



# Trend analysis of categorical data streams with a concept change method



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## ABSTRACT

This paper proposes a new method to trend analysis of categorical data streams. A data stream is partitioned into a sequence of time windows and the records in each window are assumed to carry a number of concepts represented as clusters. A data labeling algorithm is proposed to identify the concepts or clusters of a window from the concepts of the preceding window. The expression of a concept is presented and the distance between two concepts in two consecutive windows is defined to analyze the change of concepts in consecutive windows. Finally, a trend analysis algorithm is proposed to compute the trend of concept change in a data stream over the sequence of consecutive time windows. The methods for measuring the significance of an attribute that causes the concept change and the outlier degrees of objects are presented to reveal the causes of concept change. Experiments on real data sets are presented to demonstrate the benefits of the trend analysis method.

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## 1. Introduction

Many real world applications generate continuously arriving data, such as business transactions, network event logs and social networks. This type of data is known as data streams [23]. In data stream mining, most research has been focused on numerical data streams [2,1,5,12,13,18]. Recently, the mining of categorical data streams has become a research topic of growing interest [4,10,11,14,19,20].

A data stream can be considered as a sequence of data records, each representing an object with a timestamp. Given a time window, we assume that the objects represented by these records within the time window are distributed in different clusters and each cluster represents a concept. As new data records arrive over time, the structure of clusters changes, which results in change of concepts represented in the clusters. In this context, a concept change is called concept drift [23].

Two types of concept drift are illustrated in [17]. One is sudden (abrupt) concept drift and the other is gradual concept drift. Sudden concept drift is described as that the structure of clusters is changed dramatically in short time. Gradual concept drift is considered that the change of a concept occurs gradually over time. For example, in social network analysis, people in a social group or cluster are interested in a particular topic at certain time period. Some people gradually change their

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interest in the topic and some suddenly change their interests from the current topic to a new topic. The former represents a gradual concept drift and the later is a sudden concept drift.

In [6], we have defined a difference measure to compute the change of concepts between two consecutive windows. With this measure, we are able to analyze the trend of concept change over time through the change of clusters in consecutive windows. However, this measure cannot reveal the relative concept change between two time windows. To solve this problem, we have defined in [8] the new concept emerging degree and the old concept fading degree to measure the relative concept change between two consecutive windows.

In this paper, we propose a new method to trend analysis of categorical data streams by extending the work of [6,8]. In this method, we partition a data stream into a sequence of time windows. The data records in each window are assumed to carry a number of concepts represented as clusters. We propose a data labeling algorithm to identify the concepts or clusters in a window from the concepts of the preceding window. We express concepts following the idea of Node Importance [10] and define the distance between two concepts in two consecutive windows using the new concept emerging degree and the old concept fading degree to analyze the change of concepts in consecutive windows. We present the methods for measuring the significance of an attribute that causes the concept change and the outlier degrees of objects to reveal the causes of concept change. Finally, we integrate the above techniques in a trend analysis algorithm to compute the trend of concept change in a data stream over the sequence of consecutive time windows.

A series of experiments were conducted on KDD-CUP'99 data set [22]. The experimental results have shown that the proposed method can discover the trend of concept change in consecutive windows. In comparison with [6], the new method not only revealed the relative concept change between consecutive windows but also found the causes of concept change.

The rest of this paper is organized as follows. Section 2 states the research problem. Section 3 reviews some preliminaries. The trend analysis algorithm and the corresponding techniques are presented in Section 4. Experimental results on real data sets are shown in Section 5. The paper is concluded in Section 6.

## 2. Problem statement

A categorical data stream consists of a sequence of records or objects with timestamps, where each record is described by a set of categorical attributes such as Sex, Position, Location and Class. A categorical attribute takes values from a finite set of categories, for instance, Sex = {M, F}. Formally, a categorical data stream can be formulated as a table of the quintuple  $TDT = (U, A, V, f, t)$ , where  $U$  is a nonempty set of objects called the universe,  $A$  is a nonempty set of attributes,  $t$  is a sequence of timestamps,  $f: U \times A \times t \rightarrow V$  is a mapping called an information function such that for any  $x \in U, a \in A$  and  $t' \in t, f(x, a, t') \in V_a$ , where  $V_a$  is a finite and unordered set of values for attribute  $a$ .  $V = \bigcup_{a \in A} V_a$  is the union of all attribute domains.

Given a particular categorical data stream, we partition the sequence of objects into a set of consecutive time windows with respect to  $t$ , using the sliding window technique [3,9,16]. Suppose that  $N$  is the size of a sliding window, i.e., the number of records in the window, data stream  $TDT$  is partitioned into a series of subsets  $S^{T_i} (1 \leq i \leq \lceil \frac{|U|}{N} \rceil)$ , where  $T_i$  represents the  $i$ th window and  $S^{T_i} \cap S^{T_j} = \emptyset (1 \leq i \neq j \leq \lceil \frac{|U|}{N} \rceil)$ .

**Problem Statement:** Given a categorical data stream whose objects are partitioned into a set of consecutive time windows, find the concepts the objects carry in each window; find the change of concepts in two consecutive windows; find the causes of concept change; find the trend of concept change over the sequence of consecutive time windows.

## 3. Preliminaries

In this section, we briefly review some definitions, such as the new concept emerging degree, the old concept fading degree and the difference measure between two windows that are used to measure concept change. These definitions were first given in [8].

**Definition 1** [21]. Let  $TDT = (U, A, V, f, t)$  be a categorical data stream,  $P \subseteq A$  and  $X \subseteq U$ . For any  $Y \subseteq X$  and  $x \in X$ , the lower approximation and upper approximation of  $Y$  in  $X$  are defined as

$$\underline{P}Y = \{x | [x]_P \subseteq Y\} \quad (1)$$

and

$$\overline{P}Y = \{x | [x]_P \cap Y \neq \emptyset\}, \quad (2)$$

where  $[x]_P = \{y \in X | (x, y) \in IND(P)\}$ .  $IND(P)$  is an equivalence relation, which is defined as  $IND(P) = \{(x, y) \in X \times X | \forall a \in P, f(x, a) = f(y, a)\}$ .

