+	$s_{2.443}$,-	$s_{2.948}$,o	$s_{3.265}$,o	$s_{3.538}$,o	Unc
	A_5	$s_{2.058}$,-	$s_{2.943}$,o	$s_{5.489}$,+	$s_{4.248}$,+	$s_{3.69}$,+	Unc
0.9	A_1	$s_{3.709}$,+	$s_{4.668}$,+	$s_{4.583}$,+	$s_{3.786}$,+	$s_{3.912}$,+	Acc
	A_2	$s_{4.557}$,+	$s_{4.587}$,+	$s_{4.079}$,+	$s_{4.401}$,+	$s_{4.408}$,+	Acc
	A_3	$s_{3.546}$,+	$s_{3.689}$,+	$s_{3.763}$,+	$s_{3.706}$,+	$s_{3.376}$,o	Unc
	A_4	$s_{4.352}$,+	$s_{2.524}$,o	$s_{3.034}$,o	$s_{3.399}$,o	$s_{3.64}$,+	Unc
	A_5	$s_{2.17}$,-	$s_{3.003}$,o	$s_{5.549}$,+	$s_{4.282}$,+	$s_{3.792}$,+	Unc
1	A_1	$s_{3.792}$,+	$s_{4.72}$,+	$s_{4.634}$,+	$s_{3.798}$,+	$s_{3.955}$,+	Acc
	A_2	$s_{4.708}$,+	$s_{4.665}$,+	$s_{4.129}$,+	$s_{4.458}$,+	$s_{4.538}$,+	Acc
	A_3	$s_{3.596}$,+	$s_{3.746}$,+	$s_{3.815}$,+	$s_{3.747}$,+	$s_{3.444}$,o	Unc
	A_4	$s_{4.446}$,+	$s_{2.607}$,o	$s_{3.12}$,o	$s_{3.535}$,+	$s_{3.742}$,+	Unc
	A_5	$s_{2.285}$,-	$s_{3.063}$,o	$s_{5.61}$,+	$s_{4.317}$,+	$s_{3.895}$,+	Unc

For simplicity and clarity, we use + for $Region_{+}^{\gamma}(C_j)$, o for $Region_{o}^{\gamma}(C_j)$, - for $Region_{-}^{\gamma}(C_j)$, Acc for DES_{acc}^{γ}, and Unc for DES_{unc}^{γ} in this table.

Table 13
The ranking results under different risk attitude factors.

Factor	Alternatives	Overall ranking	Attributes				
			C_1	C_2	C_3	C_4	C_5
0	A_1	1	4	1	2	3	1
	A_2	2	2	2	3	2	2
	A_3	3	3	3	4	4	4
	A_4	5	1	5	5	5	5
	A_5	4	5	4	1	1	3
0.1	A_1	1	4	1	2	3	1
	A_2	2	2	2	3	2	2
	A_3	3	3	3	4	4	5
	A_4	5	1	5	5	5	4
	A_5	4	5	4	1	1	3
0.2	A_1	1	4	1	2	3	1
	A_2	2	2	2	3	2	2
	A_3	3	3	3	4	4	5
	A_4	5	1	5	5	5	4
	A_5	4	5	4	1	1	3
0.3	A_1	2	4	1	2	3	2
	A_2	1	2	2	3	2	1
	A_3	3	3	3	4	4	5
	A_4	5	1	5	5	5	4
	A_5	4	5	4	1	1	3
0.4	A_1	2	3	1	2	3	2
	A_2	1	2	2	3	1	1
	A_3	4	4	3	4	4	5
	A_4	5	1	5	5	5	4
	A_5	3	5	4	1	2	3
0.5	A_1	2	3	1	2	3	2
	A_2	1	2	2	3	1	1
	A_3	4	4	3	4	4	5
	A_4	5	1	5	5	5	4
	A_5	3	5	4	1	2	3
[0.6, 1]	A_1	2	3	1	2	3	2
	A_2	1	1	2	3	1	1
	A_3	4	4	3	4	4	5
	A_4	5	2	5	5	5	4
	A_5	3	5	4	1	2	3

Table 14
The weights of attributes based on the consistency degree under different risk attitude factors.

