

Pattern dynamics of a spatial predator–prey model with noise

Li Li · Zhen Jin

Received: 5 March 2011 / Accepted: 22 May 2011 / Published online: 16 June 2011
© Springer Science+Business Media B.V. 2011

Abstract A spatial predator–prey model with colored noise is investigated in this paper. We find that the number of the spotted pattern is increased as the noise intensity is increased. When the noise intensity and temporal correlation are in appropriate levels, the model exhibits phase transition from spotted to stripe pattern. Moreover, we show the number of the spotted and stripe pattern, with respect to both noise intensity and temporal correlation. These studies raise important questions on the role of noise in the pattern formation of the populations, which may well explain some data obtained in the ecosystems.

Keywords Predator–prey · Noise · Phase transition

1 Introduction

Interaction between species and their natural environment is the main feature in ecological systems. One

of the most important types of interaction, which have effects on population dynamics of all species, is predation. As a result, predator–prey models have received more and more attention in ecological science since the early days of this discipline [1].

It is reported that spatial inhomogeneities like the inhomogeneous distribution of nutrients as well as interactions on spatial scales like diffusion plays an important role on the dynamics of ecological populations [2–7]. Furthermore, Holling emphasized the influence of noise in the ecological dynamics [8]. There are inherent uncertainties in an ecological system such as varying rainfall or the nutrient inputs. Moreover, there are considerable anthropogenic disturbances that exacerbate the uncertainty in the way an ecosystem responds [9, 10]. Apart from random fluctuations, there are seasonal variations in various ecological parameters. Several studies consider the effects of seasonality [11–13].

In spatially extended nonlinear systems, noise-induced phenomenon have been the topic in physics and mathematics [14]. Especially, noise has organizing rather than disruptive effects in some systems. Some more important examples are noise-induced transitions [15, 16], stochastic resonance [17], and so on.

In the recent 20 years, noise-induced effects in population dynamics have been largely investigated [18–21]. In particular, the problem of the stability of complex ecological systems in the presence of noise has been widely discussed [22]. New counterintu-

L. Li (✉) · Z. Jin
Department of Mathematics, North University of China,
Taiyuan 030051, People's Republic of China
e-mail: lili831113@163.com

Z. Jin
e-mail: jinzhn@263.net

L. Li
Department of Mathematics, Taiyuan Institute
of Technology, Taiyuan 030008, People's Republic
of China

itive phenomena, such as stochastic resonance [23], noise enhanced stability [24], and noise delayed extinction [25, 26] can appear because of the presence of noise in ecosystems.

In the present paper, we performed numerical simulations of noise-induced complicated phenomenon, by using a predator–prey model in two-dimensional space with colored noise. The paper is organized as follows. In Sect. 2, we obtain a spatial predator–prey model with colored noise, and interpret the biological meaning of these parameters of the model. In Sect. 3, by performing a series of simulations, we illustrate the phase transition from spotted to stripe pattern. Finally, conclusions and discussion are given.

2 Model

Since the Holling–Tanner type predator–prey model received great attention by many theoretical and mathematical biologists [27, 28], we will focus our investigations on the following equations:

$$\frac{dn}{d\tau} = r_1 n \left(1 - \frac{n}{K} \right) - \frac{qnp}{n+c}, \tag{1a}$$

$$\frac{dp}{d\tau} = r_2 p \left(1 - \frac{p}{\gamma n} \right), \tag{1b}$$

where n and p denote the prey and predator, respectively. The parameters r_1 and r_2 represent the intrinsic growth rate. The value K represents the carrying capacity of the prey and γn takes on the role of a prey-dependent carrying capacity for the predator. The parameter γ is a measure of the quality of the prey as food for the predator. The rate at which predators remove the prey, $qn/(n+c)$, is known as a Holling type-II predator response [29–31]. And the parameter q is the maximum number of prey that can be eaten per predator per time and the parameter c is a saturation value; it corresponds to the number of prey to achieve one half the maximum rate q .

