Int. J. Mach. Learn. & Cyber. (2014) 5:233–243
DOI 10.1007/s13042-013-0160-x

ORIGINAL ARTICLE

Topological approach to multigranulation rough sets

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Received: 30 October 2012 / Accepted: 5 March 2013 / Published online: 2 April 2013

Abstract For further studying the theory of multigranulation rough sets, we attempt to investigate a new theory on multigranulation rough sets from the topological view in this paper. We firstly explore multigranulation topological rough space and its topological properties by giving some new definitions and theorems. Then, topological granularity and topological entropy are proposed to characterize the uncertainty of a multigranulation topological rough space. Finally, based on the invariance of interior and closure operators of a target concept, a granulation selection algorithm is introduced to deal with the granularity selection issue in the multigranulation rough data analysis.

Keywords Rough sets · MGRS · Topology · Multigranulation topological rough space · Topological entropy

1 Introduction

Rough set theory (RS), originated by Pawlak [28], is a new mathematical tool to deal with incomplete, imprecise, and uncertain knowledge. In the past decades, rough set theory has developed significantly due to its wide applications. Various generalized rough set models have been established and their corresponding properties or structures have been investigated intensively and extensively [1, 2, 3, 4, 16, 39, 50, 52]. These extensional ways are mainly based on either relaxing an equivalence relation or generalizing partition to covering on the universe.

Topology is an important branch of mathematics aimed at studying the invariance of a given space under topological transformation (homeomorphism) [9], whose theories and applications have been studied in [6, 7, 14, 7, 17, 18]. In [29], Pawlak has pointed out that topology is closely related to rough set theory and convinced that topology structure of the rough set is one of key issues of rough set theory. The topology theory and rough set theory have been applied in many science and engineering areas such as in chemistry, biology, image processing, knowledge acquisition and pattern recognition. Therefore, how to combine rough set theory and topology theory becomes an interesting and natural research topic, in fact which has received considerable attention from the scholars in this community [12, 21, 22, 30, 31, 27, 36, 43, 49, 46, 51]. In particular, Skowron [41] and Wiweger [45] separately discussed this topic in 1988. Lin continued to discuss this topic and established a connection between fuzzy rough sets and topology [21]. Furthermore, by using the theories of the topology and the neighborhood systems, Polkowski [26] constructed and characterized topological spaces from rough sets based on information systems. In the literature [27], Polkowski pointed out: "topological aspects of rough set theory were recognized early in the framework of topology of partitions". Lashin [15] generalized rough set theory in the frameworks of topological space and topologized information tables. Zhu [51] studied several

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covering-based rough sets on the topological view. Yang [44] investigated relationships among separation axioms and two topological spaces. Kortelainen [11] and Jarvinen [8] considered relationships among modified sets, topological spaces and rough sets based on a preorder. Qin [37] discussed the relationship between fuzzy rough set models and fuzzy topologies on a finite non-empty universe. Some other authors discussed relationships between generalized rough sets and the topology from different viewpoints (for example see [5, 10, 41]). Skowron et al. [41] generalized the classical approximation spaces to tolerance approximation spaces and discussed the problems of attribute reduction in these spaces. In addition, connections between fuzzy rough set theory and fuzzy topology were also investigated [16, 37, 40].

According to the different strategies, Qian and Liang [33] proposed multigranulation rough sets by employing multiple binary relations on the universe instead of a single one. A target concept was described through these granulations on the universe based on a user's different requirements or targets of problem solving. Since multigranulation rough set is an important research direction of the rough set theory, there have been many studies on this topic. For example, Liu et al. [24, 25] proposed covering fuzzy rough set based multigranulation rough sets. Xu et al. [42] investigated another generalized version, called variable precision multigranulation rough sets. Yang et al. [44] proposed a multigranulation rough set based on a fuzzy binary relation. Lin et al. [23] investigated a neighborhood-based multigranulation rough set model, which can be used to deal with the data sets with hybrid attributes. Liang et al. [20] proposed an efficient feature selection algorithm for large-scale data sets from the perspective of multigranulation which also demonstrates the usefulness of MGRS theory. What deserves to be mentioned is that She et al. [38] explored the topological structures and the properties of multigranulation rough sets. However, the multigranulation topological rough space and the corresponding topological properties are still not studied. The motivation of this paper is to investigate these problems and to describe the uncertainty of multigranulation topological space by proposing topological granularity measure and topological entropy.

