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Multi-granularity three-way decisions with adjustable hesitant fuzzy linguistic multigranulation decision-theoretic rough sets over two universes☆

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ABSTRACT

The notion of hesitant fuzzy linguistic term sets (HFLTSs), which enables experts to utilize a few possible linguistic terms to evaluate varieties of common qualitative information, plays a significant role in handling situations in cases where these experts are hesitant in offering linguistic expressions. For addressing the challenges of information analysis and information fusion in hesitant fuzzy linguistic (HFL) group decision making, in accordance with the multi-granularity three-way decisions paradigm, the primary purpose of this study is to develop the notion of multigranulation decision-theoretic rough sets (MG-DTRSs) into the HFL background within the two-universe framework. Having revisited the relevant literature, we first propose a hybrid model named adjustable HFL MG-DTRSs over two universes by introducing an adjustable parameter for the expected risk appetite of experts, in which both optimistic and pessimistic versions of HFL MG-DTRSs over two universes are special cases of the adjustable version. Second, some of the fundamental properties of the proposed model are discussed. Then, on the basis of the presented hybrid model, a group decision making approach within the HFL context is further constructed. Finally, a practical example, a comparative analysis, and a validity test concerning person-job fit problems are explored to reveal the rationality and practicability of the constructed decision making rule.

1. Introduction

In realistic decision making processes, owing to the innate fuzziness of human thought and the complexity of decision making contexts, experts are notably prone to express their preferences in qualitative situations. To cope with such issues, the fuzzy linguistic approach [40] expresses qualitative information as linguistic variables, and this approach enhances the flexibility and reliability of processing various linguistic expressions. However, considering that the form of linguistic variables is often expressed by single terms, the fuzzy linguistic approach is still insufficient in modeling every individual person's complicated thinking. For instance, when estimating the programming ability of a job seeker, human resource experts

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can only express “modest” aptitude, but they cannot express “higher than modest” or “between modest and competent” ones. Therefore, considering that it is beneficial for experts to describe their respective opinions in multiple linguistic terms, Rodriguez et al. [17] established the notion of HFLTSs by simultaneously overcoming the limitations of fuzzy linguistic approaches and hesitant fuzzy sets (HFSs) [6,27]. Ever since the establishment of HFLTSs, substantial academic achievements have been realized in terms of solving HFL group decision making problems [5,9,18,28,43].

In practice, experts are usually confronted with the following two challenges in HFL group decision making situations: (a) each expert needs to explain a decision with respect to the scenario under which alternative have been selected as the optimal one, i.e., the information analysis issue when experts make group decisions; (b) all experts need to reach an agreement or a conclusion by referring to each decision result, i.e., the information fusion issue in the group decision making procedure.

The rough set theory [13] is widely known as a reasonable and efficient soft computing method for handling several decision making situations via attribute selections and rule acquisitions [4,21–23]. Moreover, in the past decades, various generalized rough set models have been constructed in step with the actual demands of real-world situations [2,15,37–39]. In this research, for the sake of enhancing the applicability of generalized rough set models in handling the two challenges mentioned above in HFL group decision making, we aim to explore a novel rough set model by means of the multi-granularity three-way decisions paradigm. Multi-granularity three-way decisions, which originate from the granular computing framework [41], construct multi-level problem solving methods by providing information analysis and information fusion rules for solution spaces in different granularity levels based on the three-way decisions theory [31–36]. Among the most commonly used multi-granularity three-way decisions models that combine decision-theoretic rough sets (DTRSs) [1,32,33,37,38] with multigranulation rough sets (MGRSs) [14,15], Qian et al. [16] were the ones who initially constructed MG-DTRSs. Since then, many scholars have enriched MG-DTRSs from the viewpoints of multi-granularity three-way decisions [3,8,10,11,25,29,30,45]. In particular, the motivations of utilizing DTRSs and MGRSs over two universes to address the challenges of information analysis and information fusion in group decision making can be summarized as follows:

(1) For solving information analysis issues in group decision making, three-way decisions theory has demonstrated its superior performances in each granularity level when constructing multi-level problem solving methods. Yao [31,32] emphasized that three-way decisions can be utilized to interpret three regions in rough sets, i.e., positive, boundary, and negative regions, which are regarded the regions of acceptance, noncommitment, and rejection in ternary classifications. DTRSs, as constructed and developed by Yao [1,32,33,37,38] in the early 1990s, are regarded one of the most typical representatives of three-way decisions [31–36] that have been proposed in recent years. Moreover, DTRSs have been proposed much earlier than three-way decisions, and the essence of DTRSs can be illustrated by three-way decisions from a broad perspective. Specifically, DTRSs are established by the ideas of acceptance, noncommitment, and rejection by introducing threshold-based induction rules to permit error tolerance. Thus, DTRSs build a link between rough sets and decision theory, especially for the purpose of providing a reasonable semantic interpretation for decision risks [7,12,19,26,47,48]. With the support of DTRSs, by reasonably quantifying and minimizing the decision loss of wrong decisions, a more reliable ternary classification result can be obtained compared with those that use other existing models without focusing on decision risks.
(2) For solving information fusion issues in group decision making, MGRSs over two universes, as constructed by Sun and Ma [20] in view of classical MGRSs, are regarded one of the most reasonable and efficient tools for fusing solution space in different granularity levels. Specifically, the strengths of MGRSs over two universes are reflected in the following two levels:
    - MGRSs regard each decision maker’s preferences as a single information system that can induce its related granular structures, and thus, MGRSs are able to transform the issue of information fusion into multigranulation fusion from multiple views and levels. In view of this, Qian et al. [14,15] established theoretical foundations of MGRSs. In addition, MGRSs reportedly not only provide an effective scheme to discover knowledge by simultaneously processing multiple binary relations with actual requirements, but they also include optimistic and pessimistic MGRSs that can be utilized according to risk-seeking and risk-averse tactics.
    - The consideration of two universes [39] outperforms a single universe when depicting real-world decision making information systems, and hence, exploring decision making problems by means of rough sets over two universes is essential. Here, we aim to discuss a special correlation group decision making problem. As an important measure in data analysis, the correlation can reflect a complex relationship of two sets with the aid of a measure of interdependency of the two sets. In using granular computing to handle the aforementioned special decision making problems, particularly by developing the model of MGRSs over two universes, decision makers can conveniently and realistically describe useful inherent relationships between the two kinds of objects of these group decision making problems [24,42,44,46].

On the basis of the above discussions, to handle HFL group decision making problems according to the multi-granularity three-way decisions paradigm, this work investigates a hybrid model named adjustable HFL MG-DTRSs over two universes from the viewpoint of theoretical foundations and real-world applications. We simultaneously combine the notion of HFLTSs, DTRSs, and MGRSs over two universes to overcome the shortcomings of existing MG-DTRS models in HFL group decision making. The combination mechanism can be summarized as follows: (a) in enabling the proposed model to quantify and minimize the loss of wrong decisions in the HFL context, we construct a single membership degree by introducing DTRSs into HFL information systems; (b) in enabling the proposed model to acquire adjustable capabilities according to one expert's risk preferences when handling HFL group decision making problems, we expand the presented single membership degree to the background of multiple granulations by introducing an adjustable parameter. Then, the notion of adjustable membership degrees is established, and both optimistic and pessimistic versions of the proposed model are regarded special cases of the adjustable version. Hence, by taking advantages of DTRSs and MGRSs over two universes, the proposed model can suitably cope with HFL group decision making situations despite decision risks.

Compared with existing HFL group decision making studies, our critical contributions in this paper can be summarized as follows:

(1) According to the multi-granularity three-way decisions paradigm, we explore the notion of adjustable HFL MG-DTRSs over two universes, and some of their properties are also discussed. Different from other studies, the viewpoint of granular computing can help us design a novel three-way decision making approach. To be specific, the concept of two universes is introduced to analyze the correlation of group decision making problems, which then can depict the complex relationship of two sets. The measure of interdependency of the two sets is considered, as it is a significant measure in data analysis.
(2) By taking full advantages of DTRSs and MGRSs over two universes, the proposed model can appropriately handle group decision making situations with risks in the HFL context. Considering that comparing and computing linguistic variables are ineffective, some studies have proposed the transformation of qualitative expressions into quantitative ones. Accordingly, we design a scheme to transform hesitant fuzzy linguistic elements (HFLEs) into interval numbers. Moreover, by introducing an adjustable parameter for the expected risk preference of experts, the proposed model can help experts to make decisions based on their risk attitudes.
(3) We propose a general group decision making rule based on adjustable HFL MG-DTRSs over two universes in the background of person-job fit. The case study illustrates that the proposed group decision making rule can substantially enhance matching accuracies and decrease uncertainties of person-job fit. Moreover, we use a classical validity test to show the effectiveness of the proposed method.

To facilitate the discussion, in Section 2, we briefly present a background on the concepts of HFLTSs, DTRSs, and MGRSs over two universes. Section 3 presents the proposed notion of adjustable HFL MG-DTRSs over two universes. In Section 4, the construction of an HFL group decision making method by means of adjustable HFL MG-DTRSs over two universes is discussed. Finally, in Section 5, the findings from a case study, a comparative analysis, and a validity test are presented to prove the validity of the developed decision making rule. The paper ends with several concluding comments in Section 6.

2. Preliminaries

In the following subsections, the notions of HFLTSs, DTRSs, and MGRSs over two universes are reviewed briefly.

2.1. HFLTSs

HFLTSs are constructed on the basis of the fuzzy linguistic approach. Thus, before presenting the background on HFLTSs, several main parts of the fuzzy linguistic approach [17] aimed at utilizing linguistic values to depict some qualitative aspects with the aid of linguistic variables need to be explained.

Suppose that $S = \{s_0, s_1, \ldots, s_g\}$ is a finite and totally ordered linguistic term set with odd granularity $g + 1$, where s_i represents a possible value for a linguistic variable. For instance, when $g = 6$, a set of seven terms can be provided.

$$S = \{s_0 : \textit{very simple}, s_1 : \textit{simple}, s_2 : \textit{slightly simple}, s_3 : \textit{medium}, s_4 : \textit{slightly complicated}, s_5 : \textit{complicated}, s_6 : \textit{very complicated}\}.$$

In addition, the linguistic term set requires the following properties:

(1) The negation operator exists: $neg(s_i) = s_j$, such that $j = g - i$.
(2) The set is ordered: if $i \geq j$, then $s_i \geq s_j$.
(3) Max operator: if $s_i \geq s_j$, then $\max(s_i, s_j) = s_i$.
(4) Min operator: if $s_i \leq s_j$, then $\min(s_i, s_j) = s_i$.

Then, Rodriguez et al. [17] presented concepts concerning HFLTSs [27].

Definition 2.1. [17] Let $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set. An HFLTS H_S on S is an ordered finite subset of consecutive linguistic terms in S.

However, Definition 2.1 notably does not present the concrete mathematical formulation of HFLTSs. Subsequently, Liao et al. [9] updated the definition of HFLTSs.

Definition 2.2. [9] Let U be a non-empty finite universe, and $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set. An HFLTS on U is related to a function h that when applied to U returns a subset of S, that is,

$$\mathbb{F} = \{\langle x, h_{\mathbb{F}}(x)\rangle | x \in U\}, \tag{1}$$

where $h_{\mathbb{F}}(x)$, denoting the possible membership degrees of the element $x \in U$ to $\mathbb{F}$, is a set of several different ordered finite values in S. For the sake of convenience, we denote $h_{\mathbb{F}}(x)$ as an HFLE. Additionally, we represent the set that consists of all HFLTSs on U as $HFL(U)$.

Example 2.1. Let $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set, i.e., $S = \{s_0 : \textit{very simple}, s_1 : \textit{simple}, s_2 : \textit{slightlysimple},$ s_3: *medium,* s_4: *slightly complicated,* s_5: *complicated,* s_6: *very complicated*}. Additionally, suppose $U = \{x_1, x_2, x_3\}$ is a finite universe of discourse, in which $h_{\mathbb{F}}(x_1) = \{s_0 : \textit{very simple}, s_1 : \textit{simple}\} = \{s_0, s_1\}$, $h_{\mathbb{F}}(x_2) = \{s_6 : \textit{very complicated}\} = \{s_6\}$, and $h_{\mathbb{F}}(x_3) = \{s_2 : \textit{slightly simple}, s_3 : \textit{medium}, s_4 : \textit{slightly complicated}\} = \{s_2, s_3, s_4\}$ are the three given HFLEs of $x_i (i = 1, 2, 3)$ to $\mathbb{F}$. Then, $\mathbb{F} = \{\langle x_1, \{s_0, s_1\}\rangle, \langle x_2, \{s_6\}\rangle, \langle x_3, \{s_2, s_3, s_4\}\rangle\}$ can be obtained.

Subsequently, we present two special HFLTSs, i.e., the full HFLTS and the empty HFLTS presented by Zhang et al. [43].

(1) Full HFLTS $\mathbb{U}$: $\mathbb{F}$ is called a full HFLTS if and only if $h_{\mathbb{F}}(x) = \{s_g\}$ for all $x \in U$.
(2) Empty HFLTS $\emptyset$: $\mathbb{F}$ is called an empty HFLTS if and only if $h_{\mathbb{F}}(x) = \{s_0\}$ for all $x \in U$.

In comparing different HFLEs, Huang and Yang [5] defined a pairwise comparison method of HFLEs.