Here, we describe the lower approximation and upper approximation of  $Y$  in a set  $X$ , not the universe  $U$ .

Given a categorical data stream whose objects are partitioned into a set of consecutive windows, we can use Definition 1 to measure the change of concepts between two consecutive windows. For example, in a social media data stream, a time

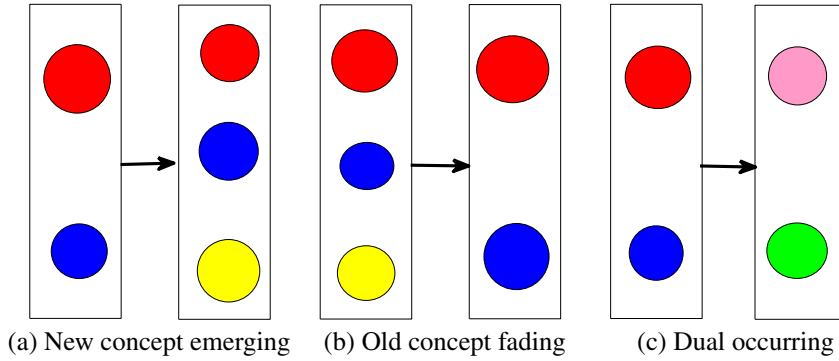


Fig. 1. Three types of concept change.

window may contain several topics (concepts). The set of topics change as a new topic emerges in the following window or an old topic disappears. The intuitive example in Fig. 1 illustrates three types of concept change. Assume the two rectangles in each subfigure represent two consecutive windows. The circles in each window indicate different concepts. Fig. 1(a) shows the concept described by the yellow circle emerged in the following window. Fig. 1(b) shows the concept described by the yellow circle disappeared in the following window. In Fig. 1(c), two old concepts faded completely and two new concepts emerged in the following window.

Using the lower approximation and upper approximation in Definition 1, we define the new concept emerging degree and the old concept fading degree in two consecutive windows as follows.

**Definition 2.** Let  $TDT = (U, A, V, f, t)$  be a categorical data stream and  $S^{T_i}, S^{T_j} \subseteq U$ , where  $S^{T_i} \cap S^{T_j} = \emptyset$  and  $S^{[T_i, T_j]} = S^{T_i} \cup S^{T_j}$ . The new concept emerging degree and the old concept fading degree from  $T_i$  to  $T_j$  with respect to  $A$  are defined as

$$NED_A \langle S^{T_i}, S^{T_j} \rangle = \frac{1}{|A|} \sum_{a \in A} NED_{(a)} \langle S^{T_i}, S^{T_j} \rangle \quad (3)$$

and

$$OFD_A \langle S^{T_i}, S^{T_j} \rangle = \frac{1}{|A|} \sum_{a \in A} OFD_{(a)} \langle S^{T_i}, S^{T_j} \rangle, \quad (4)$$

where

$$NED_{(a)} \langle S^{T_i}, S^{T_j} \rangle = \frac{|\underline{\{a\}} S^{T_j}|}{|\overline{\{a\}} S^{T_j}|},$$

$$OFD_{(a)} \langle S^{T_i}, S^{T_j} \rangle = \frac{|\underline{\{a\}} S^{T_i}|}{|\overline{\{a\}} S^{T_i}|}.$$

Here,  $\underline{\{a\}} S^{T_m}$  and  $\overline{\{a\}} S^{T_m}$  ( $m = i, j$ ) represent the lower approximation and the upper approximation of  $S^{T_m}$  in  $S^{[T_i, T_j]}$  with respect to attribute  $a$ , respectively. The objects in  $\underline{\{a\}} S^{T_m}$  can be with certainty classified as members of  $S^{T_m}$  on the basis of knowledge in  $a$ , while the objects in  $\overline{\{a\}} S^{T_m}$  can be only classified as possible members of  $S^{T_m}$  on the basis of knowledge in  $a$ .

$NED_A \langle S^{T_i}, S^{T_j} \rangle$  represents the accuracy of approximation [21] of  $S^{T_j}$  in  $S^{[T_i, T_j]}$ , while  $OFD_A \langle S^{T_i}, S^{T_j} \rangle$  is the accuracy of approximation of  $S^{T_i}$  in  $S^{[T_i, T_j]}$ . The higher the two measures, the bigger the relative concept change occurring in the two windows. That is to say, the higher the values of  $NED_A \langle S^{T_i}, S^{T_j} \rangle$  or  $OFD_A \langle S^{T_i}, S^{T_j} \rangle$  are, the bigger the difference between  $S^{T_i}$  and  $S^{T_j}$ .

If  $S^{[T_i, T_j]} / IND(\{a\}) = \{X | X = \{u\}, u \in S^{[T_i, T_j]}\}$ ,  $NED_{(a)} \langle S^{T_i}, S^{T_j} \rangle$  and  $OFD_{(a)} \langle S^{T_i}, S^{T_j} \rangle$  achieve their maximum value 1. In other words,  $NED_{(a)} \langle S^{T_i}, S^{T_j} \rangle$  and  $OFD_{(a)} \langle S^{T_i}, S^{T_j} \rangle$  are precise with respect to  $a$ .

If  $S^{[T_i, T_j]} / IND(\{a\}) = \{X | X = S^{[T_i, T_j]}\}$ ,  $NED_{(a)} \langle S^{T_i}, S^{T_j} \rangle$  and  $OFD_{(a)} \langle S^{T_i}, S^{T_j} \rangle$  achieve their minimum value 0. In other words,  $NED_{(a)} \langle S^{T_i}, S^{T_j} \rangle$  and  $OFD_{(a)} \langle S^{T_i}, S^{T_j} \rangle$  are vague with respect to  $a$ .