Following the method in [27], with the scaling

$$u = \frac{n}{K}, \quad v = \frac{p}{\gamma K}, \quad r = \frac{r_2}{r_1}, \tag{2a}$$

$$a = \frac{q\gamma}{r_1}, \quad b = \frac{c}{K}, \quad t = r_1 \tau, \tag{2b}$$

we arrive at the following equations containing dimensionless quantities:

$$\frac{du}{dt} = u(1-u) - \frac{auv}{b+u}, \tag{3a}$$

$$\frac{dv}{dt} = rv \left(1 - \frac{v}{u} \right). \tag{3b}$$

The objective of this paper is to investigate the noise effect on the system with diffusion. As a result, the model we employ is as follows:

$$\frac{\partial u}{\partial t} = u(1-u) - \frac{auv}{b+u} + d_1 \nabla^2 u, \tag{4a}$$

$$\frac{\partial v}{\partial t} = rv \left(1 - \frac{v}{u} \right) + d_2 \nabla^2 v + \eta(\mathbf{r}, t), \tag{4b}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the usual Laplacian operator in two-dimensional space and the variables u and v denote prey and predator densities, respectively. d_1, d_2 are the diffusion coefficients of prey and predator, respectively. The stochastic factors are taken into account as the term, $\eta(\mathbf{r}, t)$, obtained from microscopic interaction in the space [32–38]. The noise term $\eta(\mathbf{r}, t)$ is introduced additively in space and time, which is the Ornstein–Uhlenbeck process that obeys the following stochastic partial differential equation [39]:

$$\frac{\partial \eta(r, t)}{\partial t} = -\frac{1}{\tau} \eta(r, t) + \frac{1}{\tau} \xi(r, t), \tag{5}$$

where $\xi(r, t)$ is a Gaussian white noise with zero mean and correlation,

$$\langle \xi(r, t) \rangle = 0, \tag{6a}$$

$$\langle \xi(r, t) \xi(r', t') \rangle = 2\phi \delta(r - r') \delta(t - t'). \tag{6b}$$

The colored noise $\eta(\mathbf{r}, t)$, which is temporally correlated and white in space, satisfies

$$\langle \eta(\mathbf{r}, t) \eta(\mathbf{r}', t') \rangle = \frac{\phi}{\tau} \exp\left(-\frac{|t - t'|}{\tau} \right) \delta(\mathbf{r} - \mathbf{r}'), \tag{7}$$

where τ controls the temporal correlation, and ϕ measures the noise intensity.

3 Main results

In this paper, we rely on numerical integration of the model of (4). We consider here spatiotemporal evolution of this system while colored noise evolving

in the space and time when the system lies within the regime of Turing domain. Extensive testing was performed through numerical integration to describe model (4), and the results are shown in this section. We use XMDS (<http://www.xmds.org/>) to do the numerical simulations.

The noisy fluctuations may sometimes cause the density of the population to be less than zero. As a result, at each position in space, whenever the population densities fall below a certain prescribed value ϵ they are set to zero or a sufficiently small positive constant [40, 41]. In the simulations, different types of dynamics are observed and the distributions of prey and predator are always of the same type. As a result, we can restrict our analysis of pattern formation to one distribution (in this paper, we show the distribution of prey, for instance).

Let us first consider the case that there is no noise. From the Appendix, we can choose the proper values of the parameters for simulations. Figure 1 shows the evolution of the spatial pattern of prey population at $t = 1000$, with small random perturbation of the stationary solution E^* of the spatially homogeneous systems with the parameters set: $a = 0.75$, $b = 0.25$, $r = 0.004$, $d_1 = 0.1$, and $d_2 = 1$. One can see that the regular spotted pattern prevails in the domain.

We have checked that a large variety of distinct patterns can be obtained by making small changes in just the parameters, such as a , b , and so on. However, we are interested in how noise affects the dynamics for fixed deterministic parameters and the extent to which noise is capable of changing the patterns exhibited by the deterministic system. To this end, we fix the de-

terministic model parameters to the values and vary parameters ϕ and τ .

Figure 2 shows spontaneous formation of spotted spatial patterns emerge for different values of the noise intensity ϕ and fixed temporal correlation τ . As ϕ is increased, the number of the spotted pattern in the spatial domain is increased.