Definition 2.3. [5] Let $h_{\mathbb{F}}(x_1)$ and $h_{\mathbb{F}}(x_2)$ be two HFLEs. The pairwise comparison matrix between $h_{\mathbb{F}}(x_1)$ and $h_{\mathbb{F}}(x_2)$ is given by

$$C(h_{\mathbb{F}}(x_1), h_{\mathbb{F}}(x_2)) = [d(s_i, s_j)]_{|h_{\mathbb{F}}(x_1)| \times |h_{\mathbb{F}}(x_2)|}, \tag{2}$$

where $d(s_i, s_j) = i - j$, $s_i \in h_{\mathbb{F}}(x_1)$, $s_j \in h_{\mathbb{F}}(x_2)$. Then, the pairwise comparison matrix between $h_{\mathbb{F}}(x_1)$ and $h_{\mathbb{F}}(x_2)$ is further denoted by $C(h_{\mathbb{F}}(x_1), h_{\mathbb{F}}(x_2)) = [C_{mn}]$. Moreover, the preference relations of $h_{\mathbb{F}}(x_1)$ and $h_{\mathbb{F}}(x_2)$ are given by

$$P(h_{\mathbb{F}}(x_1) < h_{\mathbb{F}}(x_2)) = \frac{|\sum_{C_{mn}<0} C_{mn}|}{\#\{C_{mn} = 0\} + \sum |C_{mn}|}, \tag{3}$$

$$P(h_{\mathbb{F}}(x_1) > h_{\mathbb{F}}(x_2)) = \frac{|\sum_{C_{mn}>0} C_{mn}|}{\#\{C_{mn} = 0\} + \sum |C_{mn}|}, \tag{4}$$

$$P(h_{\mathbb{F}}(x_1) = h_{\mathbb{F}}(x_2)) = \frac{\#\{C_{mn} = 0\}}{\#\{C_{mn} = 0\} + \sum |C_{mn}|}, \tag{5}$$

where $\#\{C_{mn} = 0\}$ denotes the number of the element 0 in the pairwise comparison matrix.

In accordance with the preference relations mentioned above, Huang and Yang [5] further constructed the notion of non-dominance degrees to rank alternatives by virtue of preference relations.

Definition 2.4. [5] For a set of alternatives X, suppose that $P_D = [P_{ij}]$ is a preference relation with respect to X. The non-dominance degree of a certain alternative x_i is given by

$$NDD_i = \min\{1 - P_{ji}^S, j = 1, \ldots, n, j \neq i\}, \tag{6}$$

where $P_{ji}^S = \max\{P_{ji} - P_{ij}, 0\}$ represents the degree to which x_i is strictly dominated by x_j. Consequently, the optimal alternative can be chosen as

$$X^{ND} = \{x_i | x_i \in X, NDD_i = \max_{x_j \in X}\{NDD_j\}\}. \tag{7}$$

Given the two HFLTSs, as represented by $\mathbb{F}_1$ and $\mathbb{F}_2$, we develop the concept of HFL subsets to compare different HFLTSs [43].

Definition 2.5. [43] Let U be a non-empty finite universe, and $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set. $\forall \mathbb{F}_1, \mathbb{F}_2 \in HFL(U)$, $\mathbb{F}_1$ is called an HFL subset of $\mathbb{F}_2$, if $h_{\mathbb{F}_1}(x) \preceq h_{\mathbb{F}_2}(x)$ holds for each $x \in U$, such that $h_{\mathbb{F}_1}(x) \preceq h_{\mathbb{F}_2}(x) \Leftrightarrow h_{\mathbb{F}_1}^{\sigma(k)}(x) \leq h_{\mathbb{F}_2}^{\sigma(k)}(x)$, and it is denoted by $\mathbb{F}_1 \sqsubseteq \mathbb{F}_2$, where $h_{\mathbb{F}_1}^{\sigma(k)}(x)$ and $h_{\mathbb{F}_2}^{\sigma(k)}(x)$ are the kth largest element in $h_{\mathbb{F}}(x_1)$ and $h_{\mathbb{F}}(x_2)$.

Similar to HFSs, some operations on HFLTSs were proposed in [17].

Definition 2.6. [17] Let U be a non-empty finite universe, and $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set. Suppose that $\forall \mathbb{F}_1, \mathbb{F}_2 \in HFL(U)$, then for all $x \in U$,

(1) the complement of $\mathbb{F}_1$, as represented by $\mathbb{F}_1{}^c$, is provided by

$$h_{\mathbb{F}_1{}^c}(x) = \sim h_{\mathbb{F}_1}(x) = \{s_{g-i} | i \in ind(h_{\mathbb{F}_1}(x))\}, \tag{8}$$

where $ind(s_i)$ represents the index i of a linguistic term s_i in S, and $ind(h_{\mathbb{F}_1}(x))$ represents the set of indexes of several linguistic terms in $h_{\mathbb{F}_1}(x)$;

(2) the intersection of $\mathbb{F}_1$ and $\mathbb{F}_2$, as represented by $\mathbb{F}_1 \Cap \mathbb{F}_2$, is provided by

$$h_{\mathbb{F}_1 \Cap \mathbb{F}_2}(x) = h_{\mathbb{F}_1}(x) \barwedge h_{\mathbb{F}_2}(x) = \{s_i \in (h_{\mathbb{F}_1}(x) \cup h_{\mathbb{F}_2}(x)) | s_i \leq \min(h_{\mathbb{F}_1}^+(x), h_{\mathbb{F}_2}^+(x))\}; \tag{9}$$

(3) the union of $\mathbb{F}_1$ and $\mathbb{F}_2$, as represented by $\mathbb{F}_1 \Cup \mathbb{F}_2$, is provided by

$$h_{\mathbb{F}_1 \Cup \mathbb{F}_2}(x) = h_{\mathbb{F}_1}(x) \underline{\vee} h_{\mathbb{F}_2}(x) = \{s_i \in (h_{\mathbb{F}_1}(x) \cup h_{\mathbb{F}_2}(x)) | s_i \geq \max(h^-_{\mathbb{F}_1}(x), h^-_{\mathbb{F}_2}(x))\}; \tag{10}$$

(4) the ring sum of $\mathbb{F}_1$ and $\mathbb{F}_2$, as represented by $\mathbb{F}_1 \boxplus \mathbb{F}_2$, is provided by

$$h_{\mathbb{F}_1 \boxplus \mathbb{F}_2}(x) = h_{\mathbb{F}_1}(x) \oplus h_{\mathbb{F}_2}(x) = \cup_{s_\alpha \in h_{\mathbb{F}_1}(x), s_\beta \in h_{\mathbb{F}_2}(x)} \{s_{\alpha+\beta}\}; \tag{11}$$

(5) the ring product of $\mathbb{F}_1$ and $\mathbb{F}_2$, as represented by $\mathbb{F}_1 \boxtimes \mathbb{F}_2$, is provided by

$$h_{\mathbb{F}_1 \boxtimes \mathbb{F}_2}(x) = h_{\mathbb{F}_1}(x) \otimes h_{\mathbb{F}_2}(x) = \cup_{s_\alpha \in h_{\mathbb{F}_1}(x), s_\beta \in h_{\mathbb{F}_2}(x)} \{s_{\alpha\times\beta}\}, \tag{12}$$

where $h^+_{\mathbb{F}_1}(x)$ represents the upper bound of $h_{\mathbb{F}_1}(x)$, i.e., $h^+_{\mathbb{F}_1}(x) = \max\{s_i | s_i \in h_{\mathbb{F}_1}(x)\}$, and $h^-_{\mathbb{F}_1}(x)$ represents the lower bound of $h_{\mathbb{F}_1}(x)$, i.e., $h^-_{\mathbb{F}_1}(x) = \min\{s_i | s_i \in h_{\mathbb{F}_1}(x)\}$. Moreover, on the basis of the upper and lower bounds of $h_{\mathbb{F}}(x)$, Rodriguez et al. [17] further constructed the concept of envelope for $h_{\mathbb{F}}(x)$, i.e., $env(h_{\mathbb{F}}(x)) = [h^-_{\mathbb{F}}(x), h^+_{\mathbb{F}}(x)]$.

According to Rodriguez et al. [17], the comparison and computation of HFLTSs, such as for ranking and selection, are necessary in many linguistic decision making situations. However, an HFLTS is a linguistic term subset that contains several linguistic terms, and the comparison and computation of HFLTSs are not straightforward and effective. Moreover, comparing discrete linguistic terms in HFLTSs is not suitable when using other classical comparison methods. According to the definition of HFLTSs, each of its linguistic term is a possible value of the linguistic information. Furthermore, the two compared HFLTSs may have different lengths. Hence, when utilizing two HFLTSs, considering the concept of envelope for HFLEs, we design a scheme to transform HFLTSs into interval numbers and subsequently enhance the efficiency of the group decision making process. Therefore, according to the notion of $ind(h_{\mathbb{F}}(x))$, we can extract the index of $env(h_{\mathbb{F}}(x))$ by introducing the operation $ind(env(h_{\mathbb{F}}(x)))$. For example, if we have $h_{\mathbb{F}}(x) = \{s_1, s_2, s_4\}$, then $env(h_{\mathbb{F}}(x)) = [s_1, s_4]$ and $ind(env(h_{\mathbb{F}}(x))) = [1, 4]$ can be obtained.

Given that the result of $ind(env(h_{\mathbb{F}}(x)))$ is an interval number, we present some main operations of interval numbers as follows:

Definition 2.7. [17] Let $a = [a^L, a^U]$ and $b = [b^L, b^U]$ be two interval numbers with following conditions: (1) $a + b = [a^L + b^L, a^U + b^U]$. (2) If a^L, $b^L > 0$, then $ab = [a^L b^L, a^U b^U]$ and $\frac{a}{b} = [\frac{a^L}{b^U}, \frac{a^U}{b^L}]$. The preference degree of $a \geq b$ is given by

$$p(a \geq b) = \max\left\{1 - \max\left(\frac{b^U - a^L}{l_a + l_b}, 0\right), 0\right\}, \tag{13}$$

and the preference degree of $a \leq b$ is given by

$$p(a \leq b) = \max\left\{1 - \max\left(\frac{a^U - b^L}{l_a + l_b}, 0\right), 0\right\}, \tag{14}$$

where $l_a = a^U - a^L$, $l_b = b^U - b^L$.

2.2. DTRSs

As an influential model in the rough set community, the concept of DTRSs provides reasonable semantic interpretations for the decision theory based on Bayesian decision procedures [1,32,33,37,38]. Specifically, DTRSs include two states and three actions, i.e., the state $\Omega = \{X, \neg X\}$ indicates an object is in X or not in X, and the action $A = \{a_P, a_B, a_N\}$ represents the three different actions when classifying x. Then, suppose that λ_{PP}, λ_{BP}, and λ_{NP} are the losses incurred for taking actions on the aforementioned a_P, a_B, and a_N when $x \in X$ occurs. In a similar manner, suppose that λ_{PN}, λ_{BN}, and λ_{NN} are the losses triggered by taking the same actions when $x \in \neg X$ happens. In addition, $P(X|[x].)$ represents the conditional probability of $x \in X$. Then, for an object x, the expected loss $R(a_i|[x].)$ can be given by

$R(a_P|[x]) = \lambda_{PP} P(X|[x]) + \lambda_{PN} P(\neg X|[x])$,
$R(a_B|[x]) = \lambda_{BP} P(X|[x]) + \lambda_{BN} P(\neg X|[x])$,
$R(a_N|[x]) = \lambda_{NP} P(X|[x]) + \lambda_{NN} P(\neg X|[x])$.

With reference to Bayesian decision procedures, the decision rule associated with minimum-cost approaches can be obtained.

(P) If $R(a_P|[x].) \leq R(a_B|[x].)$ and $R(a_P|[x].) \leq R(a_N|[x].)$, then decide $x \in POS(X)$.
(B) If $R(a_B|[x].) \leq R(a_P|[x].)$ and $R(a_B|[x].) \leq R(a_N|[x].)$, then decide $x \in BND(X)$.
(N) If $R(a_N|[x].) \leq R(a_P|[x].)$ and $R(a_N|[x].) \leq R(a_B|[x].)$, then decide $x \in NEG(X)$.

$\lambda_{PP} \leq \lambda_{BP} < \lambda_{NP}$ and $\lambda_{NN} \leq \lambda_{BN} < \lambda_{PN}$ often hold in depicting some conditions of loss functions. In particular, if $P(X|[x]) + P(\neg X|[x]) = 1$, $(\lambda_{PN} - \lambda_{BN})(\lambda_{NP} - \lambda_{BP}) \geq (\lambda_{BP} - \lambda_{PP})(\lambda_{BN} - \lambda_{NN})$, and $\alpha > \beta$, then the simplified decision rule can be concluded.

(P_1) If $P(X|[x].) \geq \alpha$, then decide $x \in POS(X)$.
(B_1) If $\beta < P(X|[x].) < \alpha$, then decide $x \in BND(X)$.

(N_1) If $P(X|[x].) \leq \beta$, then decide $x \in NEG(X)$,
where $\alpha = \frac{\lambda_{PN}-\lambda_{BN}}{(\lambda_{PN}-\lambda_{BN})+(\lambda_{BP}-\lambda_{PP})}$ and $\beta = \frac{\lambda_{BN}-\lambda_{NN}}{(\lambda_{BN}-\lambda_{NN})+(\lambda_{NP}-\lambda_{BP})}$. On the basis of the aforementioned results, the lower and upper approximations of the target set X can be expressed as

$$\underline{apr}^{\alpha}(X) = \{x : P(X|[x]) \geq \alpha, x \in U\}, \tag{15}$$

$$\overline{apr}^{\beta}(X) = \{x : P(X|[x]) > \beta, x \in U\}. \tag{16}$$

2.3. MGRSs over two universes

To effectively describe and analyze various decision making information systems, Sun and Ma [20] studied classical MGRSs by applying the idea of two universes and developed MGRSs over two universes.