Obviously, we have  $0 \leq NED_A \langle S^{T_i}, S^{T_j} \rangle \leq 1$  and  $0 \leq OFD_A \langle S^{T_i}, S^{T_j} \rangle \leq 1$ .

Fig. 1 shows that old concept fading and new concept emerging can occur simultaneously. We use  $NED_A \langle S^{T_i}, S^{T_j} \rangle$  and  $OFD_A \langle S^{T_i}, S^{T_j} \rangle$  to define the difference measure between two consecutive windows as follows.

**Definition 3.** Let  $TDT = (U, A, V, f, t)$  be a categorical data stream and  $S^{T_i}, S^{T_j} \subseteq U$ , where  $S^{T_i} \cap S^{T_j} = \emptyset$  and  $S^{[T_i, T_j]} = S^{T_i} \cup S^{T_j}$ . The difference measure between  $S^{T_i}$  and  $S^{T_j}$  with respect to  $A$  is defined as

$$DM_A(S^{T_i}, S^{T_j}) = \frac{NED_A\langle S^{T_i}, S^{T_j} \rangle + OFD_A\langle S^{T_i}, S^{T_j} \rangle}{2}. \tag{5}$$

We can verify that  $DM_A(S^{T_i}, S^{T_j})$  is a distance metric.

#### 4. Trend analysis method

In this section, we propose a new algorithm for trend analysis of concept change in categorical data streams. We first present a data labeling algorithm to identify concepts of a given time window from the concepts in the preceding window. Then, we define a method to express concepts and a distance measure of two concepts in two consecutive windows. After that, we integrate all these methods in the trend analysis algorithm. Finally, we present methods to measure the significance of an attribute that affects the concept change and the outlier degree of objects in a time window.

##### 4.1. Data-labeling algorithm

Given the set of objects in the first window of a data stream, we can use a clustering algorithm to divide the objects into clusters and identify concepts. If the difference measure between  $T_i$  and  $T_j$  is greater than a given threshold, we consider that  $T_j$  is a concept-drifting window relative to  $T_i$  and use a clustering algorithm to find new concepts in  $T_j$ . If there is no significant change in concepts between two consecutive windows, we can use a data labeling method to quickly partition the objects in the current window by referencing the concepts in the preceding window. Inspired by the idea of Node Importance [10], we define the degree of membership of an object in the current window  $T_j$  that belongs to a cluster or concept in the preceding window  $T_i$  as follows.

**Definition 4.** Let  $TDT = (U, A, V, f, t)$  be a categorical data stream and  $S^{T_i}, S^{T_j} \subseteq U$ , where  $S^{T_i} \cap S^{T_j} = \emptyset$  and  $S^{[T_i, T_j]} = S^{T_i} \cup S^{T_j}$ . Suppose that  $C^{T_i} = \{c_1^{T_i}, c_2^{T_i}, \dots, c_{k_{T_i}}^{T_i}\}$  is the clustering results on  $S^{T_i}$ , where  $c_m^{T_i}$  is the  $m$ th cluster,  $1 \leq m \leq k_{T_i}$ . For any unlabeled object  $x \in S^{T_j}$ , the degree of membership of  $x$  belonging to  $c_m^{T_i}$  with respect to  $A$  is defined as

$$Sim_A(x, c_m^{T_i}) = \sum_{a \in A} \delta_a \times \omega_a, \tag{6}$$

where

$$\delta_a = \frac{|\{y | f(x, a) = f(y, a), y \in c_m^{T_i}\}|}{|c_m^{T_i}|}$$

and

$$\omega_a = 1 + \frac{1}{\log_2(k_{T_i})} \times \sum_{m=1}^{k_{T_i}} (q_a \times \log_2(q_a)).$$

The value of  $\delta_a$  reflects the frequency of the component  $f(x, a)$  in  $c_m^{T_i}$ . In other words, the component is important in the cluster when the frequency of the component is high in this cluster. The value of  $\omega_a$  measures the entropy of component  $f(x, a)$  between clusters, where  $q_a = \frac{|\{y | f(x, a) = f(y, a), y \in c_m^{T_i}\}|}{|\{z | f(z, a) = f(x, a), z \in C^{T_i}\}|}$ . Suppose that there is a component which occurs in all clusters uniformly, the component which contains the maximum uncertainty provides less similarity. In other words, attribute  $a$  is of no effect for the degree of membership.

We use an example to show that  $DM_A()$  can measure not only the difference between two windows, but also the relative concept change between two windows. The data is shown in Table 1.

Let  $U = \{x_1, x_2, \dots, x_{20}\}, A = \{A_1, A_2, A_3, A_4\}$ , where  $A_4$  is the timestamp. Suppose that the size of the time window is 5. We have  $S^{T_1} = \{x_1, x_2, \dots, x_5\}$ ,  $S^{T_2} = \{x_6, x_7, \dots, x_{10}\}$ ,  $S^{T_3} = \{x_{11}, x_{12}, \dots, x_{15}\}$  and  $S^{T_4} = \{x_{16}, x_{17}, \dots, x_{20}\}$ . Using Definition 3, we have  $DM_A(S^{T_1}, S^{T_2}) = 0.0333$ ,  $DM_A(S^{T_2}, S^{T_3}) = 0.2507$  and  $DM_A(S^{T_3}, S^{T_4}) = 0.2381$ . We set the threshold of concept drift to 0.2. Since  $DM_A(S^{T_1}, S^{T_2}) \leq 0.2$ , we have to allocate the most appropriate cluster label to each object of  $S^{T_2}$ . We first used the  $k$ -modes algorithm [15] to partition  $S^{T_1}$ . Assume that  $x_1, x_2$  were chosen as the initial cluster centers in  $S^{T_1}$ . We obtained the clustering results  $C^{T_1} = \{c_1^{T_1}, c_2^{T_1}\}$ , where  $c_1^{T_1} = \{x_1, x_5\}$  and  $c_2^{T_1} = \{x_2, x_3, x_4\}$ . Table 2 shows the degree of membership between each object in  $S^{T_2}$  and each cluster in  $S^{T_1}$  according to Definition 4.