We found that, for both large ϕ and τ , the transition of different type patterns of the prey population in two-dimensional space will emerge. In Fig. 3, we show the snapshots of contour pictures of the time evolution of the prey population with $\phi = 0.5$ and $\tau = 20$. One can see from this figure that the initial conditions lead to

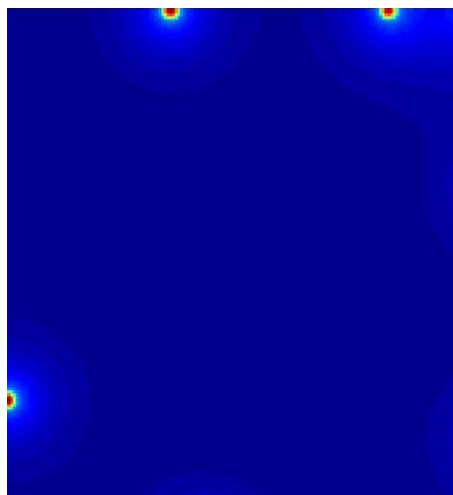
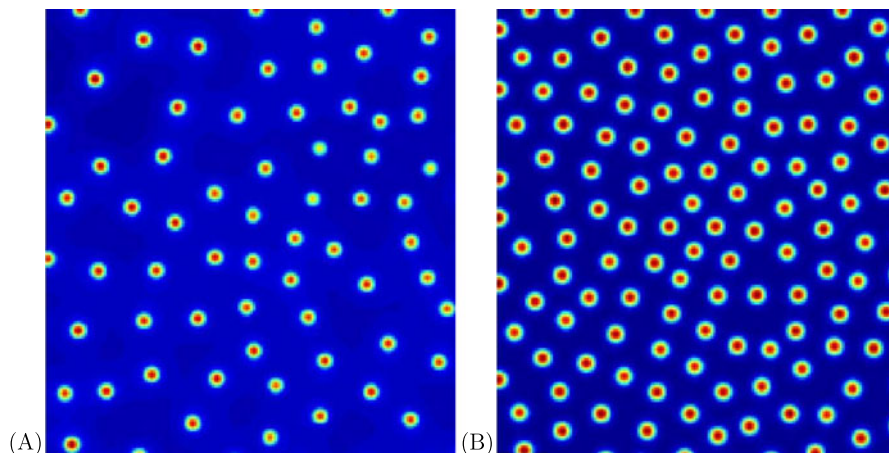


Fig. 1 (Color online) Typical spotted pattern of the prey in two-dimensional space when system (4) without the noise term. Numerical results are obtained in 400×400 size. Parameter values are used as: $a = 0.75$, $b = 0.25$, $r = 0.004$, $d_1 = 0.1$, and $d_2 = 1$

Fig. 2 (Color online) Snapshots of contour pictures of the time evolution of the prey at $t = 1000$ as the value of noise intensity ϕ being decreased. Parameter values are used as: $a = 0.75$, $b = 0.25$, $r = 0.004$, $d_1 = 0.1$, $d_2 = 1$, and $\tau = 3$. (A) $\phi = 0.001$; (B) $\phi = 0.01$



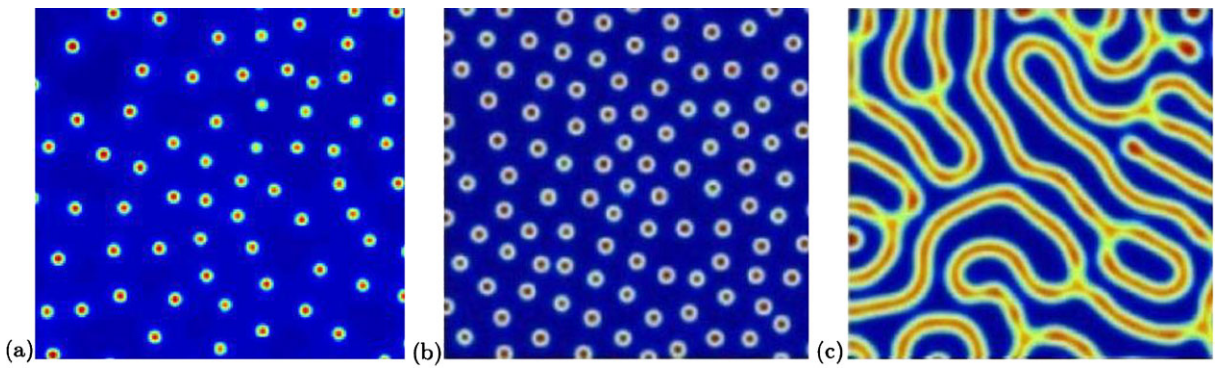


Fig. 3 (Color online) Transition from spotted patterns to the stripe pattern in two-dimensional space when system (4) with the noise term. Numerical results are obtained in 300×300 size.