Definition 2.8. [20] Let U and V be two non-empty finite universes, and $R_i (1 \leq i \leq m)$ be a binary compatibility relation from U to V. For any $X \subseteq V$, the optimistic lower and upper approximations of X, as represented by $\underline{\sum_{i=1}^{m} R_i}^{O}(X)$ and $\overline{\sum_{i=1}^{m} R_i}^{O}(X)$, are given by

$$\underline{\sum_{i=1}^{m} R_i}^{O}(X) = \{[x]_{R_1} \subseteq X \vee [x]_{R_2} \subseteq X \vee \ldots \vee [x]_{R_m} \subseteq X | x \in U\}, \tag{17}$$

$$\overline{\sum_{i=1}^{m} R_i}^{O}(X) = \left(\underline{\sum_{i=1}^{m} R_i}^{O}(X^c) \right)^c, \tag{18}$$

where $[x]_{R_i}$ represents the equivalence class of X in terms of R_i, and X^c represents the complement of X. Then, we call $(\underline{\sum_{i=1}^{m} R_i}^{O}(X), \overline{\sum_{i=1}^{m} R_i}^{O}(X))$ an optimistic MGRS over two universes. In an identical manner, a pessimistic MGRS over two universes, as represented by $(\underline{\sum_{i=1}^{m} R_i}^{P}(X), \overline{\sum_{i=1}^{m} R_i}^{P}(X))$, can also be obtained conveniently, where

$$\underline{\sum_{i=1}^{m} R_i}^{P}(X) = \{[x]_{R_1} \subseteq X \wedge [x]_{R_2} \subseteq X \wedge \ldots \wedge [x]_{R_m} \subseteq X | x \in U\}, \tag{19}$$

$$\overline{\sum_{i=1}^{m} R_i}^{P}(X) = \left(\underline{\sum_{i=1}^{m} R_i}^{P}(X^c) \right)^c. \tag{20}$$

3. Adjustable HFL MG-DTRSs over two universes

By taking into account the necessity of introducing MG-DTRSs into HFL information systems within the framework of two universes, the notion of HFL MG-DTRSs over two universes is proposed in this paper.

Moreover, the majority of existing MGRS models reportedly consist of optimistic and pessimistic types of MGRSs. The optimistic type corresponds to the situation in which at least a single granular structure can be utilized when processing multiple binary relations, whereas its pessimistic counterpart corresponds to the situation in which all granular structures should be utilized when performing the same task. Moreover, the two MGRS models can be established on the basis of maximal and minimal operators. Owing to the lack of adjustable capabilities, the utilization of maximal and minimal operators precludes MGRSs from adjusting according to a user's varying practical requirements. Correspondingly, according to the standpoint of risk-based decision making, optimistic and pessimistic types of MGRSs can only represent two extreme models in solving group decision making problems. Hence, proposing an adjustable MGRS model that enables experts to make decisions based on their risk attitudes is necessary. Thus, unlike most of the extant studies that focus on optimistic and pessimistic MGRS models, we propose the adjustable HFL MG-DTRSs over two universes, in which the optimistic and pessimistic HFL MG-DTRSs over two universes are the two special models of the proposed adjustable version. Thereafter, to facilitate the introduction of the proposed model, we initially propose the concept of HFL relations over two universes.

Definition 3.1. [43] Let U and V be two non-empty finite universes and $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set. An HFL subset of the universe $U \times V$ is named an HFL relation over two universes, as represented by $\mathbb{R}$, is given by

$$\mathbb{R} = \{\langle (x, y), h_{\mathbb{R}}(x, y) \rangle | (x, y) \in U \times V\}, \tag{21}$$

where $h_{\mathbb{R}} : U \times V \to 2^S$, and $h_{\mathbb{R}}(x, y)$ is a set of several different ordered finite values in S. Additionally, we denote the set that includes all HFL relations over $U \times V$ as $HFLR\ (U \times V)$.

Then, by introducing HFL inclusion degrees, we propose the notion of single membership degrees for some elements in their corresponding HFLTSs.

Definition 3.2. Let U and V be two non-empty finite universes, $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set, and $\mathbb{R} \in HFLR(U \times V)$ be an HFL relation. For any $\mathbb{F} \in HFL(V)$, $x \in U$, $y \in V$, the single membership degree of x in $\mathbb{F}$ with respect to $\mathbb{R}$, as represented by $\Phi_{\mathbb{F}}^{\mathbb{R}}(x)$, is given by

$$\Phi_{\mathbb{F}}^{\mathbb{R}}(x) = [1, 1] - \frac{ind(env(|\mathbb{R}(x) \Cap \mathbb{F}^c|))}{ind(env(|\mathbb{R}(x)|))}, \tag{22}$$

where $|\mathbb{F}| = \sum_{y \in V} \mathbb{F}(y)$ represents the cardinality of an HFLTS $\mathbb{F}$, and $\Phi_{\mathbb{F}}^{\mathbb{R}}(x)$ denotes the HFL inclusion degree between an element and an HFLTS $\mathbb{F}$.

From the perspective of single granulation, the concept of HFL DTRSs over two universes can be constructed by using the single membership degree. Considering that we aim to investigate the hybrid model in the view of multiple granulations, developing an adjustable membership degree by integrating those single membership degrees presented in Definition 3.2 is essential. Hence, we propose the concept of adjustable membership degrees.

Prior to the introduction of adjustable membership degrees, we present the following assumptions: according to an HFL relation over two universes $\mathbb{R}_i (i = 1, \ldots, m)$, we have the membership degree of x in $\mathbb{F}$ in terms of $\mathbb{R}_i$, as represented by $\Phi_{\mathbb{F}}^{\mathbb{R}_i}(x) = [1, 1] - \frac{ind(env(|\mathbb{R}_i(x) \Cap \mathbb{F}^c|))}{ind(env(|\mathbb{R}_i(x)|))}$. Then, we arrange all the values of $\Phi_{\mathbb{F}}^{\mathbb{R}_i}(x)$ in an increasing order, and we let $\Phi_{\mathbb{F}}^{\mathbb{R}_{\sigma(i)}}(x)$ be the ith smallest value for all the values of $\Phi_{\mathbb{F}}^{\mathbb{R}_i}(x)$.

Definition 3.3. Let U and V be two non-empty finite universes, $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set, and $\mathbb{R}_i \in HFLR(U \times V)(i = 1, \ldots, m)$ be an HFL relation. For any $\mathbb{F} \in HFL(V)$, $x \in U$, and $y \in V$, the adjustable membership degree of x in $\mathbb{F}$ with respect to $\mathbb{R}_i$, as represented by $\delta_{\mathbb{F}}^{\sum_{i=1}^m \mathbb{R}_i}(x)$, is given by

$$\delta_{\mathbb{F}}^{\sum\limits_{i=1}^m \mathbb{R}_i}(x) = \Phi_{\mathbb{F}}^{\mathbb{R}_{\sigma(i)}}(x). \tag{23}$$

The adjustable membership degree $\delta_{\mathbb{F}}^{\sum_{i=1}^m \mathbb{R}_i}(x)$ notably reduces to the maximal and minimal membership degrees of x in $\mathbb{F}$ with respect to $\mathbb{R}_i$ when $i = m$ and $i = 1$, as represented by $\xi_{\mathbb{F}}^{\sum_{i=1}^m \mathbb{R}_i}(x) = \max_{i=1}^m \Phi_{\mathbb{F}}^{\mathbb{R}_i}(x) = \Phi_{\mathbb{F}}^{\mathbb{R}_{\sigma(m)}}(x)$ and $\zeta_{\mathbb{F}}^{\sum_{i=1}^m \mathbb{R}_i}(x) = \min_{i=1}^m \Phi_{\mathbb{F}}^{\mathbb{R}_i}(x) = \Phi_{\mathbb{F}}^{\mathbb{R}_{\sigma(1)}}(x)$, respectively. By utilizing the idea of adjustable membership degrees, the adjustable HFL MG-DTRSs over two universes can then be defined.

Definition 3.4. Let U and V be two non-empty finite universes, $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set, and $\mathbb{R}_i \in HFLR(U \times V)(i = 1, \ldots, m)$ be an HFL relation. Then, $(U, V, \mathbb{R}_i)$ is an HFL multigranulation approximation space over two universes. For any $\mathbb{F} \in HFL(V)$, $x \in U$, and $y \in V$, with threshold parameter $[0, 0] \le \beta < \alpha \le [1, 1]$, the adjustable lower and upper approximations of $\mathbb{F}$ in terms of $(U, V, \mathbb{R}_i)$, as represented by $\underline{\sum_{i=1}^m \mathbb{R}_i}^{\eta,\alpha}(\mathbb{F})$ and $\overline{\sum_{i=1}^m \mathbb{R}_i}^{\eta,\beta}(\mathbb{F})$, are given by

$$\underline{\sum_{i=1}^m \mathbb{R}_i}^{\eta,\alpha}(\mathbb{F}) = \left\{ x \in U | \delta_{\mathbb{F}}^{\sum\limits_{i=1}^m \mathbb{R}_i}(x) \ge \alpha \right\}, \tag{24}$$

$$\overline{\sum_{i=1}^m \mathbb{R}_i}^{\eta,\beta}(\mathbb{F}) = U - \left\{ x \in U | \delta_{\mathbb{F}}^{\sum\limits_{i=1}^m \mathbb{R}_i}(x) \le \beta \right\}, \tag{25}$$

where the parameter $\eta = \frac{i}{m}$ indicates the expected risk preference of decision makers, a scenario to depict the gradually changing procedure from the pessimistic version to the optimistic version of HFL MG-DTRSs over two universes. Specifically, the adjustable version of HFL MG-DTRSs over two universes reduces to the pessimistic version when $\eta = \frac{1}{m}$, whereas the adjustable version reduces to the optimistic version when $\eta = 1$. Thus, when the value of parameter η increases in the interval of $[\frac{1}{m}, 1]$, the expected risk preference of decision makers changes from completely risk-averse to completely risk-seeking. In general, the parameter η can usually be obtained from decision makers' preferences or empirical researches according to practical decision making situations.

Then, we name $(\underline{\sum_{i=1}^m \mathbb{R}_i}^{\eta,\alpha}(\mathbb{F}), \overline{\sum_{i=1}^m \mathbb{R}_i}^{\eta,\beta}(\mathbb{F}))$ an adjustable HFL MG-DTRS over two universes. The corresponding positive region, boundary region, and negative region are defined as follows:

$$POS_{\alpha,\beta}^{\eta}(\mathbb{F}) = \underline{\sum_{i=1}^m \mathbb{R}_i}^{\eta,\alpha}(\mathbb{F}),$$

$$BND_{\alpha,\beta}^{\eta}(\mathbb{F}) = \overline{\sum_{i=1}^m \mathbb{R}_i}^{\eta,\beta}(\mathbb{F}) - \underline{\sum_{i=1}^m \mathbb{R}_i}^{\mu,\alpha}(\mathbb{F}),$$

$$NEG^{\eta}_{\alpha,\beta}(\mathbb{F}) = U - POS^{\eta}_{\alpha,\beta}(\mathbb{F}) - BND^{\eta}_{\alpha,\beta}(\mathbb{F}).$$

Example 3.1. Suppose that $U = \{x_1, x_2, x_3\}$ and $V = \{y_1, y_2, y_3, y_4, y_5\}$, and let $S = \{s_0, s_1, \ldots, s_6\}$ be a linguistic term set. The given HFL relations over $U \times V$ are presented as follows:

$$\mathbb{R}_1 = \begin{array}{cc} & \begin{array}{ccccc} y_1 & y_2 & y_3 & y_4 & y_5 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} & \left\{ \begin{array}{ccccc} \{s_4, s_5\} & \{s_0, s_1\} & \{s_3\} & \{s_3, s_4\} & \{s_1, s_2\} \\ \{s_1, s_2\} & \{s_0, s_1\} & \{s_4, s_5\} & \{s_1\} & \{s_3, s_4\} \\ \{s_2, s_3\} & \{s_4, s_5\} & \{s_2\} & \{s_1\} & \{s_5, s_6\} \end{array} \right\} \end{array},$$

$$\mathbb{R}_2 = \begin{array}{cc} & \begin{array}{ccccc} y_1 & y_2 & y_3 & y_4 & y_5 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} & \left\{ \begin{array}{ccccc} \{s_5\} & \{s_2, s_3\} & \{s_4, s_5\} & \{s_3\} & \{s_0, s_1\} \\ \{s_3, s_4\} & \{s_1, s_2\} & \{s_4\} & \{s_2, s_3\} & \{s_3\} \\ \{s_2, s_3\} & \{s_4, s_5\} & \{s_2, s_3\} & \{s_2\} & \{s_3, s_4\} \end{array} \right\} \end{array},$$

$$\mathbb{R}_3 = \begin{array}{cc} & \begin{array}{ccccc} y_1 & y_2 & y_3 & y_4 & y_5 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} & \left\{ \begin{array}{ccccc} \{s_4, s_5\} & \{s_2, s_3\} & \{s_3\} & \{s_3\} & \{s_2, s_3\} \\ \{s_0, s_1\} & \{s_3\} & \{s_5\} & \{s_2, s_3\} & \{s_5, s_6\} \\ \{s_3, s_4\} & \{s_5, s_6\} & \{s_1, s_2\} & \{s_2, s_3\} & \{s_6\} \end{array} \right\} \end{array}.$$

Then, let $\mathbb{F}$ be an HFLTS over V such that

$$\mathbb{F} = \{\langle y_1, \{s_2, s_3\}\rangle, \langle y_2, \{s_3, s_4\}\rangle, \langle y_3, \{s_5, s_6\}\rangle, \langle y_4, \{s_3\}\rangle, \langle y_5, \{s_0, s_1\}\rangle\}.$$

According to Definition 3.2–Definition 3.4, we have

$$\Phi^{\mathbb{R}_1}_{\mathbb{F}}(x_1) = [1, 1] - \frac{ind(env(|\mathbb{R}_1(x_1) \Cap \mathbb{F}^c|))}{ind(env(|\mathbb{R}_1(x_1)|))} = [1, 1] - \frac{ind(env([s_7, s_{11}]))}{ind(env([s_{11}, s_{15}]))} = [0, 0.53].$$

In a similar manner, we also have $\Phi^{\mathbb{R}_1}_{\mathbb{F}}(x_2) = [0, 0.62]$, $\Phi^{\mathbb{R}_1}_{\mathbb{F}}(x_3) = [0, 0.41]$, $\Phi^{\mathbb{R}_2}_{\mathbb{F}}(x_1) = [0.14, 0.53]$, $\Phi^{\mathbb{R}_2}_{\mathbb{F}}(x_2) = [0, 0.44]$, $\Phi^{\mathbb{R}_2}_{\mathbb{F}}(x_3) = [0, 0.47]$, $\Phi^{\mathbb{R}_3}_{\mathbb{F}}(x_1) = [0, 0.41]$, $\Phi^{\mathbb{R}_3}_{\mathbb{F}}(x_2) = [0.07, 0.50]$, and $\Phi^{\mathbb{R}_3}_{\mathbb{F}}(x_3) = [0, 0.43]$.