From Table 2, we can obtain that  $c_1^{T_2} = \{x_6, x_8, x_{10}\}$  and  $c_2^{T_2} = \{x_7, x_9\}$ .

**Table 1**  
An example of categorical data stream.

Object	$A_1$	$A_2$	$A_3$	$A_4$
$x_1$	A	M	C	$t_1$
$x_2$	Y	E	P	$t_2$
$x_3$	X	E	P	$t_3$
$x_4$	Y	M	P	$t_4$
$x_5$	A	M	D	$t_5$
$x_6$	A	M	C	$t_6$
$x_7$	X	M	P	$t_7$
$x_8$	A	M	D	$t_8$
$x_9$	Y	M	P	$t_9$
$x_{10}$	A	M	C	$t_{10}$
$x_{11}$	B	E	G	$t_{11}$
$x_{12}$	X	M	P	$t_{12}$
$x_{13}$	B	E	D	$t_{13}$
$x_{14}$	Y	M	P	$t_{14}$
$x_{15}$	B	F	D	$t_{15}$
$x_{16}$	Y	M	P	$t_{16}$
$x_{17}$	X	M	P	$t_{17}$
$x_{18}$	Z	N	T	$t_{18}$
$x_{19}$	X	M	P	$t_{19}$
$x_{20}$	Y	M	P	$t_{20}$

**Table 2**  
The degrees of membership between objects of  $S^{T_2}$  and clusters of  $S^{T_1}$ .

	$x_6$	$x_7$	$x_8$	$x_9$	$x_{10}$
$c_1^{T_1}$	1.5817	0.0817	1.5817	0.0817	1.5817
$c_2^{T_1}$	0.0272	1.3606	0.0272	1.6939	0.0272

The data labeling algorithm is described in [Algorithm 1](#). The time complexity for computing the degree of membership between an object and a cluster is  $O(|S^{T_i}| |A|)$ . The total computational cost of the algorithm is  $O(|S^{T_i}| |A| |S^{T_j}| k_{T_i})$ . Therefore, this algorithm is linear to the number of the objects in  $S^{T_j}$ , i.e., the size of the time window.

**Algorithm 1.** The data labeling algorithm

- 1: **Input:**
- 2: –  $C^{T_i}$  : the clustering results in  $T_i$ ;
- 3: –  $S^{T_j}$  : the objects in  $T_j$ ;
- 4: **Output:** a partition of  $S^{T_j}$ ;
- 5: **Method:**
- 6: Generate a partition  $C^{T_i} = \{c_1^{T_i}, c_2^{T_i}, \dots, c_{k_{T_i}}^{T_i}\}$  of  $S^{T_i}$  with respect to  $A$  by calling the corresponding categorical clustering algorithm;
- 7: **for**  $j' = 1$  to  $|S^{T_j}|$  **do**
- 8:   **for**  $i' = 1$  to  $k_{T_i}$  **do**
- 9:     Calculate  $Sim_A(x_{j'}, c_{i'}^{T_i})$  according to [Definition 4](#), where  $x_{j'}$  is the  $j'$ th object in  $S^{T_j}$ .
- 10:   **end for**
- 11:   Give label  $L$  to  $x_{j'}$ , where  $L = \arg \max_{i'=1, \dots, k_{T_i}} \{Sim_A(x_{j'}, c_{i'}^{T_i})\}$ ;
- 12: **end for**
- 13: Return  $C^{T_j} = \{c_1^{T_j}, c_2^{T_j}, \dots, c_{k_{T_j}}^{T_j}\}$ ;

#### 4.2. Expression of concepts

The cluster expressions contribute to the understanding of concepts. The “modes” [15] are a traditional expression of clusters for categorical data. However, “modes” are mainly focused on the intra-cluster similarity and do not take the inter-cluster similarity into account. To solve this problem, we define a new cluster expression that considers both intra- and inter-cluster similarities.

**Definition 5.** Let  $TDT = (U, A, V, f, t)$  be a categorical data stream and  $S^T \subseteq U$ . Suppose that  $C^T = \{c_1^T, c_2^T, \dots, c_{k_T}^T\}$  is the clustering results on  $S^T$ . The expression of  $c_i^T \in C^T$  is defined as

$$R(c_i^T) = \left\{ q_j | q_j = \arg \max_{q_j \in V_{a_j}} \delta'_{a_j} \times \omega'_{a_j}, j = 1, 2, \dots, |A| \right\}, \tag{7}$$

where

$$\delta'_{a_j} = \frac{|\{x | f(x, a_j) = q_j, x \in c_i^T\}|}{|c_i^T|}$$

and

$$\omega'_{a_j} = 1 + \frac{1}{\log_2(k_T)} \times \psi.$$

Here

$$\psi = \sum_{i=1}^{k_T} \left( \frac{\left| \left\{ x | f(x) = q'_j, x \in c_i^T \right\} \right|}{\left| \left\{ z | f(z) = q'_j, z \in C^T \right\} \right|} \times \log_2 \frac{\left| \left\{ x | f(x) = q'_j, x \in c_i^T \right\} \right|}{\left| \left\{ z | f(z) = q'_j, z \in C^T \right\} \right|} \right).$$

Similar to Definition 4, the value of  $\delta'_{a_j}$  reflects the frequency of  $q'_j$  in  $c_i^T$ . The value of  $\omega'_{a_j}$  measures the entropy of component  $q'_j$  between clusters.

Continuing from Example 1, we have  $c_1^{T_1} = \{x_1, x_5\}$ ,  $c_2^{T_1} = \{x_2, x_3, x_4\}$ ,  $c_1^{T_2} = \{x_6, x_8, x_{10}\}$  and  $c_2^{T_2} = \{x_7, x_9\}$ . With Definition 5, we can obtain the expression of each cluster as shown in Table 3.