Parameter values are used as: $a = 0.75, b = 0.25, r = 0.004, d_1 = 0.1, d_2 = 1, \phi = 0.5$ and $\tau = 20$. (A) $t = 100$; (B) $t = 300$; (C) $t = 1500$

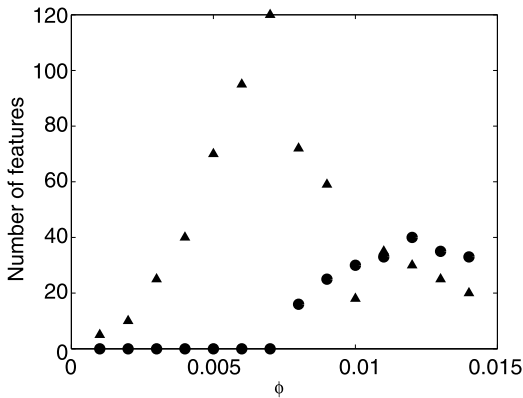


Fig. 4 The number of spotted and stripe patterns for $t = 1000$ as a function of noise intensity in the space with $\Delta x = \Delta y = 1$ and 300×300 size. Here, *triangles* and *dots* stand for spotted and stripe patterns, respectively. Parameter values are used as: $a = 0.75, b = 0.25, r = 0.004, d_1 = 0.1, d_2 = 1$, and $\tau = 30$

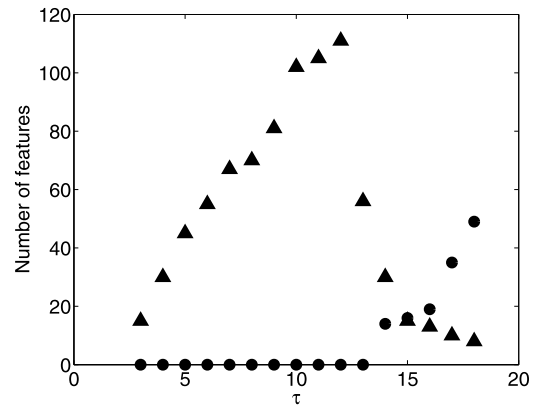


Fig. 5 The number of spotted and stripe patterns for 300,000 iterations as a function of temporal correlation in the space with $\Delta x = \Delta y = 1.25$ and 400×400 size. Here, *triangles* and *dots* stand for spotted and stripe patterns, respectively. Parameter values are used as: $a = 0.75, b = 0.25, r = 0.004, d_1 = 0.1, d_2 = 1$, and $\phi = 0.8$

the formation of the spotted pattern. And as time is increased, coexistence of the spotted and stripe patterns emerge, but the spotted patterns dominate the domain.

The question naturally arising here is how noise intensity and temporal correlation have effect on the spatial pattern. We show the effect of noise in Fig. 4, which is obtained from a statistics of 14 experiments and $t = 1000$ for different value of the noise intensity ϕ . To well see our analysis, we have also included the evolution of the number of the spotted (denoted by triangles) and the stripe pattern (denoted by dots), respectively. One can see from this figure that there exists an optimum-noise interval that guarantees the maximum number of spots for fixed τ . Moreover, when the noise intensity is small, there is no stripe

pattern. As noise intensity is increased, stripe pattern emerges.

Now, it is natural to ask what the consequence of the temporal correlation of the colored noise is, in particular, on the number of the spotted and stripe pattern. In order to well understand that mechanism, we also performed a series of simulations for fixed noise intensity ϕ , which is shown in Fig. 5. We choose 16 parameters set for numerical simulations. We find that, when τ is small, the number of the spotted pattern is increasing as τ being increased, and reaches the maximum value at $\tau \approx 12$. However, when τ is large enough, the number of the spotted pattern is decreasing as τ is further increased. Furthermore, there is also no stripe pat-

tern for small value of temporal correlation. And while the value of τ is larger than 14, stripe pattern emerge and the number of it is a increasing function of τ .