Factor	C				
	C_1	C_2	C_3	C_4	C_5
0	0.060	0.374	0.151	0.103	0.313
0.1	0.062	0.389	0.157	0.107	0.284
0.2	0.062	0.389	0.157	0.107	0.284
0.3	0.122	0.306	0.129	0.19	0.253
0.4	0.107	0.208	0.134	0.301	0.251
0.5	0.107	0.208	0.134	0.301	0.251
[0.6, 1]	0.170	0.193	0.125	0.297	0.233

$$Region_{+}^{\gamma}(*) = \{A_i \in A | I(E^{\gamma}(\tilde{Z}_i^{\gamma})) > 3.5\},$$
$$Region_{o}^{\gamma}(*) = \{A_i \in A | 2.5 \leqslant I(E^{\gamma}(\tilde{Z}_i^{\gamma})) \leqslant 3.5\},$$
$$Region_{-}^{\gamma}(*) = \{A_i \in A | I(E^{\gamma}(\tilde{Z}_i^{\gamma})) < 2.5\}.$$

To facilitate comparison and analysis, the above grading information and those obtained in Step 5 are listed in Table 16 at the same time. According to these coarse-granulation grading information, we can make the following decisions: when $\gamma < 0.2$ or $\gamma \geqslant 0.7$, A_1 and A_2 are two better choices; while when $0.2 \leqslant \gamma \leqslant 0.6$, A_2 is the best choice. From the perspective of ranking results, we can make the following decisions: when $\gamma < 0.3$, A_1 is the best choice; while when $\gamma \geqslant 0.3$, A_2 is the best choice. Fig. 5 shows the expected values of alternatives under different risk attitudes and the impact of risk attitudes on the ranking of all alternatives visually. It can be seen that risk attitude does have an impact on decision-making results.

Table 15
The multi-granulation weights of attributes under different risk attitude factors.

Factor	C				
	C_1	C_2	C_3	C_4	C_5
0	0.060	0.460	0.160	0.110	0.210
0.1	0.062	0.452	0.162	0.112	0.212
0.2	0.062	0.452	0.162	0.112	0.212
0.3	0.122	0.332	0.132	0.182	0.232
0.4	0.108	0.208	0.178	0.268	0.238
0.5	0.108	0.208	0.158	0.288	0.238
0.6	0.176	0.196	0.126	0.276	0.226
0.7	0.176	0.196	0.126	0.276	0.226
0.8	0.170	0.193	0.141	0.273	0.223
0.9	0.176	0.196	0.126	0.276	0.226
1.0	0.176	0.196	0.126	0.276	0.226

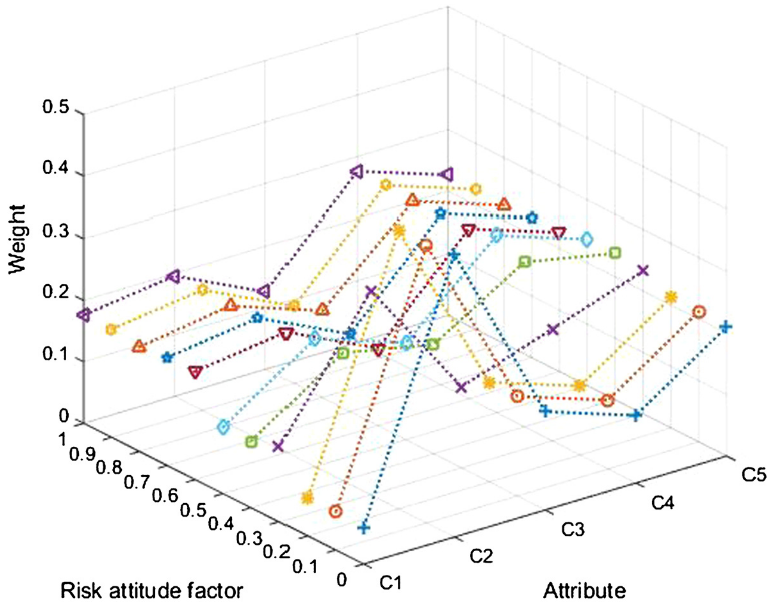

Fig. 4. The multi-granulation weights of attributes under different risk attitude factors.

Decision makers can make appropriate decisions according to specific problems by combining grading information and ranking results.

7. Comparative analysis

This section will illustrate the advantages and characteristics of the proposed method by analyzing and comparing the relationship and differences between the proposed method and other relevant methods in the following aspects.