If we take $\eta = 0.67$, then $\delta^{\sum_{i=1}^{3} \mathbb{R}_i}_{\mathbb{F}}(x_1) = [0, 0.53]$, $\delta^{\sum_{i=1}^{3} \mathbb{R}_i}_{\mathbb{F}}(x_2) = [0.07, 0.50]$, and $\delta^{\sum_{i=1}^{3} \mathbb{R}_i}_{\mathbb{F}}(x_3) = [0, 0.43]$. Moreover, when $\alpha = [0, 0.55]$ and $\beta = [0, 0.45]$, then $\underline{\sum_{i=1}^{3} \mathbb{R}_i}^{0.67,[0,0.55]}(\mathbb{F}) = \{x \in U | \delta_F^{\sum_{i=1}^{3} R_i}(x) \geq [0, 0.55]\} = \{x_2\}$ and $\overline{\sum_{i=1}^{3} \mathbb{R}_i}^{0.67,[0,0.45]}(\mathbb{F}) = U - \{x \in U | \delta_F^{\sum_{i=1}^{3} R_i}(x) \leq [0, 0.45]\} = \{x_1, x_2\}$. Therefore, we have $POS^{0.67}_{[0,0.55],[0,0.45]}(\mathbb{F}) = \underline{\sum_{i=1}^{3} \mathbb{R}_i}^{0.67,[0,0.55]}(\mathbb{F}) = \{x_2\}$, $BND^{0.67}_{[0,0.55],[0,0.45]}(\mathbb{F}) = \overline{\sum_{i=1}^{3} \mathbb{R}_i}^{0.67,[0,0.45]}(\mathbb{F}) - \underline{\sum_{i=1}^{3} \mathbb{R}_i}^{0.67,[0,0.55]}(\mathbb{F}) = \{x_1\}$, and $NEG^{0.67}_{[0,0.55],[0,0.45]}(\mathbb{F}) = U - POS^{0.67}_{[0,0.55],[0,0.45]}(\mathbb{F}) - BND^{0.67}_{[0,0.55],[0,0.45]}(\mathbb{F}) = \{x_3\}$.

Proposition 3.1. *Let U and V be two non-empty finite universes, $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set, and $\mathbb{R}_i \in HFLR(U \times V)(i = 1, \ldots, m)$ be an HFL relation. For any $\mathbb{F} \in HFL(V)$, with $[0, 0] \leq \beta < \alpha \leq [1, 1]$, then*

(1) $POS^{\eta}_{\alpha,\beta}(\mathbb{F}) \Cap BND^{\eta}_{\alpha,\beta}(\mathbb{F}) = \emptyset$,
$POS^{\eta}_{\alpha,\beta}(\mathbb{F}) \Cap NEG^{\eta}_{\alpha,\beta}(\mathbb{F}) = \emptyset$,
$BND^{\eta}_{\alpha,\beta}(\mathbb{F}) \Cap NEG^{\eta}_{\alpha,\beta}(\mathbb{F}) = \emptyset$,
(2) $POS^{\eta}_{\alpha,\beta}(\mathbb{F}) \Cup BND^{\eta}_{\alpha,\beta}(\mathbb{F}) \Cup NEG^{\eta}_{\alpha,\beta}(\mathbb{F}) = \mathbb{U}$.

Proposition 3.2. *Let U and V be two non-empty finite universes, $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set, and $\mathbb{R}_i \in HFLR(U \times V)(i = 1, \ldots, m)$ be an HFL relation. For any $\mathbb{F} \in HFL(V)$, with $[0, 0] \leq \beta < \alpha \leq [1, 1]$, then*

(1) $\underline{\sum_{i=1}^{m} \mathbb{R}_i}^{P,\alpha}(\mathbb{F}) \sqsubseteq \underline{\sum_{i=1}^{m} \mathbb{R}_i}^{\eta,\alpha}(\mathbb{F}) \sqsubseteq \underline{\sum_{i=1}^{m} \mathbb{R}_i}^{O,\alpha}(\mathbb{F})$,
(2) $\overline{\sum_{i=1}^{m} \mathbb{R}_i}^{O,\beta}(\mathbb{F}) \sqsubseteq \overline{\sum_{i=1}^{m} \mathbb{R}_i}^{\eta,\beta}(\mathbb{F}) \sqsubseteq \overline{\sum_{i=1}^{m} \mathbb{R}_i}^{P,\beta}(\mathbb{F})$,

where $\underline{\sum_{i=1}^{m} \mathbb{R}_i}^{O,\alpha}(\mathbb{F}) = \{x \in U | \xi_{\mathbb{F}}^{\sum_{i=1}^{m} \mathbb{R}_i}(x) \geq \alpha\}$ and $\overline{\sum_{i=1}^{m} \mathbb{R}_i}^{O,\beta}(\mathbb{F}) = U - \{x \in U | \xi_{\mathbb{F}}^{\sum_{i=1}^{m} \mathbb{R}_i}(x) \leq \beta\}$ represent the optimistic lower and upper approximations of $\mathbb{F}$ with respect to $(U, V, \mathbb{R}_i)$. Similarly, $\underline{\sum_{i=1}^{m} \mathbb{R}_i}^{P,\alpha}(\mathbb{F}) = \{x \in U | \zeta_{\mathbb{F}}^{\sum_{i=1}^{m} \mathbb{R}_i}(x) \geq \alpha\}$ and $\overline{\sum_{i=1}^{m} \mathbb{R}_i}^{P,\beta}(\mathbb{F}) = U - \{x \in U | \zeta_{\mathbb{F}}^{\sum_{i=1}^{m} \mathbb{R}_i}(x) \leq \beta\}$ represent the pessimistic lower and upper approximations of $\mathbb{F}$ with respect to $(U, V, \mathbb{R}_i)$.

Proposition 3.3. *Let U and V be two non-empty finite universes, $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set, and $\mathbb{R}_i \in HFLR(U \times V)(i = 1, \ldots, m)$ be an HFL relation. For any $\mathbb{F}, \mathbb{G} \in HFL(V)$, with $[0, 0] \leq \beta < \alpha \leq [1, 1]$, then*

(1) $\underline{\sum_{i=1}^{m} \mathbb{R}_i}^{\eta,\alpha}(\emptyset) = \overline{\sum_{i=1}^{m} \mathbb{R}_i}^{\eta,\beta}(\emptyset) = \emptyset$,
$\underline{\sum_{i=1}^{m} \mathbb{R}_i}^{\eta,\alpha}(\mathbb{U}) = \overline{\sum_{i=1}^{m} \mathbb{R}_i}^{\eta,\beta}(\mathbb{U}) = \mathbb{U}$,

(2) $\underline{\sum_{i=1}^{m}\mathbb{R}_i}^{\eta,\alpha}(\mathbb{F}) \sqsubseteq \overline{\sum_{i=1}^{m}\mathbb{R}_i}^{\eta,\beta}(\mathbb{F})$,

(3) $\mathbb{F} \sqsubseteq \mathbb{G} arrow \underline{\sum_{i=1}^{m}\mathbb{R}_i}^{\eta,\alpha}(\mathbb{F}) \sqsubseteq \underline{\sum_{i=1}^{m}\mathbb{R}_i}^{\eta,\alpha}(\mathbb{G}), \overline{\sum_{i=1}^{m}\mathbb{R}_i}^{\eta,\alpha}(\mathbb{F}) \sqsubseteq \overline{\sum_{i=1}^{m}\mathbb{R}_i}^{\eta,\alpha}(\mathbb{G})$.

Proof.

(1) According to Definition 3.4, we have

$$\underline{\sum_{i=1}^{m}\mathbb{R}_i}^{\eta,\alpha}(\emptyset) = \{x \in U | \delta_{\emptyset}^{\sum_{i=1}^{m}\mathbb{R}_i}(x) \geq \alpha\} = \{x \in U | \Phi_{\emptyset}^{\mathbb{R}_{\sigma(i)}}(x) \geq \alpha\}$$
$$= \{x \in U | [1,1] - \frac{ind(env(|\mathbb{R}_i(x) \Cap \mathbb{U}|))}{ind(env(|\mathbb{R}_i(x)|))} \geq \alpha\} = \emptyset.$$

Hence, $\underline{\sum_{i=1}^{m}\mathbb{R}_i}^{\eta,\alpha}(\emptyset) = \emptyset$. In a similar manner, $\overline{\sum_{i=1}^{m}\mathbb{R}_i}^{\eta,\beta}(\emptyset) = \emptyset$ can be obtained.

$$\underline{\sum_{i=1}^{m}\mathbb{R}_i}^{\eta,\alpha}(\mathbb{U}) = \{x \in U | \delta_{\mathbb{U}}^{\sum_{i=1}^{m}\mathbb{R}_i}(x) \geq \alpha\} = \{x \in U | \Phi_{\mathbb{U}}^{\mathbb{R}_{\sigma(i)}}(x) \geq \alpha\}$$
$$= \{x \in U | [1,1] - \frac{ind(env(|\mathbb{R}_i(x) \Cap \emptyset|))}{ind(env(|\mathbb{R}_i(x)|))} \geq \alpha\} = \mathbb{U}.$$

Hence, we can conclude that $\underline{\sum_{i=1}^{m}\mathbb{R}_i}^{\eta,\alpha}(\mathbb{U}) = \mathbb{U}$. In a similar manner, $\overline{\sum_{i=1}^{m}\mathbb{R}_i}^{\eta,\beta}(\mathbb{U}) = \mathbb{U}$ can be obtained.

(2) $\underline{\sum_{i=1}^{m}\mathbb{R}_i}^{\eta,\alpha}(\mathbb{F}) = \{x \in U | \delta_{\mathbb{F}}^{\sum_{i=1}^{m}\mathbb{R}_i}(x) \geq \alpha\} \sqsubseteq \{x \in U | \delta_{\mathbb{F}}^{\sum_{i=1}^{m}\mathbb{R}_i}(x) > \beta\}$

$$= U - \{x \in U | \delta_{\mathbb{F}}^{\sum_{i=1}^{m}\mathbb{R}_i}(x) \leq \beta\} = \overline{\sum_{i=1}^{m}\mathbb{R}_i}^{\eta,\beta}(\mathbb{F}).$$

Thus, $\underline{\sum_{i=1}^{m}\mathbb{R}_i}^{\eta,\alpha}(\mathbb{F}) \sqsubseteq \overline{\sum_{i=1}^{m}\mathbb{R}_i}^{\eta,\beta}(\mathbb{F})$.

(3) Given that $\mathbb{F} \sqsubseteq \mathbb{G}$, according to Definition 3.4, we have

$$\underline{\sum_{i=1}^{m}\mathbb{R}_i}^{\eta,\alpha}(\mathbb{F}) = \{x \in U | \delta_{\mathbb{F}}^{\sum_{i=1}^{m}\mathbb{R}_i}(x) \geq \alpha\} = \{x \in U | \Phi_{\mathbb{F}}^{\mathbb{R}_{\sigma(i)}}(x) \geq \alpha\}$$
$$= \{x \in U | [1,1] - \frac{ind(env(|\mathbb{R}_i(x) \Cap \mathbb{F}^c|))}{ind(env(|\mathbb{R}_i(x)|))} \geq \alpha\} \leq \{x \in U | [1,1] - \frac{ind(env(|\mathbb{R}_i(x) \Cap \mathbb{G}^c|))}{ind(env(|\mathbb{R}_i(x)|))} \geq \alpha\}$$
$$= \{x \in U | \Phi_{\mathbb{G}}^{\mathbb{R}_{\sigma(i)}}(x) \geq \alpha\} = \{x \in U | \delta_{\mathbb{G}}^{\sum_{i=1}^{m}\mathbb{R}_i}(x) \geq \alpha\} = \underline{\sum_{i=1}^{m}\mathbb{R}_i}^{\eta,\alpha}(\mathbb{G}).$$

Hence, we obtain $\mathbb{F} \sqsubseteq \mathbb{G} arrow \underline{\sum_{i=1}^{m}\mathbb{R}_i}^{\eta,\alpha}(\mathbb{F}) \sqsubseteq \underline{\sum_{i=1}^{m}\mathbb{R}_i}^{\eta,\alpha}(\mathbb{G})$, and $\mathbb{F} \sqsubseteq \mathbb{G} arrow \overline{\sum_{i=1}^{m}\mathbb{R}_i}^{\eta,\alpha}(\mathbb{F}) \sqsubseteq \overline{\sum_{i=1}^{m}\mathbb{R}_i}^{\eta,\alpha}(\mathbb{G})$ is concluded in an identical manner. □

4. Group decision making method by using adjustable HFL MG-DTRSs over two universes

A general group decision making rule is proposed by virtue of adjustable HFL MG-DTRSs over two universes. Furthermore, we intend to study the newly proposed group decision making rule by exploring a person-job fit issue, which is regarded as an attractive study subject in the discipline of human resources. The person-job fit group decision making approach not only reasonably matches job seekers' abilities with a corresponding position, but it also promotes the sound development of various business organizations, and eventually achieves a balance between corporate and personal interests.