4 Discussion and conclusion

It has been observed in the literature that the effect of color noise on the dynamic behavior of spatial predator–prey models [42, 43], especially spatial patterns, had been not well understood despite their potential ecological reality and intrinsic theoretical interest. These structures may, in fact, correspond to the real ecosystem. For such reason, we have investigated the spatial patterns of a Holling–Tanner model with both noise and diffusion.

In this paper, we have presented a numerical analysis of the effect of noise on the pattern formation of the predator–prey model. In [3], we only considered the spatial model with diffusion and did not obtain the transition from one type pattern to another. Moreover, predator–prey models with noise and diffusion can well understand the contradiction between the population dynamics predicted by deterministic models and those observed data in marine and for terrestrial animals [44].

A question of interest is whether our results are robust to the density dependent noise. That is to say, the model will be changed as the following form:

$$\frac{\partial u}{\partial t} = u(1 - u) - \frac{auv}{b + u} + d_1 \nabla^2 u, \tag{8a}$$

$$\frac{\partial v}{\partial t} = rv \left(1 - \frac{v}{u} \right) + d_2 \nabla^2 v + v\eta(\mathbf{r}, t). \tag{8b}$$

To address this issue, we performed extensive computer simulations for different parameter values (in total, more than 100 parameter sets were examined). We found that, as the noise intensity is increased, the number of the spotted pattern is increased; see Fig. 6. However, there is no noise-induced phase transition from the spotted to stripe pattern.

In this paper, the results are obtained with respect to the choice of initial conditions as small random perturbation of the stationary solution N^* and P^* . But our computer simulations indicate that the number of the spotted or stripe pattern is rather sensitive to the details of the initial distribution [45]. This issue, however, should be given a more careful consideration and will become a subject of a separate study.

Appendix: Stability analysis of system (4)

To study patterns of the cross diffusion system given by (4), we must consider a spatially homogeneous system. So, we first find the steady state as follows:

- (i) $E_0 = (1, 0)$, which is corresponding to extinction of the predator;
- (ii) interior equilibrium point $E^* = (u^*, v^*)$, which is corresponding to coexistence of prey and predator, and given by

$$u^* = v^*, \tag{9a}$$

$$v^* = \frac{1 - a - b + \sqrt{(a + b - 1)^2 + 4b}}{2}. \tag{9b}$$

There has been some work on the stability analysis of the model (1) [29, 46]. However, the main purpose of the present paper is to investigate the effect of noise on the spatial pattern. As a result, we are interested to study the stability behavior of the interior equilibrium point E^* , as in this case both the population exist simultaneously. We address the temporal stability of the uniform states which is associated with nonuniform perturbations

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} u^* \\ v^* \end{pmatrix} + \varepsilon \begin{pmatrix} u_\kappa \\ v_\kappa \end{pmatrix} e^{\lambda t + i\kappa x} + C + \mathcal{O}(\varepsilon^2), \tag{10}$$

where λ is the perturbation growth rate, κ is the wavenumber, and C stands for complex conjugate. The linear instability ($\varepsilon \ll 1$) of each one of the uniform states, is deduced from the dispersion relations. Substituting expression (10) into (4) and neglecting all nonlinear terms in u and v , one finds the characteristic equation for the growth rate λ as determinant of \mathcal{J} , where

$$\mathcal{J} = \begin{pmatrix} \lambda - a_{11} + d_1 \kappa^2 & -a_{12} \\ -a_{21} & \lambda - a_{22} + d_2 \kappa^2 \end{pmatrix}, \tag{11}$$

where

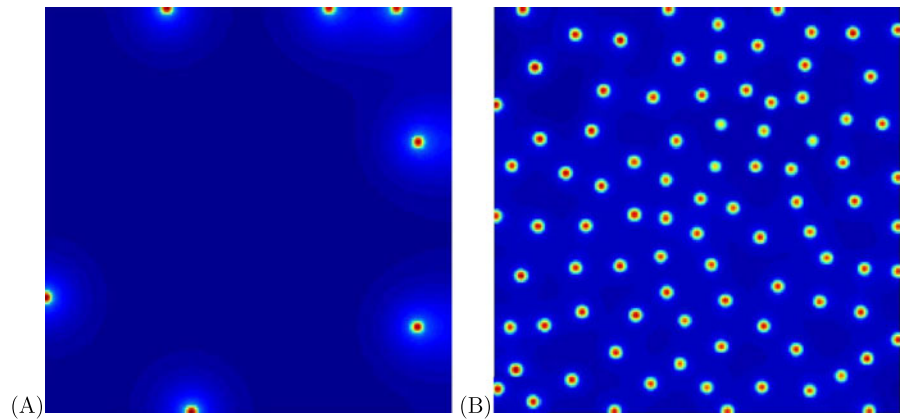
$$a_{11} = \frac{2P}{(-1 - a - b - M)^2}, \tag{12}$$