### 4.3. Distance between two concepts in consecutive windows

With the difference measure in Definition 3, we define a new distance between two concepts (clusters) in consecutive windows as follows.

**Definition 6.** Let  $TDT = (U, A, V, f, t)$  be a categorical data stream and  $S^{T_i}, S^{T_j} \subseteq U$ , where  $S^{T_i} \cap S^{T_j} = \emptyset$  and  $S^{[T_i, T_j]} = S^{T_i} \cup S^{T_j}$ . Suppose that  $C^{T_i} = \{c_1^{T_i}, c_2^{T_i}, \dots, c_{k_{T_i}}^{T_i}\}$  and  $C^{T_j} = \{c_1^{T_j}, c_2^{T_j}, \dots, c_{k_{T_j}}^{T_j}\}$  are the clustering results on  $S^{T_i}$  and  $S^{T_j}$ , respectively. The distance between  $c_i^{T_i}$  and  $c_j^{T_j}$  with respect to  $A$  is defined as

**Table 3**  
The expressions of clusters in 2 consecutive windows.

Clusters	Cluster expression
$c_1^{T_1} = \{x_1, x_5\}$	$R(c_1^1) = \{A, M, C\}$
$c_2^{T_1} = \{x_2, x_3, x_4\}$	$R(c_2^1) = \{Y, E, P\}$
$c_1^{T_2} = \{x_6, x_8, x_{10}\}$	$R(c_1^2) = \{A, M, C\}$
$c_2^{T_2} = \{x_7, x_9\}$	$R(c_2^2) = \{X, M, P\}$

**Table 4**  
The distances between clusters in two consecutive windows of 4 windows.

	$c_1^{T_1} = \{x_1, x_5\}$	$c_2^{T_1} = \{x_2, x_3, x_4\}$
$c_1^{T_2} = \{x_6, x_8, x_{10}\}$	0	0.7222
$c_2^{T_2} = \{x_7, x_9\}$	0.6667	0.0667
$c_1^{T_3} = \{x_{11}, x_{13}, x_{15}\}$	$c_1^{T_2} = \{x_6, x_8, x_{10}\}$ 0.7750	$c_2^{T_2} = \{x_7, x_9\}$ 1
$c_2^{T_3} = \{x_{12}, x_{14}\}$	0.6667	0
$c_1^{T_4} = \{x_{16}, x_{17}, x_{19}, x_{20}\}$	$c_1^{T_3} = \{x_{11}, x_{13}, x_{15}\}$ 1	$c_2^{T_3} = \{x_{12}, x_{14}\}$ 0
$c_2^{T_4} = \{x_{18}\}$	1	1

$$d_A(c_i^{T_i}, c_j^{T_j}) = \frac{NEDA\langle c_i^{T_i}, c_j^{T_j} \rangle + OFDA\langle c_i^{T_i}, c_j^{T_j} \rangle}{2}, \tag{8}$$

where  $1 \leq i' \leq k_{T_i}, 1 \leq j' \leq k_{T_j}$ .

Continuing from Example 1, we have  $DM_A(S^{T_2}, S^{T_3}) > 0.2$  and  $DM_A(S^{T_3}, S^{T_4}) > 0.2$ . We consider that  $T_3$  and  $T_4$  are two concept-drifting windows. Suppose that the clustering results of  $S^{T_3}$  and  $S^{T_4}$  are  $C^{T_3} = \{c_1^{T_3}, c_2^{T_3}\}$  and  $C^{T_4} = \{c_1^{T_4}, c_2^{T_4}\}$ , where  $c_1^{T_3} = \{x_{11}, x_{13}, x_{15}\}, c_2^{T_3} = \{x_{12}, x_{14}\}, c_1^{T_4} = \{x_{16}, x_{17}, x_{19}, x_{20}\}$  and  $c_2^{T_4} = \{x_{18}\}$ . With Definition 6, we can compute the distances of clusters in  $S^{T_i}$  and  $S^{T_{i+1}}$  ( $1 \leq i \leq 3$ ) as shown in Table 4.

4.4. Significance of attributes and outlier degree of objects

To find the causes of concept change in consecutive windows, we measure the significance of an attribute the change of whose values affects the change of clusters in the following window. If the value distributions of an attribute remain the same in the two consecutive windows, then this attribute has little effect on the concept change. The significance of an attribute is measured as follows.

**Definition 7.** Let  $TDT = (U, A, V, f, t)$  be a categorical data stream and  $S^{T_i}, S^{T_j} \subseteq U$ , where  $S^{T_i} \cap S^{T_j} = \emptyset$  and  $S^{[T_i, T_j]} = S^{T_i} \cup S^{T_j}$ . For any  $a \in A$ , the significance of  $a$  between  $S^{T_i}$  and  $S^{T_j}$  is defined as

$$Sig_{\{a\}}(S^{T_i}, S^{T_j}) = \frac{DM_{\{a\}}(S^{T_i}, S^{T_j})}{\sum_{c \in A} DM_{\{c\}}(S^{T_i}, S^{T_j})}. \tag{9}$$

Continuing from Example 1, we can use Definition 7 to compute the significance of each attribute in two consecutive windows as shown in Table 5.

From Table 5, we can see that  $A_1$  and  $A_3$  has no effect on concept change from  $T_1$  to  $T_2$ .  $A_1, A_2$  and  $A_3$  have the same contributions for concept change from  $T_3$  to  $T_4$ .

Similarly, different objects provide different contributions for the concept change. If the attribute values of an object rarely occur in two consecutive windows, the object provides the maximal contribution to concept change and can be considered as an outlier [24]. We measure the degree of an object as an outlier as follows.

