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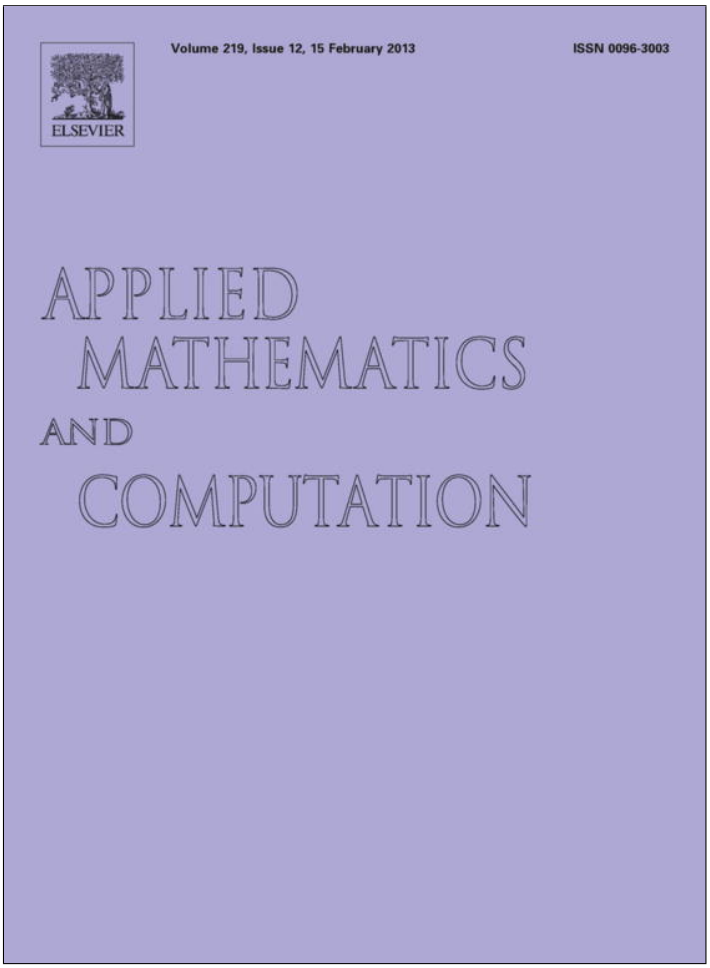

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Dynamics in a predator–prey model with space and noise

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ABSTRACT

In this paper, we presented a predator–prey model with spatial diffusion and density-dependent noise. It was found that the spatial model without noise has spotted pattern. However, combined with density-dependent noise, the predator–prey model exhibits transition from spotted to labyrinth pattern. Furthermore, when noise intensity and temporal correlation are in appropriate levels, the populations will extinct. The obtained results suggest that stochastic factors may play an important role in populations dynamics, which well enrich the findings in predator–prey models.

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1. Introduction

In biological systems, one important type of interactions which have effect population dynamics of all species is predation. Thus, researches on predator–prey relationships have been and continues to be the focus of ecological science [17]. Since that long time series of the density of both predator and prey is needed, it is difficult to identify the population dynamics from the observed data. Consequently, it may be useful by constructing mathematical models to explain the phenomenon observation in the real world.

On one hand, predator–prey interactions are strongly influenced by space because of the localized nature of predation or other forms of interaction [24,21,23,19,20,22], which implies that mathematical models with time and space are useful in predator–prey systems [30,28,32]. Based on spatial predator–prey models, we can predict how populations distribute in space and time, estimate the spread speed and provide some policy decisions to protect endangered populations [37,36,9,31].

On the other hand, noise widely exists in the actual population dynamics [10]. It was reported that songbirds experience decreased predation rate in the environments with noise [8], and hermit crabs were distracted by boat motor noise and hence less vigilant against approaching predators [3,25]. What is more, the impact of noise on breeding numbers and species richness has been well demonstrated by empirical results [1,8,12].

In the fields of theoretical ecology, noise effects have been assumed to be a source of disorder and thus understanding the role of noise in population dynamics has been a hot area of extensive study [29]. Noise-induced effects in ecosystems have been well investigated [26]. Particularly, the interactions of biological factors and noise has been widely discussed. New counterintuitive phenomena, such as stochastic resonance, noise enhanced stability and noise delayed extinction can emerge due to the presence of noise in living systems [11,5,2,27,6,14,34]. For such reason, noise should be included when modelling the interactions of predator and prey.