$$a_{12} = \frac{a(-1 + a + b - M)}{1 - a + b + M}, \tag{13}$$

$$a_{21} = r, \tag{14}$$

$$a_{22} = -r, \tag{15}$$

Fig. 6 (Color online) Snapshots of contour pictures of the time evolution of the prey at $t = 1000$ as the value of noise intensity ϕ being decreased. Parameter values are used as: $a = 0.75$, $b = 0.25$, $r = 0.004$, $d_1 = 0.1$, $d_2 = 1$ and $\tau = 3$. (A) $\phi = 0.001$; (B) $\phi = 0.01$



with $P = -1 + 4a - 2b - b^2 + ab - 5a^2 + 2a^3 + ab^2 + 3a^2b - M - bM + 3aM - abM - 2a^2M$, and $M = \sqrt{(a + b - 1)^2 + 4b}$.

We obtain that the eigenvalue is the root of the following equation:

$$\lambda^2 + A_\kappa \lambda + B_\kappa = 0, \tag{16}$$

where

$$A_\kappa = (d_1 + d_2)\kappa^2 - (a_{11} + a_{22}), \tag{17a}$$

$$B_\kappa = d_1 d_2 \kappa^4 - (d_2 a_{11} + d_1 a_{22})\kappa^2 + a_{11} a_{22} - a_{12} a_{21}. \tag{17b}$$

Therefore, the solution of (16) is given by

$$\lambda_\kappa = \frac{-A_\kappa \pm \sqrt{A_\kappa^2 - 4B_\kappa}}{2}. \tag{18}$$

Diffusion-driven instability requires that the stable, homogeneous steady state is driven unstable by the interaction of the dynamics and diffusion of the species and, therefore, from the arguments given above we are interested in obtaining conditions such that, without diffusion and with diffusion, respectively,

$$\text{Re}(\lambda_{\kappa^2=0}) < 0, \tag{19a}$$

$$\text{Re}(\lambda_{\kappa^2>0}) > 0, \tag{19b}$$

for some $\kappa^2 > 0$.

The onset of Hopf instability corresponds to the case, when a pair of imaginary eigenvalues cross the real axis from the negative to the positive side. And this situation occurs only when the diffusion vanishes.

Mathematically speaking, the Hopf bifurcation occurs when

$$\text{Im}(\lambda_k) \neq 0, \quad \text{Re}(\lambda_k) = 0 \quad \text{at } k = 0. \tag{20}$$

And the Turing bifurcation occurs when

$$\text{Im}(\lambda_k) = 0, \quad \text{Re}(\lambda_k) = 0 \quad \text{at } k = k_T \neq 0, \tag{21}$$

and the wavenumber k_T satisfies

$$k_T^2 = \sqrt{\frac{B_0}{d_1 d_2}}. \tag{22}$$

A general linear analysis [3, 5] shows that the necessary conditions for yielding Turing patterns are given by

$$a_{11} + a_{22} < 0,$$

$$a_{11} a_{22} - a_{12} a_{21} > 0,$$

$$d_1 a_{22} + d_2 a_{11} > 0,$$

$$(d_2 a_{11} + d_1 a_{22})^2 > 4d_1 d_2 (a_{11} a_{22} - a_{12} a_{21}).$$

References

1. Baumann, M., Gross, T., Feudel, U.: Instabilities in spatially extended predator–prey systems: spatio-temporal patterns in the neighborhood of Turing–Hopf bifurcations. *J. Theor. Biol.* **245**, 220–229 (2007)
2. Medvinsky, A.B., Petrovskii, S.V., Tikhonova, I.A., Malchow, H., Li, B.-L.: Spatio-temporal complexity of plankton and fish dynamics in simple model ecosystems. *SIAM Rev.* **44**, 311–370 (2002)
3. Sun, G.-Q., Jin, Z., Liu, Q.-X., Li, L.: Pattern formation induced by cross-diffusion in a predator–prey system. *Chin. Phys. B* **17**, 3936–3941 (2008)