(1) From the perspective of experts' weights, the weight information obtained by the proposed method is multi-granulation and finest-granulation, which is more flexible, more comprehensive and more practical than that obtained by other MAGDM methods.

Firstly, the experts' weights in the proposed method are different under different attributes w.r.t. different alternatives, that is to say, they are finest-granulation, which makes the method can be used to solve incomplete or even heterogeneous MAGDM problems. There are three specific situations in the process of aggregating group evaluation information: for the case that only one expert gives evaluation information for one alternative under one attribute, the weight of the expert under the attribute w.r.t. the alternative can be set to 1, that is, the individual evaluation value of the expert is taken as the collective evaluation value of the group; if there are two experts evaluating the performance of one alternative under one attribute, we only need to calculate the weights of experts based on uncertainty; when three or more experts evaluate the performance of one alternative under one attribute, the method proposed in this paper is used to calculate the weights of experts. In comparison, the MAGDM methods based on coarse or finer granulation weights cannot deal with the above situations directly.

Table 16
The overall evaluation values, expected values, ranking results and grading information under different risk attitude factors.

Factor	A	$\tilde{Z}_i^\gamma$	$E^\gamma(\tilde{Z}_i^\gamma)$	Ranking results	Grade
0	A_1	$\langle[s_{4.985}, s_{5.143}], [0.742, 0.819], [0.109, 0.158]\rangle$	$s_{3.949}$	1	Unc/+
	A_2	$\langle[s_{4.966}, s_{5.458}], [0.699, 0.796], [0.000, 0.199]\rangle$	$s_{3.724}$	2	Unc/+
	A_3	$\langle[s_{4.168}, s_{4.360}], [0.701, 0.796], [0.103, 0.190]\rangle$	$s_{3.148}$	3	Unc/o
	A_4	$\langle[s_{3.409}, s_{4.048}], [0.584, 0.707], [0.159, 0.260]\rangle$	$s_{2.414}$	5	Unc/-
	A_5	$\langle[s_{3.823}, s_{4.286}], [0.747, 0.822], [0.000, 0.149]\rangle$	$s_{3.054}$	4	Unc/o
0.1	A_1	$\langle[s_{4.980}, s_{5.138}], [0.742, 0.819], [0.109, 0.158]\rangle$	$s_{3.989}$	1	Unc/+
	A_2	$\langle[s_{4.960}, s_{5.453}], [0.699, 0.797], [0.000, 0.199]\rangle$	$s_{3.832}$	2	Unc/+
	A_3	$\langle[s_{4.166}, s_{4.360}], [0.701, 0.796], [0.103, 0.190]\rangle$	$s_{3.201}$	3	Unc/o
	A_4	$\langle[s_{3.417}, s_{4.058}], [0.585, 0.708], [0.159, 0.260]\rangle$	$s_{2.346}$	5	Unc/-
	A_5	$\langle[s_{3.827}, s_{4.295}], [0.747, 0.822], [0.000, 0.149]\rangle$	$s_{3.140}$	4	Unc/o
0.2	A_1	$\langle[s_{4.980}, s_{5.138}], [0.742, 0.819], [0.109, 0.158]\rangle$	$s_{4.033}$	1	Unc/+
	A_2	$\langle[s_{4.960}, s_{5.453}], [0.699, 0.797], [0.000, 0.199]\rangle$	$s_{3.945}$	2	Acc/+
	A_3	$\langle[s_{4.166}, s_{4.360}], [0.701, 0.96], [0.103, 0.190]\rangle$	$s_{3.254}$	3	Unc/o
	A_4	$\langle[s_{3.417}, s_{4.058}], [0.585, 0.708], [0.158, 0.260]\rangle$	$s_{2.428}$	5	Unc/-
	A_5	$\langle[s_{3.827}, s_{4.295}], [0.747, 0.822], [0.000, 0.149]\rangle$	$s_{3.222}$	4	Unc/o
0.3	A_1	$\langle[s_{4.880}, s_{5.029}], [0.736, 0.808], [0.111, 0.165]\rangle$	$s_{3.962}$	2	Unc/+
	A_2	$\langle[s_{4.867}, s_{5.397}], [0.704, 0.803], [0.000, 0.191]\rangle$	$s_{4.021}$	1	Acc/+
	A_3	$\langle[s_{4.132}, s_{4.337}], [0.708, 0.800], [0.105, 0.185]\rangle$	$s_{3.301}$	3	Unc/o
	A_4	$\langle[s_{3.562}, s_{4.263}], [0.605, 0.717], [0.158, 0.256]\rangle$	$s_{2.664}$	5	Unc/o