4.1. Problem statement

With the rapid development of the society and the economy, the reasonable employment and positioning of talents have gained important strategic significance among enterprises, especially since enterprise competition is often depicted as talent competition. Person-job fit, a theory that explores the extent to which job seekers or employees have preferences for job characteristics that are consistent with the actual job requirements, provides an efficient scheme for ensuring a rational utilization of personnel.

Person-job fit involves three fundamental processes: (1) position capability analysis, which aims to construct relationships between positions and required capabilities; (2) professional capability analysis, which aims to evaluate the required capabilities by a job seeker; (3) the matching procedure based on the previous two steps. An increasing number of enterprises and individuals have become much more willing to analyze position capabilities and professional capabilities by using HFL information. For instance, according to the qualifications of a banking and financial service organization posted on a popular job-hunting website in mainland China, job seekers should meet the following requirements for the position of contact center service and sales representative: (a) good spoken & written English and Mandarin; (b) excellent communication skills, and polite and friendly at all times; (c) expertise in specialized software packages and applications. Some common linguistic terms, such as "proficiency", "excellent" and "good", can be extracted from the post job requirements, and they can comprise a linguistic term set S as follows:

$$S = \{s_0 : extremely\ limited\ user, s_1 : limited\ user, s_2 : modest\ user, s_3 : competent\ user, s_4 : good\ user,$$
$$s_5 : excellent\ user, s_6 : expert\ user\}.$$

Thus, with the aid of the constructed linguistic term set S, one can complete the position capability analysis and the professional capability analysis by means of HFL information. Consequently, by using adjustable HFL MG-DTRSs over two universes, a person-job fit matching approach based on the results of position capability analysis and professional capability analysis is proposed.

4.2. Application model

According to the aforementioned person-job fit processes, relationships between positions and required capabilities should be constructed in the position capability analysis. This step involves two kinds of objects concerning person-job fit, i.e., the set of positions and the set of required capabilities. We let $U = \{x_1, x_2, \ldots, x_k\}$ be a position set and $V = \{y_1, y_2, \ldots, y_n\}$ be a required capability set. In person-job fit processes, multiple capabilities may be required to qualify for a position, while each required capability may be listed as one of the qualifications for multiple positions. Suppose that the person responsible for the business invites multiple human resource professionals to establish relationships between positions and required capabilities, and each of the professionals expresses a relationship by virtue of HFL information that is represented by $\mathbb{R}_i \in HFLR(U \times V)$. Then, in the step of the professional capability analysis, we let $\mathbb{F} \in HFL(V)$ be the set of professional competence evaluation results. According to the above two steps, an HFL multigranulation information system over two universes $(U, V, \mathbb{R}_i, \mathbb{F})$ can be obtained.

Subsequently, we present a group decision making approach by using adjustable HFL MG-DTRSs over two universes. Prior to the development of such an approach, in effectively describing complicated linguistic expressions in many situations, the introduction of a transformation function [18] is necessary to change various linguistic expressions H obtained by the context-free grammar G_H into an HFLE $h_{\mathbb{F}}(x)$, as represented by $E_{G_H} : Harrowh_{\mathbb{F}}(x)$, where

$$
\begin{aligned}
&E_{G_H}(s_i) = \{s_i | s_i \in S\},\\
&E_{G_H}(greater\ than\ s_i) = \{s_j | s_j \in S\ and\ s_j > s_i\},\\
&E_{G_H}(lower\ than\ s_i) = \{s_j | s_j \in S\ and\ s_j < s_i\},\\
&E_{G_H}(at\ most\ s_i) = \{s_j | s_j \in S\ and\ s_j \le s_i\},\\
&E_{G_H}(at\ least\ s_i) = \{s_j | s_j \in S\ and\ s_j \ge s_i\},\\
&E_{G_H}(between\ s_i\ and\ s_j) = \{s_k | s_k \in S\ and\ s_i \le s_k \le s_j\}.
\end{aligned}
$$

Then, a matching approach for person-job fit based on adjustable HFL MG-DTRSs over two universes is presented. The problem for a position with regard to the set of professional competence evaluation results involves determining whether the position can be selected, needs extra assessment, or can be discarded. Thus, this procedure can be depicted as deciding on the acceptance region, noncommitment region, and rejection region of the set of professional competence evaluation results $\mathbb{F}$ in terms of $(U, V, \mathbb{R}_i)$.

According to the idea of DTRSs, suppose that $\mathbb{F}$ and $\mathbb{F}^c$ denote a position x belonging to $\mathbb{F}$ and not belonging to $\mathbb{F}$. We also let $\lambda_{PP} = [\lambda_{PP}^-, \lambda_{PP}^+]$, $\lambda_{BP} = [\lambda_{BP}^-, \lambda_{BP}^+]$, and $\lambda_{NP} = [\lambda_{NP}^-, \lambda_{NP}^+]$ be the losses incurred for taking actions a_P, a_B, and a_N when $x \in \mathbb{F}$, while $\lambda_{PN} = [\lambda_{PN}^-, \lambda_{PN}^+]$, $\lambda_{BN} = [\lambda_{BN}^-, \lambda_{BN}^+]$, and $\lambda_{NN} = [\lambda_{NN}^-, \lambda_{NN}^+]$ be the losses triggered by taking the same actions when $x \in \mathbb{F}^c$. Then, suppose that $\Pr(\mathbb{F}|x)$ represents the conditional probability of a position belonging to the set of professional competence evaluation results $\mathbb{F}$ given that the position is expressed by x. Hence, for a position x, the expected loss $\mathcal{R}(a_i|x)$ can be given by

$$
\begin{aligned}
&\mathcal{R}(a_P|x) = \lambda_{PP}\Pr(\mathbb{F}|x) + \lambda_{PN}\Pr(\mathbb{F}^c|x) = \lambda_{PP}\delta_{\mathbb{F}}^{\sum_{i=1}^m \mathbb{R}_i}(x) + \lambda_{PN}\delta_{\mathbb{F}^c}^{\sum_{i=1}^m \mathbb{R}_i}(x),\\
&\mathcal{R}(a_B|x) = \lambda_{BP}\Pr(\mathbb{F}|x) + \lambda_{BN}\Pr(\mathbb{F}^c|x) = \lambda_{BP}\delta_{\mathbb{F}}^{\sum_{i=1}^m \mathbb{R}_i}(x) + \lambda_{BN}\delta_{\mathbb{F}^c}^{\sum_{i=1}^m \mathbb{R}_i}(x),\\
&\mathcal{R}(a_N|x) = \lambda_{NP}\Pr(\mathbb{F}|x) + \lambda_{NN}\Pr(\mathbb{F}^c|x) = \lambda_{NP}\delta_{\mathbb{F}}^{\sum_{i=1}^m \mathbb{R}_i}(x) + \lambda_{NN}\delta_{\mathbb{F}^c}^{\sum_{i=1}^m \mathbb{R}_i}(x).
\end{aligned}
$$

In light of Bayesian decision procedures, the decision rule associated with minimum-cost approaches can be obtained.

(P_2) If $\mathcal{R}(a_P|x) \le \mathcal{R}(a_B|x)$ and $\mathcal{R}(a_P|x) \le \mathcal{R}(a_N|x)$, then decide $x \in POS(\mathbb{F})$.

(B_2) If $\mathcal{R}(a_B|x) \le \mathcal{R}(a_P|x)$ and $\mathcal{R}(a_B|x) \le \mathcal{R}(a_N|x)$, then decide $x \in BND(\mathbb{F})$.

(N_2) If $\mathcal{R}(a_N|x) \le \mathcal{R}(a_P|x)$ and $\mathcal{R}(a_N|x) \le \mathcal{R}(a_B|x)$, then decide $x \in NEG(\mathbb{F})$.

For the person-job fit, the risk of classifying the suitable position as the positive region and the risk of classifying the unsuitable position as the negative region are minimal, whereas the risk of classifying the suitable position as the negative region and the risk of classifying the unsuitable position as the positive region are maximal. Thus, we have $\lambda_{PP}^- \le \lambda_{BP}^- < \lambda_{NP}^-$, $\lambda_{PP}^+ \le \lambda_{BP}^+ < \lambda_{NP}^+$, $\lambda_{NN}^- \le \lambda_{BN}^- < \lambda_{PN}^-$, and $\lambda_{NN}^+ \le \lambda_{BN}^+ < \lambda_{PN}^+$. In particular, if $\delta_{\mathbb{F}}^{\sum_{i=1}^m \mathbb{R}_i}(x) + \delta_{\mathbb{F}^c}^{\sum_{i=1}^m \mathbb{R}_i}(x) = [1, 1]$, $(\lambda_{PN}^- - \lambda_{BN}^-)(\lambda_{NP}^- - \lambda_{BP}^-) \ge (\lambda_{BP}^- - \lambda_{PP}^-)(\lambda_{BN}^- - \lambda_{NN}^-)$, $(\lambda_{PN}^+ - \lambda_{BN}^+)(\lambda_{NP}^+ - \lambda_{BP}^+) \ge (\lambda_{BP}^+ - \lambda_{PP}^+)(\lambda_{BN}^+ - \lambda_{NN}^+)$, and $\alpha > \beta$, then the decision rules with respect to adjustable HFL MG-DTRSs over two universes can be simplified.

(P_3) If $\delta_{\mathbb{F}}^{\sum_{i=1}^m \mathbb{R}_i}(x) \ge \alpha$, decide $x \in POS_{\alpha,\beta}^{\eta}(\mathbb{F})$, then the position x is a suitable position for the job seeker.

(B_3) If $\beta < \delta_{\mathbb{F}}^{\sum_{i=1}^m \mathbb{R}_i}(x) < \alpha$, decide $x \in BND_{\alpha,\beta}^{\eta}(\mathbb{F})$, then human resource experts are not sure whether the position x is suitable for the job seeker or not, and they need additional available information to make a decision.

(N_3) If $\delta_{\mathbb{F}}^{\sum_{i=1}^m \mathbb{R}_i}(x) \le \beta$, decide $x \in NEG_{\alpha,\beta}^{\eta}(\mathbb{F})$, then the position x is an unsuitable position for the job seeker. Here, $\alpha = [\frac{\lambda_{PN}^- - \lambda_{BN}^-}{(\lambda_{PN}^- - \lambda_{BN}^-) + (\lambda_{BP}^- - \lambda_{PP}^-)}, \frac{\lambda_{PN}^+ - \lambda_{BN}^+}{(\lambda_{PN}^+ - \lambda_{BN}^+) + (\lambda_{BP}^+ - \lambda_{PP}^+)}]$, $\beta = [\frac{\lambda_{BN}^- - \lambda_{NN}^-}{(\lambda_{BN}^- - \lambda_{NN}^-) + (\lambda_{NP}^- - \lambda_{BP}^-)}, \frac{\lambda_{BN}^+ - \lambda_{NN}^+}{(\lambda_{BN}^+ - \lambda_{NN}^+) + (\lambda_{NP}^+ - \lambda_{BP}^+)}]$.

Table 1
Linguistic relationship between positions and required capabilities given by the first human resource expert.

$\mathbb{R}_1$	y_1	y_2	y_3	y_4	y_5	y_6
x_1	{*competent user*}	{*good user*}	{*excellent user*}	{*competent user*}	{*expert user*}	{*competent user*}
x_2	{*excellent user*}	{*expert user*}	{*competent user*}	{*expert user*}	{*competent user*}	{*good user*}
x_3	{*excellent user*}	{*excellent user*}	{*good user*}	{*competent user*}	{*excellent user*}	{*good user*}
x_4	{*competent user*}	{*good user*}	{*excellent user*}	{*expert user*}	{*good user*}	{*excellent user*}

Table 2
Linguistic relationship between positions and required capabilities given by the second human resource expert.

$\mathbb{R}_2$	y_1	y_2	y_3	y_4	y_5	y_6
x_1	{*good user*}	{*good user*}	{*expert user*}	{*good user*}	{*expert user*}	{*good user*}
x_2	{*excellent user*}	{*expert user*}	{*good user*}	{*competent user*}	{*good user*}	{*excellent user*}
x_3	{*expert user*}	{*excellent user*}	{*good user*}	{*competent user*}	{*good user*}	{*good user*}
x_4	{*competent user*}	{*competent user*}	{*good user*}	{*expert user*}	{*good user*}	{*excellent user*}

Table 3
Linguistic relationship between positions and required capabilities given by the third human resource expert.