4. Sun, G.-Q., Jin, Z., Liu, Q.-X., Li, L.: Dynamical complexity of a spatial predator–prey model with migration. *Ecol. Model.* **219**, 248–255 (2008)
5. Murray, J.D.: *Mathematical Biology*. Springer, Berlin (1989)
6. Garvie, M.R.: Finite-difference schemes for reaction–diffusion equations modeling predator–prey interactions in Matlab. *Bull. Math. Biol.* **69**, 931–956 (2007)
7. Garvie, M.R., Trenchea, C.: Finite element approximations of spatially extended predator–prey interactions with the Holling type II functional response. *Numer. Math.* **107**, 641–667 (2007)
8. Holling, C.S.: Resilience and stability of ecological systems. *Ann. Rev. Ecol. Syst.* **4**, 1–23 (1973)
9. Folke, C., Carpenter, S.R., Walker, B., Scheffer, M., Elmqvist, T., Gunderson, L.H., Holling, C.: Regime shifts resilience, and biodiversity in ecosystem management. *Annu. Rev. Ecol. Evol. Syst.* **35**, 557–581 (2004)
10. Richter, O.: Spatio-temporal patterns of gene flow and dispersal under temperature increase. *Math. Biosci.* **218**, 15–23 (2009)
11. Scheffer, M., Rinaldi, S., Kuznetsov, Y.A., van Nes, E.H.: Seasonal dynamics of daphnia and algae explained as a periodically forced predator–prey system. *Oikos* **80**, 519–532 (1997)
12. Scheffer, M., Rinaldi, S.: Minimal models of top-down control of phytoplankton. *Freshw. Biol.* **45**, 265–283 (2000)
13. Guttal, V., Jayaprakash, C.: Impact of noise on bistable ecological systems. *Ecol. Model.* **201**, 420–428 (2007)
14. García-Ojalvo, J., Sancho, J.M.: *Noise in Spatially Extended Systems*. Springer, New York (1999)
15. Horsthemke, W., Lefever, R.: *Noise-Induced Transitions*. Springer, Berlin (1984)
16. Lesmes, F., Hochberg, D., Morán, F., Pérez-Mercader, J.: Noise-controlled self-replicating patterns. *Phys. Rev. Lett.* **91**, 238301 (2003)
17. Gammaitoni, L., Hanggi, P., Jung, P., Marchesoni, F.: Stochastic resonance. *Rev. Mod. Phys.* **70**, 223–287 (1998)
18. Vilar, J.M.G., Sole, R.V.: Effects of noise in symmetric two-species competition. *Phys. Rev. Lett.* **80**, 4099 (1998)
19. Scheffer, M., Carpenter, S., Foley, J., Folke, C., Walker, B.: Catastrophic shifts in ecosystems. *Nature* **413**, 591–596 (2001)
20. Giardina, I., Bouchaud, J.P., Mezard, M.: Proliferation assisted transport in a random environment. *J. Phys. A, Math. Gen.* **34**, L245–252 (2001)
21. Staliunas, K.: Spatial and temporal noise spectra of spatially extended systems with order–disorder phase transitions. *Int. J. Bifurc. Chaos Appl. Sci. Eng.* **11**, 2845–2852 (2001)
22. Bjornstad, O.N., Grenfell, B.T.: Noisy clockwork: time series analysis of population fluctuations in animals. *Science* **293**, 638–643 (2001)
23. Mantegna, R.N., Spagnolo, B.: Stochastic resonance in a tunnel diode. *Phys. Rev. E* **49**, R1792–R1795 (1994)
24. Mantegna, R.N., Spagnolo, B.: Noise enhanced stability in an unstable system. *Phys. Rev. Lett.* **76**, 563–566 (1996)
25. Spagnolo, B., Fiasconaro, A., Valenti, D.: Noise induced phenomena in Lotka–Volterra systems. *Fluct. Noise Lett.* **3**, L177–L185 (2003)
26. Valenti, D., Fiasconaro, A., Spagnolo, B.: Stochastic resonance and noise delayed extinction in a model of two competing species. *Physica A* **331**, 477–486 (2004)