	A_5	$\langle[s_{3.765}, s_{4.308}], [0.748, 0.816], [0.000, 0.147]\rangle$	$s_{3.270}$	4	Unc/o
0.4	A_1	$\langle[s_{4.786}, s_{4.931}], [0.745, 0.810], [0.110, 0.157]\rangle$	$s_{3.955}$	2	Unc/+
	A_2	$\langle[s_{4.786}, s_{5.292}], [0.714, 0.811], [0.000, 0.182]\rangle$	$s_{4.105}$	1	Acc/+
	A_3	$\langle[s_{4.162}, s_{4.361}], [0.709, 0.794], [0.107, 0.189]\rangle$	$s_{3.365}$	4	Unc/o
	A_4	$\langle[s_{3.588}, s_{4.351}], [0.619, 0.723], [0.156, 0.254]\rangle$	$s_{2.815}$	5	Unc/o
	A_5	$\langle[s_{4.013}, s_{4.553}], [0.769, 0.823], [0.000, 0.137]\rangle$	$s_{3.613}$	3	Unc/+
0.5	A_1	$\langle[s_{4.777}, s_{4.916}], [0.745, 0.808], [0.110, 0.157]\rangle$	$s_{3.981}$	2	Unc/+
	A_2	$\langle[s_{4.798}, s_{5.292}], [0.714, 0.813], [0.000, 0.180]\rangle$	$s_{4.222}$	1	Acc/+
	A_3	$\langle[s_{4.163}, s_{4.358}], [0.709, 0.795], [0.107, 0.190]\rangle$	$s_{3.416}$	4	Unc/o
	A_4	$\langle[s_{3.579}, s_{4.361}], [0.622, 0.724], [0.155, 0.254]\rangle$	$s_{2.914}$	5	Unc/o
	A_5	$\langle[s_{3.995}, s_{4.536}], [0.767, 0.821], [0.000, 0.137]\rangle$	$s_{3.680}$	3	Unc/+
0.6	A_1	$\langle[s_{4.770}, s_{4.913}], [0.734, 0.798], [0.113, 0.167]\rangle$	$s_{3.977}$	2	Unc/+
	A_2	$\langle[s_{4.778}, s_{5.326}], [0.712, 0.811], [0.000, 0.182]\rangle$	$s_{4.337}$	1	Acc/+
	A_3	$\langle[s_{4.128}, s_{4.332}], [0.714, 0.802], [0.107, 0.183]\rangle$	$s_{3.463}$	4	Unc/o
	A_4	$\langle[s_{3.681}, s_{4.461}], [0.627, 0.726], [0.156, 0.252]\rangle$	$s_{3.096}$	5	Unc/o
	A_5	$\langle[s_{3.797}, s_{4.392}], [0.754, 0.812], [0.000, 0.144]\rangle$	$s_{3.596}$	3	Unc/+
0.7	A_1	$\langle[s_{4.770}, s_{4.913}], [0.734, 0.798], [0.113, 0.167]\rangle$	$s_{4.017}$	2	Acc/+
	A_2	$\langle[s_{4.778}, s_{5.326}], [0.712, 0.811], [0.000, 0.182]\rangle$	$s_{4.456}$	1	Acc/+
	A_3	$\langle[s_{4.128}, s_{4.332}], [0.714, 0.802], [0.107, 0.183]\rangle$	$s_{3.515}$	4	Unc/+
	A_4	$\langle[s_{3.681}, s_{4.461}], [0.627, 0.726], [0.156, 0.252]\rangle$	$s_{3.195}$	5	Unc/o
	A_5	$\langle[s_{3.797}, s_{4.392}], [0.754, 0.812], [0.000, 0.144]\rangle$	$s_{3.690}$	3	Unc/+
0.8	A_1	$\langle[s_{4.772}, s_{4.918}], [0.735, 0.800], [0.113, 0.165]\rangle$	$s_{4.067}$	2	Acc/+
	A_2	$\langle[s_{4.779}, s_{5.320}], [0.713, 0.811], [0.000, 0.183]\rangle$	$s_{4.572}$	1	Acc/+
	A_3	$\langle[s_{4.135}, s_{4.338}], [0.714, 0.801], [0.107, 0.184]\rangle$	$s_{3.570}$	4	Unc/+
	A_4	$\langle[s_{3.674}, s_{4.449}], [0.626, 0.726], [0.156, 0.253]\rangle$	$s_{3.287}$	5	Unc/o
	A_5	$\langle[s_{3.836}, s_{4.423}], [0.758, 0.814], [0.000, 0.143]\rangle$	$s_{3.820}$	3	Unc/+
0.9	A_1	$\langle[s_{4.770}, s_{4.913}], [0.734, 0.798], [0.113, 0.167]\rangle$	$s_{4.099}$	2	Acc/+
	A_2	$\langle[s_{4.778}, s_{5.326}], [0.712, 0.811], [0.000, 0.182]\rangle$	$s_{4.699}$	1	Acc/+
	A_3	$\langle[s_{4.128}, s_{4.332}], [0.714, 0.802], [0.107, 0.183]\rangle$	$s_{3.619}$	4	Unc/+
	A_4	$\langle[s_{3.681}, s_{4.461}], [0.627, 0.726], [0.156, 0.252]\rangle$	$s_{3.399}$	5	Unc/o
	A_5	$\langle[s_{3.797}, s_{4.392}], [0.754, 0.812], [0.000, 0.144]\rangle$	$s_{3.882}$	3	Unc/+
1	A_1	$\langle[s_{4.770}, s_{4.913}], [0.734, 0.798], [0.113, 0.167]\rangle$	$s_{4.140}$	2	Acc/+
	A_2	$\langle[s_{4.778}, s_{5.326}], [0.712, 0.811], [0.000, 0.182]\rangle$	$s_{4.823}$	1	Acc/+
	A_3	$\langle[s_{4.128}, s_{4.332}], [0.714, 0.802], [0.107, 0.183]\rangle$	$s_{3.672}$	4	Unc/+
	A_4	$\langle[s_{3.681}, s_{4.461}], [0.627, 0.726], [0.156, 0.252]\rangle$	$s_{3.503}$	5	Unc/+
	A_5	$\langle[s_{3.797}, s_{4.392}], [0.754, 0.812], [0.000, 0.144]\rangle$	$s_{3.979}$	3	Unc/+