However, spatial predator–prey models with density-dependent noise had been generally overlooked despite its potential ecological reality and intrinsic theoretical interest. As a result, in the present paper, we want to check the impact of noise

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on the dynamical behavior of a spatial predator–prey model in terms of phase transition from persistence to extinction. To this end, we will investigate pattern formation of a spatial predator–prey model in the Holling–Tanner form. More specifically, we will reveal that how noise intensity and temporal correlation have influence on the spatial distribution of the populations.

The paper is organized as follows. In Section 2, we obtain a predator–prey model with spatial diffusion and density-dependent noise, and interpret the biological meaning of these parameters of the model. In Section 3, by performing a series of simulations, we showed that there are transition from spotted to labyrinth pattern. What is more, we give the regions between persistence and extinction of populations in parameters space. At last, conclusions and discussion are given.

2. Main model

Lotka–Volterra model is famous in mathematical ecology. However, it has the unavoidable limitations to describe many realistic phenomena. In order to describe the real ecological interactions between the predator–prey species, Tanner proposed the following predator–prey model [33]:

$$\frac{dU}{dT} = RU\left(1 - \frac{U}{K}\right) - \frac{MUV}{U + C}, \tag{1a}$$

$$\frac{dV}{dT} = V\left[\theta\left(1 - \frac{HV}{U}\right)\right], \tag{1b}$$

where U and V are the prey and predator populations. In absence of predation, the prey grows logistically with carrying capacity K and intrinsic growth rate R. The saturating predator functional response $MU/(C + U)$ used in (1) is of Holling type. The parameter M is the maximum specific rate of product formation, and C (the half-saturation constant) is the substrate density at which the rate of product formation is half maximal. It assumes predators grow logistically with intrinsic growth rate θ and carrying capacity proportional to the prey populations size U. The parameter H is the number of prey required to support one predator at equilibrium when V equals to U/H [35].

By taking that

$$u = \frac{U}{K}, \quad v = \frac{MV}{RK}, \quad t = RT, \tag{2a}$$

$$\alpha = \frac{C}{K}, \quad \beta = \frac{\theta}{R}, \quad \gamma = \frac{HR}{M}, \tag{2b}$$

we arrive at the following equations containing dimensionless quantities:

$$\frac{du}{dt} = u(1 - u) - \frac{uv}{u + \alpha}, \tag{3a}$$

$$\frac{dv}{dt} = v\left[\beta\left(1 - \frac{\gamma v}{u}\right)\right]. \tag{3b}$$

The stochastic factors are taken into account as the term, $\eta(r, t)$, obtained from microscopic interaction in the space [15,16,13]. The noise term $\eta(r, t)$ is introduced additively in space and time, which is the Ornstein–Uhlenbech process that obeys the following equation:

$$\langle \eta(\mathbf{r}, t)\eta(\mathbf{r}', t')\rangle = \frac{\phi}{\tau}\exp\left(-\frac{-|t - t'|}{\tau}\right)\delta(\mathbf{r} - \mathbf{r}'), \tag{4}$$

where τ controls the temporal correlation, and ϕ measures the noise intensity.

In this paper, we want to investigate the dynamics of a predator–prey model with density-dependent noise, and thus we will focus our attentions on the following model:

$$\frac{\partial u}{\partial t} = u(1 - u) - \frac{uv}{u + \alpha} + \nabla^2 u + u\eta(r, t), \tag{5a}$$

$$\frac{\partial v}{\partial t} = v\left[\beta\left(1 - \frac{\gamma v}{u}\right)\right] + \varepsilon\nabla^2 v + v\eta(r, t), \tag{5b}$$

Before proceeding to the spatially explicited case, the first step is to have a look at the properties of the local dynamics. System (3) has the following stationary steady:

(i) $E_0 = (1, 0)$, which is corresponding to extinct of the predator;
(ii) interior equilibrium point $E^* = (u^*, v^*)$, which is corresponding to coexistence of prey and predator, and given by

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$$u^* = \gamma v^*, \tag{6a}$$

$$v^* = \frac{\gamma - 1 - \gamma\alpha + \sqrt{(\gamma - 1 - \gamma\alpha)^2 + 4\alpha\gamma^2}}{2\gamma^2}. \tag{6b}$$

3. Pattern formation

In the following part, we rely on numerical integration of the model of Eq. (5) and parameters values lies within the regime of Turing domain. Extensive testing was performed through numerical integration to describe model (5), and the results are shown in this section. Noise term may cause the density of the prey or predator to be negative. As a result, if the population densities are less than a certain prescribed value at each position in space, they are set to zero. During the simulations, different types of dynamics are obtained and the distributions of prey and predator are always of the same type. Consequently, we can restrict our analysis of pattern formation to predator population.