**Definition 8.** Let  $TDT = (U, A, V, f, t)$  be a categorical data stream and  $S^{T_i}, S^{T_j} \subseteq U$ , where  $S^{T_i} \cap S^{T_j} = \emptyset$  and  $S^{[T_i, T_j]} = S^{T_i} \cup S^{T_j}$ . For any  $x \in S^{[T_i, T_j]}$ , the outlier degree of the object  $x$  with respect to  $A$  is defined as

$$OD_A(x) = \frac{1}{|A|} \sum_{a \in A} \left( 1 + \frac{w(a)}{|S^{[T_i, T_j]}|} \times \log_2 \frac{w(a)}{|S^{[T_i, T_j]}|} \right), \tag{10}$$

where  $w(a) = |\{z | f(x, a) = f(z, a), z \in S^{[T_i, T_j]}\}|$ .

Continuing from Example 1, the outlier degree of each object in  $T_1 \rightarrow T_2, T_2 \rightarrow T_3$  and  $T_3 \rightarrow T_4$  is shown in Table 6.

From Table 6, we can see that objects  $x_3, x_{11}$  and  $x_{18}$  have the maximum outlier degree in  $T_1 \rightarrow T_2, T_2 \rightarrow T_3$  and  $T_3 \rightarrow T_4$ , respectively.

**Table 5**  
The significance of each attribute in two consecutive windows of 4 windows.

Windows	$A_1$	$A_2$	$A_3$
$T_1 \rightarrow T_2$	0	1	0
$T_2 \rightarrow T_3$	0.5698	0.1994	0.2308
$T_3 \rightarrow T_4$	0.3333	0.3333	0.3333

**Table 6**  
The outlier degree of each object in two consecutive windows of 4 windows.

$T_1 \rightarrow T_2$	$x_1$ 0.5950	$x_2$ 0.8157	$x_3$ 0.8434	$x_4$ 0.5950	$x_5$ 0.6226	$x_6$ 0.5950	$x_7$ 0.6226	$x_8$ 0.6226	$x_9$ 0.5950	$x_{10}$ 0.5950
$T_2 \rightarrow T_3$	$x_6$ 0.7350	$x_7$ 0.7025	$x_8$ 0.7074	$x_9$ 0.7025	$x_{10}$ 0.7350	$x_{11}$ 0.9322	$x_{12}$ 0.7025	$x_{13}$ 0.8845	$x_{14}$ 0.7025	$x_{15}$ 0.9046
$T_3 \rightarrow T_4$	$x_{11}$ 0.9322	$x_{12}$ 0.6410	$x_{13}$ 0.9122	$x_{14}$ 0.6410	$x_{15}$ 0.9322	$x_{16}$ 0.6410	$x_{17}$ 0.6410	$x_{18}$ 1.0000	$x_{19}$ 0.6410	$x_{20}$ 0.6410

4.5. Trend analysis algorithm

Integrating the techniques discussed in the previous sections, we define the trend analysis algorithm in Algorithm 2. The total computational cost of this algorithm is  $O(|S^{T_i}||A|k_{T_i} + |S^{T_j}||A|k_{T_j} + k_{T_i}k_{T_j}|S^{T_i} \cup S^{T_j}||A|) = O(k_{T_i}k_{T_j}|S^{T_i} \cup S^{T_j}||A|)$ .

**Algorithm 2.** The trend analysis algorithm

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```

1: Input:
2: -  $C^{T_i}$  : the clustering results in  $T_i$ ;