For simplicity and clarity, we use + for $Region_+^\gamma(*)$, o for $Region_o^\gamma(*)$, - for $Region_-^\gamma(*)$, Acc for DES_{acc}^γ, and Unc for DES_{unc}^γ in this table.

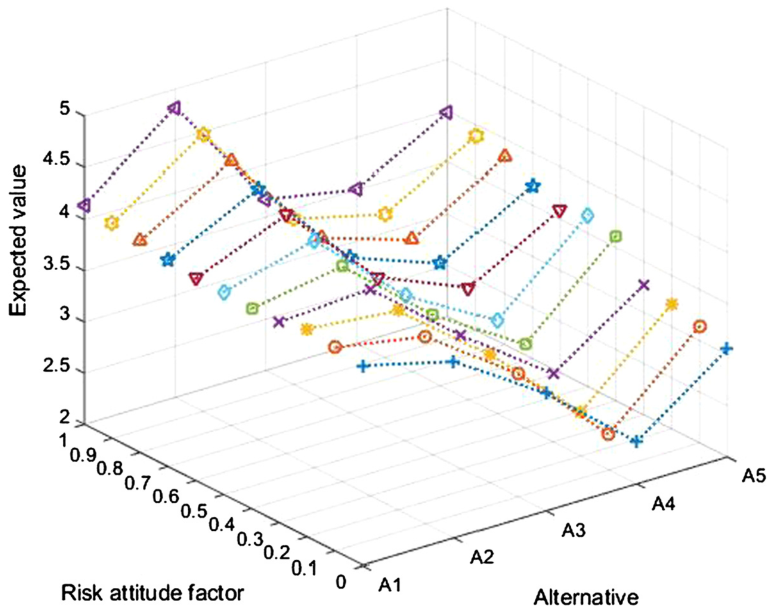

Fig. 5. The expected values for all alternatives under different risk attitude factors.