We firstly consider the case that there is no noise. Fig. 1 shows the evolution of the spatial pattern of predator population at $t = 0$, 100, 200, and 500 with small random perturbation of the stationary solution E^* of the spatially homogeneous systems with the parameters set: $\alpha = 0.8$, $\beta = 0.2$, $\gamma = 1$ and $\varepsilon = 0.01$. One can see that the regular spotted pattern emerges in the two-dimensional space.

It is checked that a large variety of distinct patterns can be obtained by making small changes in parameters ϕ and τ. In Fig. 2, we find that the distribution of predator is random at the early time. As time increases, the system has spotted pattern.

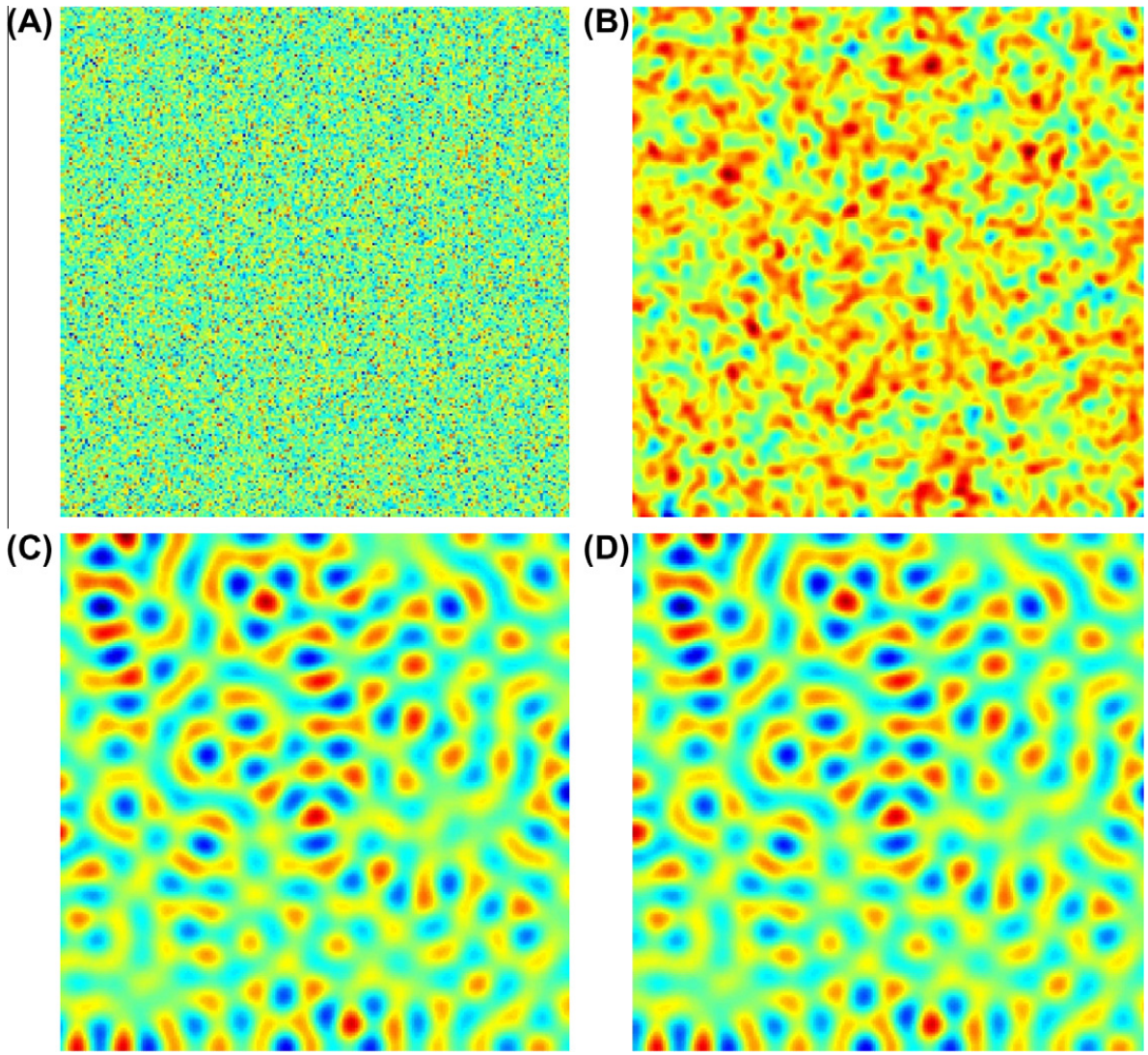

Fig. 1. Typical spatial pattern diagram of the predator density in two-dimension space for the system (5) without noise. Parameter values: $\alpha = 0.8$, $\beta = 0.2$, $\gamma = 1$ and $\varepsilon = 0.01$. (A) $t = 0$; (B) $t = 100$; (C) $t = 200$; (D) $t = 500$. Prey density exhibits similar properties.