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3: -  $C^{T_j}$  : the clustering results in  $T_j$ ;
4: -  $\gamma$  : the specified threshold;
5: Output: the trend of concept change from  $T_i$  to  $T_j$ ;
6: Method:
7: Obtain clustering results  $C^{T_i} = \{c_1^{T_i}, c_2^{T_i}, \dots, c_{k_{T_i}}^{T_i}\}$  and  $C^{T_j} = \{c_1^{T_j}, c_2^{T_j}, \dots, c_{k_{T_j}}^{T_j}\}$  with respect to  $A$ ;
8: for  $i' = 1$  to  $k_{T_i}$  do
9:   Generate  $R(c_{i'}^{T_i})$  according to Definition 5;
10: end for
11: for  $j' = 1$  to  $k_{T_j}$  do
12:   Generate  $R(c_{j'}^{T_j})$  according to Definition 5;
13: end for
14: for  $i' = 1$  to  $k_{T_i}$  do
15:   for  $j' = 1$  to  $k_{T_j}$  do
16:     if  $d_A(c_{i'}^{T_i}, c_{j'}^{T_j}) \leq \gamma$  then
17:       Connect  $c_{i'}^{T_i}, c_{j'}^{T_j}$  with line;
18:     end if
19:   end for
20: end for

```

---

We use the trend analysis algorithm to analyze the trend of concept change in the data stream of Table 1. We set the threshold  $\gamma$  to 0.2. The trend of concept change in 4 consecutive time windows is shown in Fig. 2. The horizontal axis is consecutive time windows. The blue and red circles in each column indicate the clusters in the time window. The size of the circle represents the number of objects. The content in each circle is the expression of concept in each cluster. Similar concepts are linked with the green lines. From this figure, we can understand how concepts change in consecutive windows.

In comparison with the result in [6], Fig. 3 shows the relative concept change between windows. We computed the new concept emerging degree and the old concept fading degree in consecutive time windows as shown in Fig. 3. We can see that concept change was caused by emerging new concepts or fading old concepts or both. From  $T_2$  to  $T_3$ , the new concept emerging degree was greater than the old concept fading degree. This indicates that more new concepts emerged than old concepts

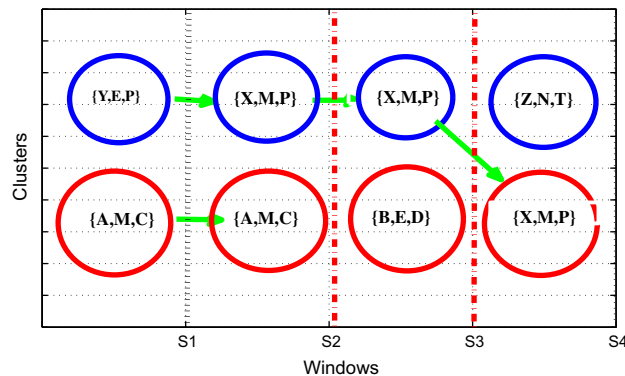


Fig. 2. The trend of concept change in 4 consecutive time windows.



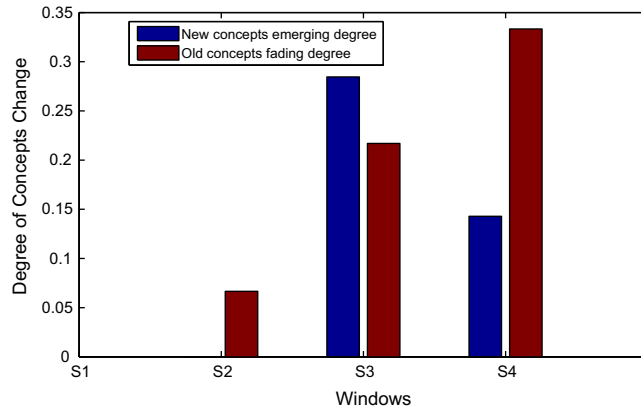


Fig. 3. The change of two measures in 4 consecutive windows.

faded. However, from  $T_3$  to  $T_4$ , more old concepts faded than new concepts emerged. This phenomenon was caused by the fact that a new cluster of  $x_{11}, x_{13}$  and  $x_{15}$  emerged in  $T_3$  from  $T_2$  and an old cluster of  $x_{11}, x_{13}$  and  $x_{15}$  in  $T_3$  faded in  $T_4$ .

### 5. Experimental results

A series of experiments was conducted on real data for evaluation of the proposed trend analysis algorithm. In this section, we present the results of trend analysis on a real data stream for network intrusion detection and investigate the causes of concept change through significance of attributes and discuss the property of parameter  $\gamma$ .

#### 5.1. Network stream data

KDD-CUP'99 was used as a test data for The Third International Knowledge Discovery and Data Mining Tools Competition. The data set contained 494,021 records, each having a timestamp. The records were classified into 23 classes. One class indicated the normal connection and other 22 classes were network attack types. Each record was described by 41 attributes, in which 34 attributes were continuous and 7 were categorical. We used uniform quantization to convert these continuous attributes into discrete values, each attribute with 5 categories. We also aggregated 22 attack classes into one general attack class.

#### 5.2. Trends analysis

The first 15,000 records in the network data set were selected as a sample data to show trend analysis. We choose 3000 records as the size of the time window and divided the sample data into 5 consecutive time windows. We first used the  $k$ -modes algorithm [15] to cluster the records in the first window into two clusters, each representing a concept. Before executing the  $k$ -modes algorithm, we used the method in [7] to obtain its initial cluster centers. Then, we used  $DM_A$  distance

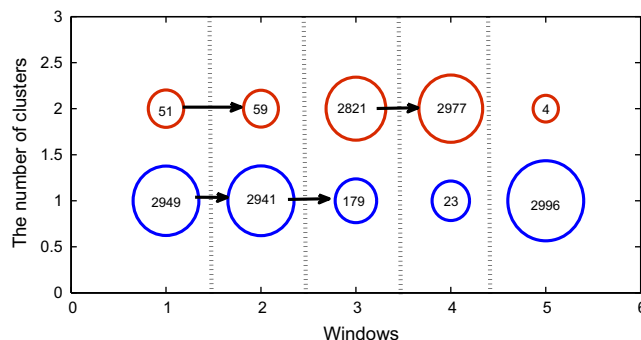


Fig. 4. The trend of concept change on the sample set by the proposed method.



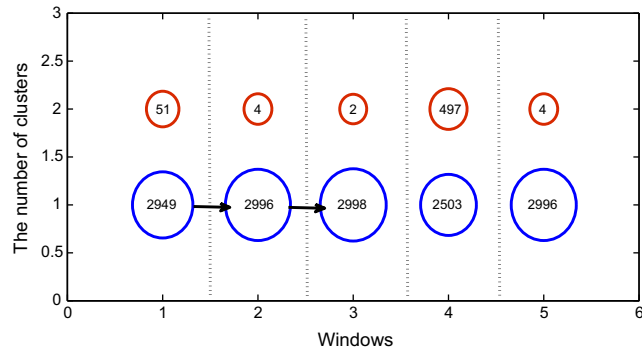


Fig. 5. The trend of concept change on the sample set by the method in [6].

Table 8  
The significance of attributes in 5 consecutive time windows

Attr	$T_1 \rightarrow T_2$	$T_2 \rightarrow T_3$	$T_3 \rightarrow T_4$	$T_4 \rightarrow T_5$
1	0.0345	0.0004	0	0
2	0	0	0	0.2445
3	0	0.0032	0.1195	0.2607
4	0.0345	0.0040	0.0298	0.0008
5	0	0	0	0
6	0	0	0	0
7	0	0	0	0
8	0	0	0	0
9	0	0	0	0
10	0.0690	0	0.0448	0.0003
11	0	0	0.0149	0.0001
12	0	0	0	0
13	0	0	0	0
14	0	0.0016	0	0.0003
15	0	0	0	0
16	0	0	0	0
17	0	0	0	0
18	0.0690	0.0008	0	0
19	0.0690	0.0008	0.0149	0.0001
20	0	0	0	0
21	0	0	0	0
22	0	0	0	0.0009
23	0	0.4834	0.1642	0.2319
24	0	0.4834	0.1642	0.2319
25	0.0690	0	0.0298	0.0003