$\mathbb{R}_3$	y_1	y_2	y_3	y_4	y_5	y_6
x_1	{*good user*}	{*excellent user*}	{*excellent user*}	{*excellent user*}	{*expert user*}	{*good user*}
x_2	{*expert user*}	{*expert user*}	{*competent user*}	{*expert user*}	{*competent user*}	{*competent user*}
x_3	{*good user*}	{*excellent user*}	{*good user*}	{*excellent user*}	{*excellent user*}	{*good user*}
x_4	{*good user*}	{*good user*}	{*excellent user*}	{*good user*}	{*good user*}	{*good user*}

4.3. Algorithm for person-job fit

Input HFL multigranulation approximation space over two universes $(U, V, \mathbb{R}_i)$ and set of professional competence evaluation results $\mathbb{F}$.

Output Optimal position for the matching procedure of person-job fit.

Step 1 Determine the risk coefficients $\lambda_{PP} = [\lambda_{PP}^-, \lambda_{PP}^+]$, $\lambda_{BP} = [\lambda_{BP}^-, \lambda_{BP}^+]$, $\lambda_{NP} = [\lambda_{NP}^-, \lambda_{NP}^+]$, $\lambda_{PN} = [\lambda_{PN}^-, \lambda_{PN}^+]$, $\lambda_{BN} = [\lambda_{BN}^-, \lambda_{BN}^+]$, and $\lambda_{NN} = [\lambda_{NN}^-, \lambda_{NN}^+]$.

Step 2 Calculate the thresholds α and β.

Step 3 Calculate the adjustable membership degree $\delta_{\mathbb{F}}^{\sum_{i=1}^{m}\mathbb{R}_i}(x)$.

Step 4 Calculate the adjustable lower approximation $\underline{\sum_{i=1}^{m}\mathbb{R}_i}^{\eta,\alpha}(\mathbb{F})$ and upper approximation $\overline{\sum_{i=1}^{m}\mathbb{R}_i}^{\eta,\beta}(\mathbb{F})$.

Step 5 Determine the corresponding positive region $POS_{\alpha,\beta}^{\eta}(\mathbb{F})$, boundary region $BND_{\alpha,\beta}^{\eta}(\mathbb{F})$, and negative region $NEG_{\alpha,\beta}^{\eta}(\mathbb{F})$ with respect to the adjustable lower and upper approximations.

Step 6 Determine the optimal position based on decision rules (P_3), (B_3), and (N_3).

5. Numerical example

We study a group decision making problem related to a person-job fit problem (adapted from Zhang et al. [43]) to show the steps of the proposed method described in Section 4. Additionally, to further prove the validity and practicability of the established group decision making rule, a comparative analysis along with several necessary discussions is conducted by using other existing approaches in the background of the same numerical example.

5.1. Application of adjustable HFL MG-DTRSs over two universes

Let $U = \{sales\ representative\ (x_1), algorithm\ engineer\ (x_2), finance\ analyst\ (x_3), administrative\ assistant\ (x_4)\}$ be four positions for a banking and financial services organization, $V = \{mathematical\ skills\ (y_1), computerskills\ (y_2), English\ skills\ (y_3), writing\ skills\ (y_4), communication\ skills\ (y_5), management\ skills\ (y_6)\}$ be six required capabilities for evaluating whether a position is suitable for the job seeker. Suppose that the person responsible for the business invites three human resource experts to construct relationships between positions and required capabilities, each of them expresses such a relationship in the form of linguistic expressions shown in Tables 1–3, based on a linguistic term set S, where $S = \{s_0 : extremely\ limited\ user, s_1 : limited\ user, s_2 : modest\ user, s_3 : competent\ user, s_4 : good\ user, s_5 : excellent\ user, s_6 : expert\ user\}$. Then, we convert the given linguistic expressions to HFL relations $\mathbb{R}_i \in HFLR(U \times V)(i = 1, 2, 3)$, as presented in Tables 4–6, Thereafter, we present the results in the six tables.

According to the transformation function presented in Section 4.2, we can transform linguistic relationships into HFL relationships between positions and required capabilities given by three human resource experts.

Table 4
HFL relationship between positions and required capabilities given by the first human resource expert.

$\mathbb{R}_1$	y_1	y_2	y_3	y_4	y_5	y_6
x_1	$\{s_3\}$	$\{s_4\}$	$\{s_5\}$	$\{s_3\}$	$\{s_6\}$	$\{s_3\}$
x_2	$\{s_5\}$	$\{s_6\}$	$\{s_3\}$	$\{s_6\}$	$\{s_3\}$	$\{s_4\}$
x_3	$\{s_5\}$	$\{s_5\}$	$\{s_4\}$	$\{s_3\}$	$\{s_5\}$	$\{s_4\}$
x_4	$\{s_3\}$	$\{s_4\}$	$\{s_5\}$	$\{s_6\}$	$\{s_4\}$	$\{s_5\}$

Table 5
HFL relationship between positions and required capabilities given by the second human resource expert.

$\mathbb{R}_2$	y_1	y_2	y_3	y_4	y_5	y_6
x_1	$\{s_4\}$	$\{s_4\}$	$\{s_6\}$	$\{s_4\}$	$\{s_6\}$	$\{s_4\}$
x_2	$\{s_5\}$	$\{s_6\}$	$\{s_4\}$	$\{s_3\}$	$\{s_4\}$	$\{s_5\}$
x_3	$\{s_6\}$	$\{s_5\}$	$\{s_4\}$	$\{s_3\}$	$\{s_4\}$	$\{s_4\}$
x_4	$\{s_3\}$	$\{s_3\}$	$\{s_4\}$	$\{s_6\}$	$\{s_4\}$	$\{s_5\}$

Table 6
HFL relationship between positions and required capabilities given by the third human resource expert.

$\mathbb{R}_3$	y_1	y_2	y_3	y_4	y_5	y_6
x_1	$\{s_4\}$	$\{s_5\}$	$\{s_5\}$	$\{s_5\}$	$\{s_6\}$	$\{s_4\}$
x_2	$\{s_6\}$	$\{s_6\}$	$\{s_3\}$	$\{s_6\}$	$\{s_3\}$	$\{s_3\}$
x_3	$\{s_4\}$	$\{s_5\}$	$\{s_4\}$	$\{s_5\}$	$\{s_5\}$	$\{s_4\}$
x_4	$\{s_4\}$	$\{s_4\}$	$\{s_5\}$	$\{s_4\}$	$\{s_4\}$	$\{s_4\}$

Subsequently, the set of professional competence evaluation results $\mathbb{F}$ for a job seeker is provided by the invited human resource experts in the form of linguistic expressions.

$$\mathbb{F} = \{\langle y_1, \{between\ good\ user\ and\ excellent\ user\}\rangle, \langle y_2, \{between\ excellent\ user\ and\ expert\ user\}\rangle,$$
$$\langle y_3, \{competent\ user\}\rangle, \langle y_4, \{between\ good\ user\ and\ excellent\ user\}\rangle, \langle y_5, \{excellent\ user\}\rangle, \langle y_6, \{modest\ user\}\rangle\}.$$

On the basis of the transformation function, we transform the above evaluation results into the form of HFLTSs as follows:

$$\mathbb{F} = \Big\{\langle y_1, \{s_4, s_5\}\rangle, \langle y_2, \{s_5, s_6\}\rangle, \langle y_3, \{s_3\}\rangle, \langle y_4, \{s_4, s_5\}\rangle, \langle y_5, \{s_5\}\rangle, \langle y_6, \{s_2\}\rangle\Big\}.$$

In what follows, we utilize the group decision method based on adjustable HFL MG-DTRSs over two universes to solve the person-job fit problem.

First, we assume that the six risk coefficients are $\lambda_{PP} = [\lambda_{PP}^-, \lambda_{PP}^+] = [0.28, 0.36]$, $\lambda_{BP} = [\lambda_{BP}^-, \lambda_{BP}^+] = [0.57, 0.64]$, $\lambda_{NP} = [\lambda_{NP}^-, \lambda_{NP}^+] = [0.68, 0.73]$, $\lambda_{PN} = [\lambda_{PN}^-, \lambda_{PN}^+] = [0.62, 0.64]$, $\lambda_{BN} = [\lambda_{BN}^-, \lambda_{BN}^+] = [0.23, 0.24]$, and $\lambda_{NN} = [\lambda_{NN}^-, \lambda_{NN}^+] = [0.11, 0.12]$. Then, the thresholds α and β can be obtained as follows:

$$\alpha = \left[\frac{\lambda_{PN}^- - \lambda_{BN}^-}{(\lambda_{PN}^- - \lambda_{BN}^-) + (\lambda_{BP}^- - \lambda_{PP}^-)}, \frac{\lambda_{PN}^+ - \lambda_{BN}^+}{(\lambda_{PN}^+ - \lambda_{BN}^+) + (\lambda_{BP}^+ - \lambda_{PP}^+)}\right] = [0.57, 0.59],$$

$$\beta = \left[\frac{\lambda_{BN}^- - \lambda_{NN}^-}{(\lambda_{BN}^- - \lambda_{NN}^-) + (\lambda_{NP}^- - \lambda_{BP}^-)}, \frac{\lambda_{BN}^+ - \lambda_{NN}^+}{(\lambda_{BN}^+ - \lambda_{NN}^+) + (\lambda_{NP}^+ - \lambda_{BP}^+)}\right] = [0.52, 0.57].$$

Subsequently, we compute the membership degree of each position in the set of professional competence evaluation results with respect to each HFL relationship between positions and required capabilities.

$$\Phi_{\mathbb{F}}^{\mathbb{R}_1}(x_1) = [1, 1] - \frac{ind(env(|\mathbb{R}_1(x_1) \Cap \mathbb{F}^c|))}{ind(env(|\mathbb{R}_1(x_1)|))} = [1, 1] - \frac{ind(env([s_9, s_{12}]))}{ind(env(s_{24}))} = [0.50, 0.63].$$

In an identical manner, we further have $\Phi_{\mathbb{F}}^{\mathbb{R}_1}(x_2) = [0.52, 0.63]$, $\Phi_{\mathbb{F}}^{\mathbb{R}_1}(x_3) = [0.50, 0.62]$, $\Phi_{\mathbb{F}}^{\mathbb{R}_1}(x_4) = [0.52, 0.63]$, $\Phi_{\mathbb{F}}^{\mathbb{R}_2}(x_1) = [0.54, 0.64]$, $\Phi_{\mathbb{F}}^{\mathbb{R}_2}(x_2) = [0.52, 0.63]$, $\Phi_{\mathbb{F}}^{\mathbb{R}_2}(x_3) = [0.50, 0.62]$, $\Phi_{\mathbb{F}}^{\mathbb{R}_2}(x_4) = [0.48, 0.60]$, $\Phi_{\mathbb{F}}^{\mathbb{R}_3}(x_1) = [0.55, 0.66]$, $\Phi_{\mathbb{F}}^{\mathbb{R}_3}(x_2) = [0.56, 0.67]$, $\Phi_{\mathbb{F}}^{\mathbb{R}_3}(x_3) = [0.52, 0.63]$, and $\Phi_{\mathbb{F}}^{\mathbb{R}_3}(x_4) = [0.48, 0.60]$.

According to the preferences of experts in this study and the findings from the case studies of previous empirical research on person-job fit, parameter $\eta = 0.67$. Then, the adjustable membership degree for each position can be obtained.

$$\delta_{\mathbb{F}}^{\sum_{i=1}^{3}\mathbb{R}_i}(x_1) = [0.54, 0.64], \delta_{\mathbb{F}}^{\sum_{i=1}^{3}\mathbb{R}_i}(x_2) = [0.52, 0.63], \delta_{\mathbb{F}}^{\sum_{i=1}^{3}\mathbb{R}_i}(x_3) = [0.50, 0.62], \delta_{\mathbb{F}}^{\sum_{i=1}^{3}\mathbb{R}_i}(x_4) = [0.48, 0.60].$$

By means of the obtained thresholds and the adjustable membership degrees, we further calculate the adjustable lower and upper approximations of $\mathbb{F}$.

$$\underline{\sum_{i=1}^{3}\mathbb{R}_i}^{0.67,[0.57,0.59]}(\mathbb{F}) = \left\{ x \in U \,\middle|\, \delta_F^{\sum_{i=1}^{3}R_i}(x) \geq [0.57, 0.59] \right\} = \{x_1\},$$

$$\overline{\sum_{i=1}^{3}\mathbb{R}_i}^{0.67,[0.52,0.57]}(\mathbb{F}) = U - \left\{ x \in U \,\middle|\, \delta_F^{\sum_{i=1}^{3}R_i}(x) \leq [0.52, 0.57] \right\} = \{x_1, x_2, x_3\}.$$

The positive region, boundary region, and negative region correspond to the adjustable lower and upper approximations that can be calculated as follows:

$$POS^{0.67}_{[0.57,0.59],[0.52,0.57]}(\mathbb{F}) = \underline{\sum_{i=1}^{3}\mathbb{R}_i}^{0.67,[0.57,0.59]}(\mathbb{F}) = \{x_1\},$$

$$BND^{0.67}_{[0.57,0.59],[0.52,0.57]}(\mathbb{F}) = \overline{\sum_{i=1}^{3}\mathbb{R}_i}^{0.67,[0.52,0.57]}(\mathbb{F}) - \underline{\sum_{i=1}^{3}\mathbb{R}_i}^{0.67,[0.57,0.59]}(\mathbb{F}) = \{x_2, x_3\},$$

$$NEG^{0.67}_{[0.57,0.59],[0.52,0.57]}(\mathbb{F}) = U - POS^{0.67}_{[0.57,0.59],[0.52,0.57]}(\mathbb{F}) - BND^{0.67}_{[0.57,0.59],[0.52,0.57]}(\mathbb{F}) = \{x_4\}.$$

The following conclusions for the aforementioned person-job fit process can be obtained by referring to decision rules (P_3), (B_3), and (N_3), as presented in Section 4.2.