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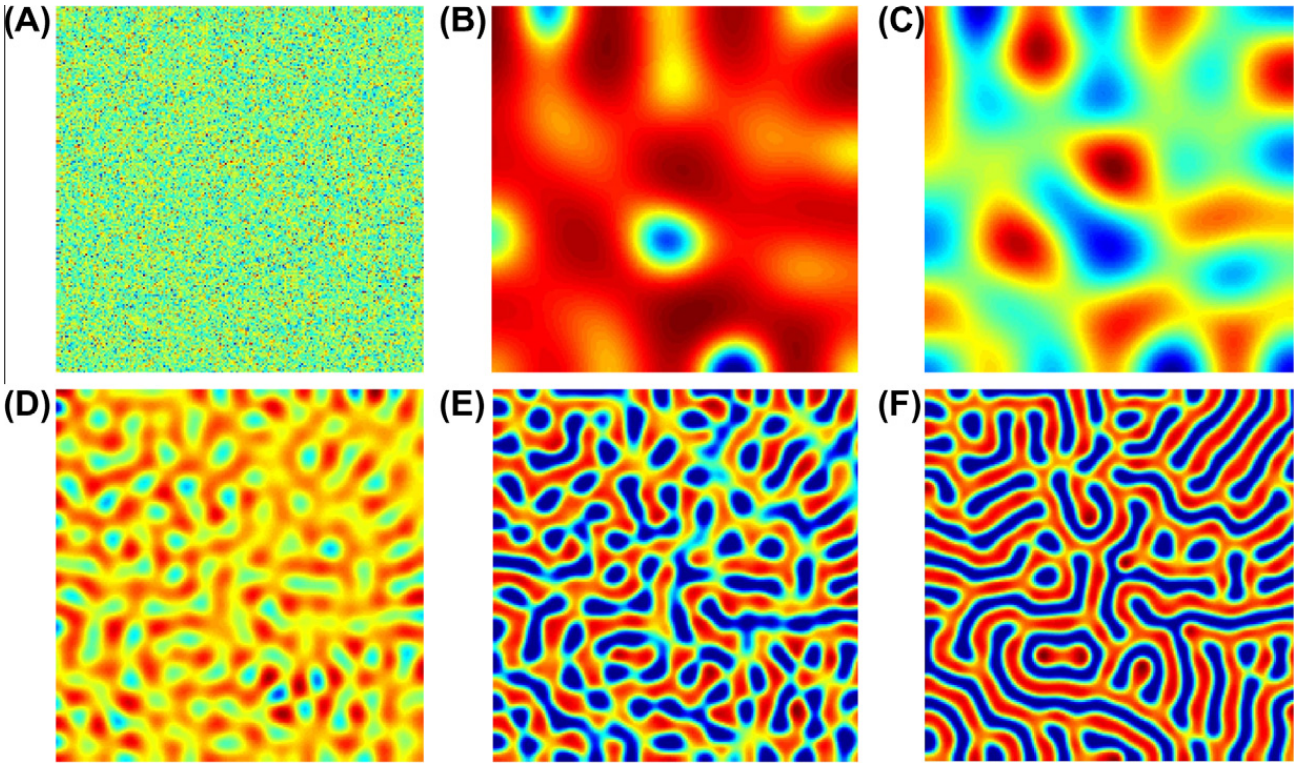

Fig. 2. Typical spatial pattern diagram of the predator density in two-dimension space for the system (5) eter values: $\alpha = 0.8$, $\beta = 0.2$, $\gamma = 1$, $\varepsilon = 0.01$, $\phi = 0.176$ and $\tau = 10$. (A) $t = 0$; (B) $t = 50$; (C) $t = 120$; (D) $t = 200$; (E) $t = 450$; (F) $t = 850$. Prey density exhibits similar properties.