26	0.0690	0.0004	0.0448	0.0001
27	0	0.0016	0	0.0004
28	0	0.0016	0	0.0004
29	0.0345	0.0028	0.0298	0.0025
30	0.0690	0.0028	0.0149	0.0025
31	0	0	0	0
32	0	0	0	0
33	0	0	0	0
34	0	0	0	0.0199
35	0.1034	0.0024	0.0746	0.0012
36	0.2069	0.0032	0.1194	0
37	0.0690	0.0012	0.0149	0.0004
38	0.0690	0.0008	0.0597	0.0002
39	0	0.0016	0.0597	0
40	0	0.0016	0	0.0004
41	0.0345	0.0020	0	0.0004

Table 9  
The value distributions of attribute 4.

$T_1$	$T_2$	$T_3$	$T_4$	$T_5$
{2,7,10}	{2,7,8,10}	{6,10}	{10}	{2,6,7,10}

### 5.3. Significance of attributes

To investigate the causes of concept change between two windows, we analyzed the significance of attributes for the changed concepts. The result is shown in Table 8. The first column is attribute and the other four columns are the significance measure of each attribute on the transition of two consecutive windows. Value 0 implies that the concept change in consecutive windows was not caused by that attribute. The values greater than 0 indicate that the attributes contributed to the concept change in the consecutive windows.

From Table 8, we can see that a few attributes contributed significantly to the change of concepts in consecutive windows, such as attributes 4, 19, 26, 29, 30, 35, 37 and 38. These attributes were the main causes of the concept change in 5 consecutive windows. Some attributes such as 23 and 24 show significant impact on the concept change in consecutive windows  $T_2 \rightarrow T_3, T_3 \rightarrow T_4$  and  $T_4 \rightarrow T_5$ .

Further investigating the causes of concept change, we looked into the value distributions of an attribute in consecutive windows. Table 9 shows the example of attribute 4 in 5 time windows. We can see that the bigger the difference of value distributions in two consecutive windows, the more significant the causes of concept change by the attribute. From this observation, we can monitor the concept change of a data stream by looking into the value change of significant attributes in the data stream.

Computing the new concept emerging degree and the old concept fading degree defined in Definition 2, we investigated the relative concept change in 5 consecutive time windows. The result is shown in Fig. 6. We can see a dramatic drop of the fading degree and an obvious rise of the emerging degree from  $T_4$  and  $T_5$ . This is an indication that the change was mainly caused by emerging new concepts and the fact was that there were 2488 attack records in  $T_4$  which disappeared in  $T_5$  whereas 3000 normal connection records emerged in  $T_5$ .

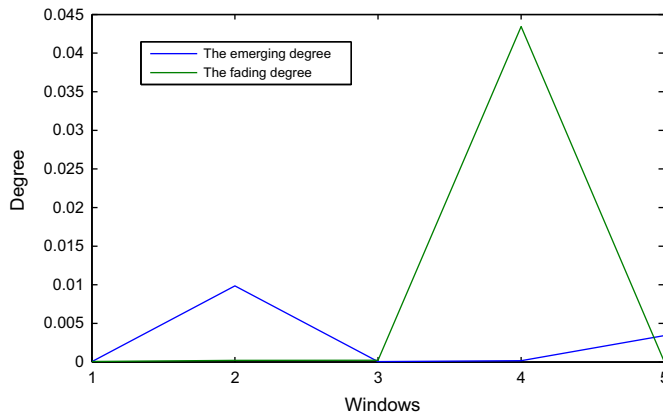


Fig. 6. The changes of the new concept emerging degree and the old concept fading degree in 5 consecutive time windows.

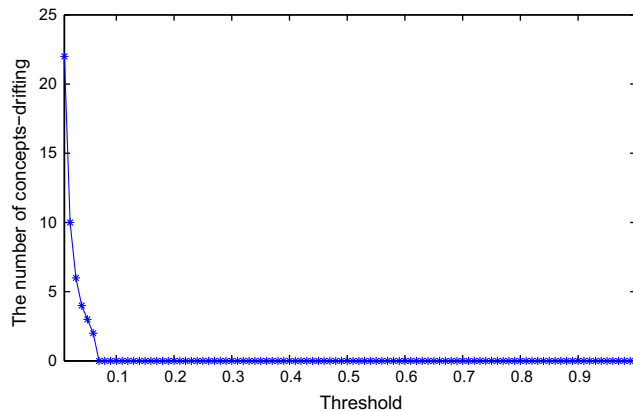


Fig. 7. The relationships between the number of concepts-drifting and  $\gamma$ .

#### 5.4. Impact of $\gamma$

Using the trend analysis algorithm [Algorithm 2](#), we need to specify a threshold  $\gamma$  that determines whether two concepts in consecutive windows are the same concept or one concept in one window has drifted to another concept in the following window. We used the entire data set to investigate  $\gamma$ . The time window size was set as 3000 records and 164 consecutive windows were obtained. We ran the trend analysis algorithm with different values of  $\gamma$  and counted the number of pairs of consecutive windows which had concept change measured by the distance of two consecutive windows which was greater than  $\gamma$ .

[Fig. 7](#) shows the relationships between the number of concept drifts and the value of  $\gamma$ . We can see that the number of drifting-concepts decreases as  $\gamma$  increases. When  $\gamma$  is greater than 0.07, the number of drifting-concepts drops to zero, which means no concept change was identified. Therefore,  $\gamma$  cannot be greater than 0.07 in this data set. To better reveal concept change patterns, we set  $\gamma = 0.01$  as default.

## 6. Conclusions

In this paper, we have presented a new method for trend analysis of categorical data streams. In this method, a data labeling method has been proposed by considering both the intra-cluster similarity and the inter-cluster similarity. We have defined a new distance between concepts in two consecutive windows that is used to measure the concept change. The significance measure of attributes has also been defined to reveal the causes of concept change. We have used a real data stream to demonstrate the usefulness of the new algorithm in trend analysis.

The trend analysis algorithm proposed in this paper is applied to categorical data streams. Our future work is to study the trend of concept change in the case of continuous data by using the neighborhood rough set because continuous data streams are widely available in real applications.

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