Secondly, the expert weight determination method proposed in this paper considers both the uncertainty of evaluation information itself and the closeness between evaluation information, which is an objective weighting method with strong comprehensiveness. And the proportion factor used to integrate the two kinds of finest-granulation objective weights is mined according to the distribution of the weights, thus avoiding the subjectivity of human effectively. This multi-granulation weight determination method can reflect the importance of experts more comprehensively and reasonably. In comparison, most of the expert weighting methods are based on single-granulation [42,41], that is to say, the importance of experts is described only from a single perspective and the weight information obtained is one-sided. The existing multi-granulation expert weight determination methods are relatively few, and most of them are based on coarse-granulation and need to specify parameters artificially [43,44], which has some limitations in flexibility and objectivity. However, compared with Meng [35], the proposed method does not consider the interactive characteristics or correlations between experts, which is one of the places to be perfected in the future.

(2) From the perspective of attribute weights, the weight information obtained by the proposed method is multi-granulation, which takes into account both the value and ranking of decision information simultaneously. In addition, the proposed method synthesizes the two kinds of attribute weights through an optimization model, eliminating the link of parameter setting and avoiding subjectivity.

In comparison, most of the existing attribute weighting methods are based on single-granulation, and the relationship between alternative rankings under different attributes is not taken into account. Liu [45] presented a combined weighting method which considers both subjective weights and objective weights based on statistical variance. In a sense, it can also be regarded as a multi-granulation weighting method, but the objective weight of attribute is still one-sided, and it needs to set two parameters to integrate subjective and objective weights. However, the attribute weight information obtained in this paper is coarse-granulation, which is lack of flexibility compared with the finer-granulation attribute weight obtained in Wan [38,39]. And compared with Meng [35,36] and Liang [27], the proposed method does not take into account the interactions between attributes. In the future, we will carry out in-depth thinking and research in these aspects.

(3) An expected value with risk attitude factor and an accuracy function with risk attitude factor are defined to compare IVIULVs in this paper. Based on which, the multi-granulation attribute weights and the final ranking results and grading information under different risk attitudes can be obtained. At present, most of the existing MAGDM methods overlook the influence of risk attitude in decision-making process, and only a few of them take it into account. Xu [54] defined the expected additive linguistic preference relation of an uncertain additive linguistic preference relation by introducing an index that reflects the expert's risk-bearing attitude. However, the method of Xu [54] is only applicable to the GDM problems based on linguistic preference relations. Yue [55] introduced an optimistic coefficient into MAGDM problem based on interval numbers. Liang [27] utilized prospect theory to describe decision makers' risk attitude. But the above three methods cannot be directly used to solve MAGDM problems based on IVIULVs. Meng [36] introduced the parameter γ, which reflects the expert's risk attitude, into the definition of interval-valued intuitionistic uncertain linguistic aggregation operator. The method can only reflect the impact of risk attitude on aggregation results, but can not reflect its impact on attribute weight.

In brief, introducing the idea of granular computing in the process of MAGDM can analyze, measure and mine the decision-making information from multi-view and multi-level, which is helpful to obtain more flexible and adaptable weight information and decision results. Therefore, the proposed method has the characteristics of easy programming, strong operability and wide application range.

8. Conclusions

In this paper, a novel data-driven MAGDM method considering risk attitude under interval-valued intuitionistic uncertain linguistic environment is established based on the idea of multi-granulation and three-way decisions. The multi-granulation weight information has considerable objectivity and flexibility, so the proposed method can be directly used to solve the MAGDM problem with completely unknown weight or even incomplete decision information. The coarse-granulation grading information based on three-way decisions provides more abundant and interpretable feedback information for decision makers. In addition, due to the integration of risk attitude factor, the proposed method can obtain the attribute weights, alternative grading and ranking results under different risk attitudes, and meet the actual needs of different decision makers. This paper gives a valuable attempt to combining multi-granulation, three-way decisions and MAGDM under interval-valued intuitionistic uncertain linguistic environment.

The prominent characteristics of the proposed method are that it can utilize the decision information sufficiently and has considerable practicability. The research results of this paper enrich decision analysis theory and methodology and provide new ideas for solving complex MAGDM problems. In the future, we will make full use of the advantages of three-way decisions and rough set to solve the large-scale MAGDM problems, focusing on designing some decision mechanisms considering correlations between experts and interactions between attributes. We are expecting to develop the potential applications of the present work in various domains.

Declaration of competing interest

We declare that we do not have any commercial or associative interest that represents a conflict of interest in connection with the work submitted.

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