(1) The most suitable position for the job seeker is sales representative, which should be an important consideration in this person-job fit case.
(2) Human resource experts are not sure about the suitability of the positions of algorithm engineer and finance analyst for the job seeker, and they need additional available information to make a decision.
(3) The positions of administrative assistant is considered an unsuitable position for the job seeker.

5.2. Comparison analysis and discussions

The above subsection presents a detailed person-job fit procedure based on adjustable HFL MG-DTRSs over two universes. In this subsection, the comparative analysis and its corresponding discussions are discussed to illustrate the quality and efficiency of the above decision results.

5.2.1. Comparison analysis with the approach proposed in Ref. [44]

In Ref. [44], in accordance with the multi-granularity computing paradigm, a group decision making approach based on interval-valued hesitant fuzzy MGRSs over two universes is constructed to deal with a steam turbine fault diagnosis problem. By using the method in Ref. [44], the following steps can be conducted to obtain the most suitable position for the job seeker.

The optimistic and the pessimistic HFL multigranulation rough approximations of $\mathbb{F}$ in terms of $(U, V, \mathbb{R}_i)$ can be calculated initially as follows:

$$\underline{\sum_{i=1}^{3}\mathbb{R}_i}^{O}(\mathbb{F}) = \{\langle x_1, \{s_3\}\rangle, \langle x_2, \{s_3\}\rangle, \langle x_3, \{s_2\}\rangle, \langle x_4, \{s_2\}\rangle\},$$

$$\overline{\sum_{i=1}^{3}\mathbb{R}_i}^{O}(\mathbb{F}) = \{\langle x_1, \{s_5\}\rangle, \langle x_2, \{s_5, s_6\}\rangle, \langle x_3, \{s_5\}\rangle, \langle x_4, \{s_4\}\rangle\},$$

$$\underline{\sum_{i=1}^{3}\mathbb{R}_i}^{P}(\mathbb{F}) = \{\langle x_1, \{s_2\}\rangle, \langle x_2, \{s_2\}\rangle, \langle x_3, \{s_2\}\rangle, \langle x_4, \{s_2\}\rangle\},$$

$$\overline{\sum_{i=1}^{3}\mathbb{R}_i}^{P}(\mathbb{F}) = \{\langle x_1, \{s_5\}\rangle, \langle x_2, \{s_5, s_6\}\rangle, \langle x_3, \{s_5\}\rangle, \langle x_4, \{s_4, s_5\}\rangle\}.$$

Second, the following sets can be obtained by virtue of $\underline{\sum_{i=1}^{3}\mathbb{R}_i}^{O}(\mathbb{F})$, $\overline{\sum_{i=1}^{3}\mathbb{R}_i}^{O}(\mathbb{F})$, $\underline{\sum_{i=1}^{3}\mathbb{R}_i}^{P}(\mathbb{F})$ and $\overline{\sum_{i=1}^{3}\mathbb{R}_i}^{P}(\mathbb{F})$:

$$\underline{\sum_{i=1}^{3}\mathbb{R}_i}^{O}(\mathbb{F}) \boxplus \overline{\sum_{i=1}^{3}\mathbb{R}_i}^{O}(\mathbb{F}) = \{\langle x_1, \{s_8\}\rangle, \langle x_2, \{s_8, s_9\}\rangle, \langle x_3, \{s_7\}\rangle, \langle x_4, \{s_6\}\rangle\},$$

Table 7
Aggregated HFL relationship between positions and required capabilities given by three human resource experts.

$\mathbb{R}$	y_1	y_2	y_3	y_4	y_5	y_6
x_1	$\{s_4\}$	$\{s_4\}$	$\{s_5\}$	$\{s_4\}$	$\{s_6\}$	$\{s_4\}$
x_2	$\{s_5\}$	$\{s_6\}$	$\{s_3\}$	$\{s_5\}$	$\{s_3\}$	$\{s_4\}$
x_3	$\{s_5\}$	$\{s_5\}$	$\{s_4\}$	$\{s_5\}$	$\{s_5\}$	$\{s_4\}$
x_4	$\{s_3\}$	$\{s_4\}$	$\{s_5\}$	$\{s_5\}$	$\{s_4\}$	$\{s_5\}$

$$\underline{\sum_{i=1}^{3}\mathbb{R}_i}^{P}(\mathbb{F}) \boxplus \overline{\sum_{i=1}^{3}\mathbb{R}_i}^{P}(\mathbb{F}) = \{\langle x_1, \{s_7\}\rangle, \langle x_2, \{s_7, s_8\}\rangle, \langle x_3, \{s_7\}\rangle, \langle x_4, \{s_6, s_7\}\rangle\},$$

$$\left(\underline{\sum_{i=1}^{3}\mathbb{R}_i}^{O}(\mathbb{F}) \boxplus \overline{\sum_{i=1}^{3}\mathbb{R}_i}^{O}(\mathbb{F})\right) \boxplus \left(\underline{\sum_{i=1}^{3}\mathbb{R}_i}^{P}(\mathbb{F}) \boxplus \overline{\sum_{i=1}^{3}\mathbb{R}_i}^{P}(\mathbb{F})\right)$$
$$= \{\langle x_1, \{s_{15}\}\rangle, \langle x_2, \{s_{15}, s_{16}, s_{17}\}\rangle, \langle x_3, \{s_{14}\}\rangle, \langle x_4, \{s_{12}, s_{13}\}\rangle\}.$$

Third, we calculate the index sets of $\underline{\sum_{i=1}^{3}\mathbb{R}_i}^{O}(\mathbb{F}) \boxplus \overline{\sum_{i=1}^{3}\mathbb{R}_i}^{O}(\mathbb{F})$, $\underline{\sum_{i=1}^{3}\mathbb{R}_i}^{P}(\mathbb{F}) \boxplus \overline{\sum_{i=1}^{3}\mathbb{R}_i}^{P}(\mathbb{F})$ and $(\underline{\sum_{i=1}^{3}\mathbb{R}_i}^{O}(\mathbb{F}) \boxplus \overline{\sum_{i=1}^{3}\mathbb{R}_i}^{O}(\mathbb{F})) \boxplus (\underline{\sum_{i=1}^{3}\mathbb{R}_i}^{P}(\mathbb{F}) \boxplus \overline{\sum_{i=1}^{3}\mathbb{R}_i}^{P}(\mathbb{F}))$ by virtue of Definition 2.3 and Definition 2.4 as follows:

$$Q_1 = \left\{ l \,|\, \max_{x_l \in U} \left\{ \underline{\sum_{i=1}^{3}\mathbb{R}_i}^{O}(\mathbb{F})(x_l) \boxplus \overline{\sum_{i=1}^{3}\mathbb{R}_i}^{O}(\mathbb{F})(x_l) \right\} \right\} = \{2\},$$

$$Q_2 = \left\{ l' \,|\, \max_{x_{l'} \in U} \left\{ \underline{\sum_{i=1}^{3}\mathbb{R}_i}^{P}(\mathbb{F})(x_{l'}) \boxplus \overline{\sum_{i=1}^{3}\mathbb{R}_i}^{P}(\mathbb{F})(x_{l'}) \right\} \right\} = \{2\},$$

$$Q_3 = \left\{ l'' \,|\, \max_{x_{l''} \in U} \left\{ \left(\underline{\sum_{i=1}^{3}\mathbb{R}_i}^{O}(\mathbb{F})(x_{l''}) \boxplus \overline{\sum_{i=1}^{3}\mathbb{R}_i}^{O}(\mathbb{F})(x_{l''}) \right) \boxplus \left(\underline{\sum_{i=1}^{3}\mathbb{R}_i}^{P}(\mathbb{F})(x_{l''}) \boxplus \overline{\sum_{i=1}^{3}\mathbb{R}_i}^{P}(\mathbb{F})(x_{l''}) \right) \right\} \right\} = \{2\}.$$

Finally, on the basis of the decision making methods that have originated from the risk decision making guideline in classical operational research, and because $Q_1 \cap Q_2 \cap Q_3 = \{2\} \neq \emptyset$, the most suitable position for the job seeker is algorithm engineer.

5.2.2. Comparison analysis with the approach proposed in Refs. [28] and [43]

In Ref. [28], Wei et al. defined a novel hesitant fuzzy linguistic ordered weighted averaging (HFLOWA) operator to aggregate several HFLEs, and further established the corresponding group decision making rule by utilizing the presented HFLOWA operator. By virtue of HFLOWA operators, we first aggregate the HFL relationships between positions and required capabilities given by three human resource experts in Tables 1–3 . Then, by using the approach in Ref. [43], the most suitable position for the job seeker can be obtained.

Specifically, suppose that the weight vector of each human resource expert is equal, then the HFL relationships between positions and required capabilities given by the three human resource experts can be aggregated on the basis of the HFLOWA operator, i.e., the HFL relations $\mathbb{R}_1$, $\mathbb{R}_2$, and $\mathbb{R}_3$ are aggregated into a single HFL relation $\mathbb{R}$, as listed in Table 7.

Subsequently, we calculate the HFL single-granulation rough approximations of $\mathbb{F}$ in terms of $(U, V, \mathbb{R})$.

$$\underline{\mathbb{R}}(\mathbb{F}) = \{\langle x_1, \{s_2\}\rangle, \langle x_2, \{s_2\}\rangle, \langle x_3, \{s_2\}\rangle, \langle x_4, \{s_2\}\rangle\},$$
$$\overline{\mathbb{R}}(\mathbb{F}) = \{\langle x_1, \{s_5\}\rangle, \langle x_2, \{s_5, s_6\}\rangle, \langle x_3, \{s_5\}\rangle, \langle x_4, \{s_4, s_5\}\rangle\}.$$

According to the decision making rule proposed in Ref. [43], it is not difficult to obtain the set $\underline{\mathbb{R}}(\mathbb{F}) \boxplus \overline{\mathbb{R}}(\mathbb{F}) = \{\langle x_1, \{s_7\}\rangle, \langle x_2, \{s_7, s_8\}\rangle, \langle x_3, \{s_7\}\rangle, \langle x_4, \{s_6, s_7\}\rangle\}$. In a similar manner, based on the index sets of $\underline{\mathbb{R}}(\mathbb{F})$, $\overline{\mathbb{R}}(\mathbb{F})$ and $\underline{\mathbb{R}}(\mathbb{F}) \boxplus \overline{\mathbb{R}}(\mathbb{F})$, we can obtain $Q_1 = \{l|\max_{x_l \in U}\{\underline{\mathbb{R}}(\mathbb{F})(x_l)\}\} = \{1, 2, 3, 4\}$, $Q_2 = \{l'|\max_{x_{l'} \in U}\{\overline{\mathbb{R}}(\mathbb{F})(x_{l'})\}\} = \{2\}$ and $Q_3 = \{l''|\max_{x_{l''} \in U}\{\overline{\mathbb{R}}(\mathbb{F})(x_{l''}) \boxplus \underline{\mathbb{R}}(\mathbb{F})(x_{l''})\}\} = \{2\}$. Moreover, given that $Q_1 \cap Q_2 \cap Q_3 = \{2\} \neq \emptyset$, the optimal choice for the job seeker is algorithm engineer.

Table 8
Comparison of the three models.

Different values of η	Risk preferences	Optimal person-job fit result
$\eta = 0.33$	pessimistic (completely risk-averse)	algorithm engineer
$\eta = 0.67$	adjustable (risk-neutral)	sales representative
$\eta = 1$	optimistic (completely risk-seeking)	algorithm engineer

5.2.3. Discussions

In view of the person-job fit result based on adjustable HFL MG-DTRSs over two universes and the comparative analysis mentioned above, by utilizing our proposed approach, the most suitable position for the job seeker is sales representative when $\eta = 0.67$. Moreover, the same job seeker is suggested to select algorithm engineer by virtue of the approach proposed in Ref. [44] and the approach proposed in Refs. [28] and [43]. Hence, the decision result is different in the above cases. In analyzing these results, a sensitivity analysis can be conducted by changing the parameter η. By adjusting the appropriate thresholds α and β, we regard the largest adjustable membership degree for each position as the optimal choice for the job seeker, mainly by increasing or decreasing the value of η. Specifically, we let η be 0.33, 0.67, and 1, after which the most suitable position for the job seeker can be obtained, as described in Table 8.