27. Braza, P.A.: The bifurcation structure of the Holling–Tanner model for predator–prey interactions using two-timing. *SIAM J. Appl. Math.* **63**, 889–904 (2003)
28. Collings, J.B.: Bifurcation and stability analysis of a temperature-dependent mite predator–prey interaction model incorporating a prey refuge. *Bull. Math. Biol.* **57**, 63–76 (1995)
29. Hsu, S.B., Huang, T.W.: Global stability for a class of predator–prey systems. *SIAM J. Appl. Math.* **55**, 763–783 (1995)
30. Wollkind, D.J., Collings, J.B., Logan, J.A.: Metastability in a temperature-dependent model system for predator–prey mite outbreak interactions on fruit flies. *Bull. Math. Biol.* **50**, 379–409 (1988)
31. May, R.M.: *Stability and Complexity in Model Ecosystems*. Princeton University Press, Princeton (1973)
32. Reichenbach, T., Mobilia, M., Frey, E.: Noise and correlations in a spatial population model with cyclic competition. *Phys. Rev. Lett.* **99**, 238105 (2007)
33. Reichenbach, T., Mobilia, M., Frey, E.: Mobility promotes and jeopardizes biodiversity in rock–paper–scissors games. *Nature* **448**, 1046–1049 (2007)
34. Liu, Q.-X., Li, B.-L., Jin, Z.: Resonance and frequency-locking phenomena in a spatially extended phytoplankton–zooplankton system with additive noise and periodic forces. *J. Stat. Mech. Theory Exp.* **5**, P05011 (2008)
35. Blasius, B., Huppert, A., Stone, L.: Complex dynamics and phase synchronization in spatially extended ecological systems. *Nature* **399**, 354–359 (1999)
36. Mankin, R., Ainsaar, A., Haljas, A., Reiter, E.: Trichotomous-noise-induced catastrophic shifts in symbiotic ecosystems. *Phys. Rev. E* **65**, 051108 (2002)
37. Mankin, R., Sauga, A., Ainsaar, A., Haljas, A., Paanel, K.: Colored-noise-induced discontinuous transitions in symbiotic ecosystems. *Phys. Rev. E* **69**, 061106 (2004)
38. Sun, G.-Q., Li, L., Jin, Z., Li, B.-L.: Effect of noise on the pattern formation in an epidemic model. *Num. Methods Partial Differ. Equ.* **26**, 1168–1179 (2010)
39. Higham, D.J.: An algorithmic introduction to numerical simulation of stochastic differential equations. *SIAM Rev.* **43**, 525–546 (2001)
40. Petrovskii, S., Li, B.L., Malchow, H.: Transition to spatiotemporal chaos can resolve the paradox of enrichment. *Ecol. Complex.* **1**, 37–47 (2004)
41. Malchow, H., Petrovskii, S.V., Venturino, E.: *Spatiotemporal Patterns in Ecology and Epidemiology: Theory, Models, and Simulations*. Chapman & Hall/CRC, London (2008)
42. Sun, G.-Q., Jin, Z., Li, L., Liu, Q.-X.: The role of noise in a predator–prey model with allee effect. *J. Biol. Phys.* **35**, 185–196 (2009)
43. Sun, G.-Q., Zhang, G., Jin, Z., Li, L.: Predator cannibalism can give rise to regular spatial pattern in a predator–prey system. *Nonlinear Dyn.* **58**, 75–84 (2009)
44. Freund, J.A., Schimansky-Geier, L., Beisner, B., Neiman, A., Russell, D.F., Yakusheva, T., Moss, F.: Behavioral stochastic resonance: how the noise from a daphnia swarm enhances individual prey capture by juvenile paddlefish. *J. Theor. Biol.* **214**, 71–83 (2002)

45. Sun, G.-Q., Liu, Q.-X., Jin, Z., Chakraborty, A., Li, B.-L.: Influence of infection rate and migration on extinction of disease in spatial epidemics. *J. Theor. Biol.* **264**, 95–103 (2010)
46. Tanner, J.T.: The stability and the intrinsic growth rates of prey and predator populations. *Ecology* **56**, 855–867 (1975)