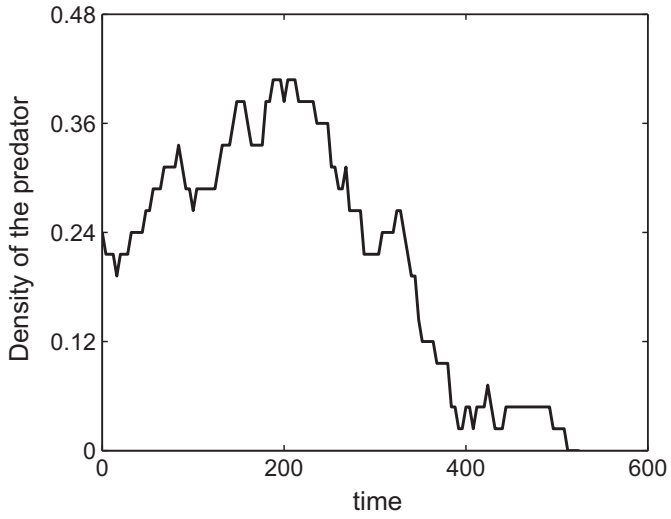

Fig. 3. Time series of the predator with fixed noise intensity and temporal correlation. It indicates that the predator will extinct for a long time. Parameter values: $\alpha = 0.8$, $\beta = 0.2$, $\gamma = 1$, $\varepsilon = 0.01$, $\phi = 0.842$ and $\tau = 7$.

However, as time is largh enough, labyrinth pattern emerges in the whole domain. That is to say, for some appropriate values, system (5) exhibits a pattern transition from spotted pattern to labyrinth pattern.

The previous work showed that even if the environment can ensure positive growth on average of populations, noise may cause the populations to extinct [18]. Now, it is natural to ask whether the density-dependent noise has the same effect or not. For fixed noise intensity and temporal correlation, we find that the density of predator will decrease to zero for a long time which means noise will induce the extinction of populations (see Fig. 3). In particular, we need to give the extinction and persistence regime of populations with respect to the noise intensity and temporal correlation. In Fig. 4, we can conclude that, for large values of noise intensity and small values of temporal correlation, the population run a high risk of extinction.

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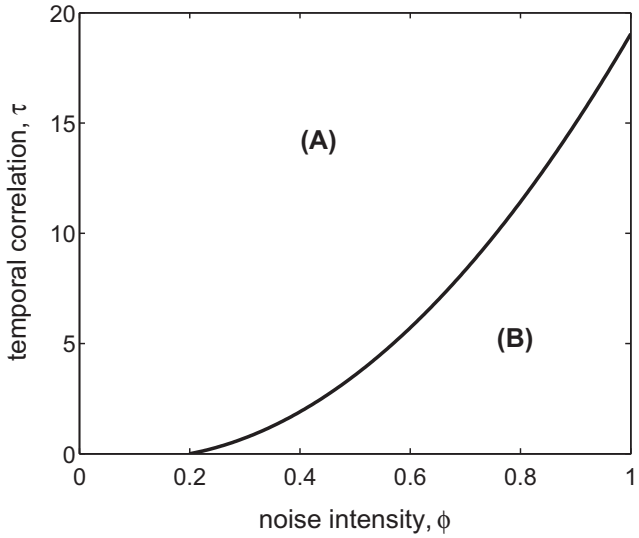

Fig. 4. Extinction and persistence regime of predator. Parameter values: $\alpha = 0.8$, $\beta = 0.2$, $\gamma = 1$ and $\varepsilon = 0.01$. (A) Persistent regime; (B) Extinct regime.

4. Discussion and conclusion

Noise has been regarded in ecological systems as perturbations that may have impact on population dynamics [26]. As a result, we investigated a predator–prey model with diffusion and density-dependent noise. A series of numerical simulations showed that pattern transition will emerge when density-dependent noise is added. What is more, the populations will extinct for some values of noise intensity and temporal correlation. The results imply that stochastic terms may be important factors in population dynamics.

Sun et al. investigated a predator–prey model with Holling III functional response and density-independent noise [29]. They found that noise can induce chaotic patterns and resonant patterns. However, pattern transition did not be observed in their work. In the present paper, we found that density-dependent noise can cause transition from spotted to labyrinth pattern.

In this paper, we assume that the motion of individuals of given population is random and isotropic. In fact, Lévy random walks strategy has been adopted by many species in response to patchy resource. Abnormal diffusion has been applied in the spatial population dynamics [4,7], which is a more general form for describing the diffusion phenomena in the nature. This issue needs further investigation and discussion.

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