The person-job fit results determined by the approach proposed in Ref. [44] and the approach proposed in Refs. [28] and [43] can be considered as the special cases of the person-job fit result determined by adjustable HFL MG-DTRSs over two universes, i.e., the pessimistic (completely risk-averse) result and the optimistic (completely risk-seeking) result. Our proposed approach can reflect the completely risk-averse, risk-neutral, and completely risk-seeking results when changing the parameter η. Thus, the proposed approach is more precise and flexible in handling group decision making problems than the other approaches. The main reasons are as follows:

- Different from the approach proposed in Ref. [44], in reasonably quantifying and minimizing the loss of wrong decisions, our proposed approach incorporates the merits of DTRSs when interpreting and coping with various decision risks in group decision making procedures. Furthermore, the absence of addressing decision risks may preclude experts from reaching a reasonable and reliable conclusion. Hence, the constructed method is more applicable and appropriate for risk decision making issues than the approach provided in Ref. [44].
- Different from the approach proposed in Refs. [28] and [43], in addition to the superiorities of DTRSs, our proposed approach also takes advantages of MGRSs over two universes, which increases the solving efficiency of group decision making problems based on parallel strategies. In general, the proposed approach outperforms the approach given in Ref. [44] in terms of approximating concepts and obtaining decision making rules.
- The other existing models based on multi-granularity three-way decisions [10,16] reportedly can only deal with crisp or fuzzy information systems and present some restrictions in handling various extended fuzzy information systems. In view of the significance of analyzing HFL decision making information systems, exploring a new multi-granularity three-way decisions model in the HFL background is meaningful. Thus, our proposed approach expands the applicability of the multi-granularity three-way decisions paradigm.
- As opposed to other HFL decision making approaches that use correlation coefficients [9], several correlation coefficients of HFLTSs are investigated. In Ref. [9], Liao et al. established some correlation coefficients of HFLTSs to solve a traditional Chinese medical diagnosis problem. Suppose a doctor intends to conduct a medical diagnosis. Let $U = \{viral\ fever(x_1), typhoid(x_2),.pneumonia(x_3),\ stomach\,problem(x_4)\}$ be a disease set, $V = \{temperature(y_1), headache.(y_2), cough(y_3),\ stomach\,pain(y_4)\}$ be a symptom set. Then, the doctor provides an opinion on the degree of relevance between U and V by seeing, smelling, asking, and touching, and then the opinion is described as an HFL relation over two universes $\mathbb{R}$, where

$$S = \{s_0 : none, s_1 : very\ slight, s_2 : slight, s_3 : a\ little\ slight, s_4 : terrible, s_5 : very\ terrible, s_6 : insufferable\},$$

$$\mathbb{R} = \left\{ \begin{array}{lcccc} & y_1 & y_2 & y_3 & y_4 \\ x_1 & \{s_4, s_5, s_6\} & \{s_3, s_4, s_5\} & \{s_4, s_5, s_6\} & \{s_0\} \\ x_2 & \{s_5, s_6\} & \{s_4, s_5, s_6\} & \{s_4, s_5, s_6\} & \{s_0, s_1\} \\ x_3 & \{s_3, s_4\} & \{s_2, s_3\} & \{s_5, s_6\} & \{s_0\} \\ x_4 & \{s_3\} & \{s_0\} & \{s_0\} & \{s_4, s_5, s_6\} \end{array} \right\}.$$

Subsequently, a patient is checked by using the symptoms in V, as denoted by the following HFL information:

$$\mathbb{F} = \left\{ \langle y_1, \{s_3\} \rangle, \langle y_2, \{s_0\} \rangle, \langle y_3, \{s_0\} \rangle, \langle y_4, \{s_4, s_5\} \rangle \right\}.$$

Thereafter, we compute the membership degree of each disease with respect to the given HFL relationship between diseases and symptoms, and we take $\alpha = [0, 0.8]$, $\beta = [0, 0.3]$. Notably, the considered patient is most likely to have a stomach problem, the doctor is not sure whether the patient is suffering from typhoid or not, and the patient is not suffering from viral fever and pneumonia. The derived result is identical with the medical diagnosis result obtained by HFL correlation coefficients. As opposed to the HFL decision making approaches that use correlation coefficients, the proposed approach

Table 9
Worse HFL relationship between positions and required capabilities given by the first human resource expert.

$\mathbb{R}_1$	y_1	y_2	y_3	y_4	y_5	y_6
x_1	$\{s_3\}$	$\{s_4\}$	$\{s_5\}$	$\{s_3\}$	$\{s_6\}$	$\{s_3\}$
x_2	$\{s_5\}$	$\{s_6\}$	$\{s_3\}$	$\{s_6\}$	$\{s_3\}$	$\{s_4\}$
x_3	$\{s_5\}$	$\{s_5\}$	$\{s_4\}$	$\{s_3\}$	$\{s_5\}$	$\{s_4\}$
x_4	$\{s_2\}$	$\{s_3\}$	$\{s_4\}$	$\{s_5\}$	$\{s_3\}$	$\{s_4\}$

Table 10
Worse HFL relationship between positions and required capabilities given by the second human resource expert.

$\mathbb{R}_2$	y_1	y_2	y_3	y_4	y_5	y_6
x_1	$\{s_4\}$	$\{s_4\}$	$\{s_6\}$	$\{s_4\}$	$\{s_6\}$	$\{s_4\}$
x_2	$\{s_5\}$	$\{s_6\}$	$\{s_4\}$	$\{s_3\}$	$\{s_4\}$	$\{s_5\}$
x_3	$\{s_6\}$	$\{s_5\}$	$\{s_4\}$	$\{s_3\}$	$\{s_4\}$	$\{s_4\}$
x_4	$\{s_2\}$	$\{s_2\}$	$\{s_3\}$	$\{s_5\}$	$\{s_3\}$	$\{s_4\}$

Table 11
Worse HFL relationship between positions and required capabilities given by the third human resource expert.

$\mathbb{R}_3$	y_1	y_2	y_3	y_4	y_5	y_6
x_1	$\{s_4\}$	$\{s_5\}$	$\{s_5\}$	$\{s_5\}$	$\{s_6\}$	$\{s_4\}$
x_2	$\{s_6\}$	$\{s_6\}$	$\{s_3\}$	$\{s_6\}$	$\{s_3\}$	$\{s_3\}$
x_3	$\{s_4\}$	$\{s_5\}$	$\{s_4\}$	$\{s_5\}$	$\{s_5\}$	$\{s_4\}$
x_4	$\{s_3\}$	$\{s_3\}$	$\{s_4\}$	$\{s_3\}$	$\{s_3\}$	$\{s_3\}$

Table 12
First smaller HFL relationship between positions and required capabilities given by the first human resource expert.

$\mathbb{R}_1$	y_1	y_2	y_3	y_4	y_5	y_6
x_1	$\{s_3\}$	$\{s_4\}$	$\{s_5\}$	$\{s_3\}$	$\{s_6\}$	$\{s_3\}$
x_2	$\{s_5\}$	$\{s_6\}$	$\{s_3\}$	$\{s_6\}$	$\{s_3\}$	$\{s_4\}$
x_3	$\{s_5\}$	$\{s_5\}$	$\{s_4\}$	$\{s_3\}$	$\{s_5\}$	$\{s_4\}$

not only deals with group decision making situations with HFL information, but it also provides a multi-strategy decision making result by virtue of three-way decisions.

5.3. Validity test of the proposed approach

In showing which alternative is optimal for the job seeker, we use a classical validity test to prove the effectiveness of the proposed person-job fit approach [6]. We list the two main test criteria as follows.

In the first test criterion, an effective decision making approach should not change the selection of the best alternative by replacing a non-optimal alternative with another worse alternative. According to this test criterion, we change a non-optimal alternative x_4, which is provided by three human resource experts. In Table 4, we replace x_4 as $\{s_2\}$, $\{s_3\}$, $\{s_4\}$, $\{s_5\}$, $\{s_3\}$, and $\{s_4\}$. In Table 5, we replace x_4 as $\{s_2\}$, $\{s_2\}$, $\{s_3\}$, $\{s_5\}$, $\{s_3\}$, and $\{s_4\}$. In Table 6, we replace x_4 as $\{s_3\}$, $\{s_3\}$, $\{s_4\}$, $\{s_3\}$, $\{s_3\}$, and $\{s_3\}$. Then, we use the proposed approach to deal with the new HFL relationships between positions and required capabilities (Tables 9, 10 and 11). The result also shows that the most suitable position for the job seeker is sales representative, human resource experts are not sure about the suitability of the positions of algorithm engineer and finance analyst for the job seeker, and the positions of administrative assistant is considered an unsuitable position for the job seeker. Thus, the proposed approach passes the first test criterion.

In the second test criterion, decomposing a group decision making problem into smaller problems is necessary. Furthermore, the same person-job fit method is applied to smaller problems for the ranking of the alternatives, and then the combination of ranking results for the alternatives should be identical to the original ranking of the un-decomposed problem. After applying this test criterion, we decompose the original group decision making problem into two smaller group decision making problems $\{x_1, x_2, x_3\}$ (Tables 12, 13 and 14) and $\{x_1, x_3, x_4\}$ (Tables 15, 16 and 17). By utilizing the proposed approach, the result of $\{x_1, x_2, x_3\}$ shows that sales representative is the optimal choice, human resource experts are not

Table 13
First smaller HFL relationship between positions and required capabilities given by the second human resource expert.

$\mathbb{R}_2$	y_1	y_2	y_3	y_4	y_5	y_6
x_1	$\{s_4\}$	$\{s_4\}$	$\{s_6\}$	$\{s_4\}$	$\{s_6\}$	$\{s_4\}$
x_2	$\{s_5\}$	$\{s_6\}$	$\{s_4\}$	$\{s_3\}$	$\{s_4\}$	$\{s_5\}$
x_3	$\{s_6\}$	$\{s_5\}$	$\{s_4\}$	$\{s_3\}$	$\{s_4\}$	$\{s_4\}$

Table 14
First smaller HFL relationship between positions and required capabilities given by the third human resource expert.

$\mathbb{R}_3$	y_1	y_2	y_3	y_4	y_5	y_6
x_1	$\{s_4\}$	$\{s_5\}$	$\{s_5\}$	$\{s_5\}$	$\{s_6\}$	$\{s_4\}$
x_2	$\{s_6\}$	$\{s_6\}$	$\{s_3\}$	$\{s_6\}$	$\{s_3\}$	$\{s_3\}$
x_3	$\{s_4\}$	$\{s_5\}$	$\{s_4\}$	$\{s_5\}$	$\{s_5\}$	$\{s_4\}$

Table 15
Second smaller HFL relationship between positions and required capabilities given by the first human resource expert.

$\mathbb{R}_1$	y_1	y_2	y_3	y_4	y_5	y_6
x_1	$\{s_3\}$	$\{s_4\}$	$\{s_5\}$	$\{s_3\}$	$\{s_6\}$	$\{s_3\}$
x_3	$\{s_5\}$	$\{s_5\}$	$\{s_4\}$	$\{s_3\}$	$\{s_5\}$	$\{s_4\}$
x_4	$\{s_3\}$	$\{s_4\}$	$\{s_5\}$	$\{s_6\}$	$\{s_4\}$	$\{s_5\}$

Table 16
Second smaller HFL relationship between positions and required capabilities given by the second human resource expert.

$\mathbb{R}_2$	y_1	y_2	y_3	y_4	y_5	y_6
x_1	$\{s_4\}$	$\{s_4\}$	$\{s_6\}$	$\{s_4\}$	$\{s_6\}$	$\{s_4\}$
x_3	$\{s_6\}$	$\{s_5\}$	$\{s_4\}$	$\{s_3\}$	$\{s_4\}$	$\{s_4\}$
x_4	$\{s_3\}$	$\{s_3\}$	$\{s_4\}$	$\{s_6\}$	$\{s_4\}$	$\{s_5\}$

Table 17
Second smaller HFL relationship between positions and required capabilities given by the third human resource expert.

$\mathbb{R}_3$	y_1	y_2	y_3	y_4	y_5	y_6
x_1	$\{s_4\}$	$\{s_5\}$	$\{s_5\}$	$\{s_5\}$	$\{s_6\}$	$\{s_4\}$
x_3	$\{s_4\}$	$\{s_5\}$	$\{s_4\}$	$\{s_5\}$	$\{s_5\}$	$\{s_4\}$
x_4	$\{s_4\}$	$\{s_4\}$	$\{s_5\}$	$\{s_4\}$	$\{s_4\}$	$\{s_4\}$

sure about the suitability of algorithm engineer and finance analyst for the job seeker. In addition, the result of $\{x_1, x_3, x_4\}$ shows that the job seeker should select sales representative, human resource experts are not sure whether finance analyst is the best choice or not, and administrative assistant is an unsuitable position for the same job seeker. Subsequently, we merge the above two results, and the final result is identical with the original person-job fit result. Thus, the proposed approach passes the second test criterion.

In light of the above analysis, we can conclude that the constructed approach takes full advantages of DTRSs and MGRSs over two universes, it is much more appropriate for group decision making problems with risks in the HFL context. Consequently, the superiorities of adjustable HFL MG-DTRSs over two universes can largely improve matching accuracies and reduce uncertainties of person-job fit.

6. Conclusions

In the area of qualitative decision making, the establishment of HFLTSs opens a new door for depicting the subjective and hesitant opinions of decision makers by utilizing multiple linguistic terms, particular with the studies on HFL decision making problems that are presently one of the most active study directions of HFLTS theory. In the present study, to effectively cope with the challenges of information analysis and information fusion in HFL group decision making, a novel rough set model named adjustable HFL MG-DTRSs over two universes is proposed within the multi-granularity three-way decisions paradigm. Specifically, by combining the advantages of the multi-granularity computing with three-way decisions, the notion of adjustable HFL MG-DTRSs over two universes is explored to cope with realistic risk-based uncertain group decision making problems. Then, several basic properties of the proposed model are explored in detail. By using the proposed model, we study a person-job fit approach with HFL information based on adjustable HFL MG-DTRSs over two universes. Furthermore, a practical case study, a comparative analysis, and a validity test concerning person-job fit problems are conducted to validate the principle and steps of the established group decision making rule.

Future research may investigate more theoretical foundations on adjustable HFL MG-DTRSs over two universes, such as attribute reduction algorithms, granular structures, and uncertainty measures. Integrating the weights of decision makers' judgments into the proposed model and developing new approaches to aid other complicated group decision making situations are also desirable.

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