With respect to the application of the multigranulation rough sets in a multi-source information system and the distributive information system where information is obtained from different sources [13], it is important to consider two interesting issues, i.e., the granular selection and the granulation selection in the view of granular computing [35]. They are two different important ways to reduce the redundant information in data analysis. The theory of granular selection is the same as that of covering reduct [53]. So we only concentrate on the issue of the granulation selection in this paper. Given two multigranulation spaces of a universe, two multigranulation topological rough spaces will be generated. In the view of granular computing, the issue in this paper is that for a family of single granulation spaces which induced a multigranulation topological rough space, what would be the corresponding "smallest" subset of that family of granulations, which can produce the same multigranulation topological rough space? That is, they have the same interior and closure operators of a target concept.

Throughout the research, some new results and achievements proposed in the paper may enrich the theories of the multigranulation rough sets and the topology, which may form the theoretical basis for further applications of multigranulation rough set theory and topology.

The main objective of this paper is to study the multigranulation rough set theory via topology theory. The rest of this paper is organized as follows. Some basic concepts of topology and multigranulation rough sets are briefly reviewed in Sect. 2. In Sect. 3, the multigranulation topological rough space is constructed and some of its important properties are investigated. In Sect. 4, the topological granularity and the topological entropy are introduced to characterize the uncertainty of multigranulation topological rough space. The concept of granulation selection is proposed and a granulation selection algorithm based on the invariance of interior and closure operators of a target concept is given to select the necessary granulation in multigranulation rough data analysis. Finally, Section 5 concludes with some remarks.

2 Preliminaries

In this section, we introduce some fundamental key concepts of topology and rough set theory [9, 29]. Throughout this paper, we suppose that the universe Ω is a non-empty finite set.

2.1 Basic concepts of topology

We present a brief overview of topological space, a closure operator, an interior operator, and a topology based on a set. They are all important concepts in topology theory and they were used to study rough sets [22, 32, 47]. In this paper, these topological tools are also employed to investigate multigranulation rough sets.

Definition 2.1 (A topological space) [9] A topological space is a pair (Ω, τ) consisting of a set Ω and a family τ of subset of Ω satisfying the following conditions:

(T1) $\phi \in \tau$ and $\Omega \in \tau$,
(T2) τ is closed under arbitrary union,

(T3) τ is closed under finite intersection.

The pair (Ω, τ) is called a topological space. The elements of Ω are called points of the space. The subsets of Ω belonging to τ are called the open set in the space, and the complement of the subsets of Ω belonging to τ are called the closed set in the space and the family of open subsets of Ω is also called a topology for Ω.

Definition 2.2 (Closure and interior operators). For an operator $cl : 2^{\Omega} \rightarrow 2^{\Omega}$, if it satisfies the following conditions, then we call it a closure operator on $\Omega. \forall X, Y \subseteq \Omega$,

(C1) $cl\ (\emptyset) = \emptyset$,
(C2) $X \subseteq cl(X)$,
(C3) $cl(cl(X)) = cl(X)$,
(C4) $cl(X \cup Y) = cl(X) \cup cl(Y)$.

For an operator $int : 2^{\Omega} \rightarrow 2^{\Omega}$, if it satisfies the following rules, then we call it an interior operator on $\Omega. \forall X, Y \subseteq \Omega$,

(I1) $int(\Omega) = \Omega$,
(I2) $int(X) \subseteq X$,
(I3) $int(int(X)) = int(X)$,
(I4) $int(X \cap Y) = int(X) \cap int(Y)$.

It is well known that an interior operator int on Ω can induce a topology τ such that in the topological space $(\Omega, \tau), int(X)$ is just the interior of X for each $X \subseteq \Omega$. The similar statement is also true for a closure operator [31].

In a topological space (Ω, τ), a family $\beta \subseteq \tau$ of sets is called a base for the topology τ if for each point x of the space, and each neighborhood X of x, there is a member V of β such that $x \in V \subseteq \Omega$. We know that a subfamily β of a topology τ is a base for τ if and only if each member of τ is the union of members of β. Moreover, $\beta \subseteq 2^{\Omega}$ forms a base for some topology on Ω if and only if β satisfies the following conditions:

(B1) $\Omega = \cup \{B | B \in \beta\}$,
(B2) For every two members X and Y of β and each point $x \in X \cap Y$ there is $Z \in \beta$ such that $x \in Z \subseteq X \cap Y$.

2.2 Fundamentals of multigranulation rough sets

In [33], Qian analyzed some restrictions of Pawlak classical rough set in practice and proposed a new extension of rough set i.e., multigranulation rough sets, in which a target concept can be approximated by multiple equivalence relations according to a user's different requirements. In other words, a target concept can be approximated by multiple granulation spaces in the view of granular computing [34].

Assume that Ω is a finite non-empty universe of discourse. Let R be an equivalence relation on $\Omega, \Omega/R$ is a corresponding partition of Ω, denoted by $\Omega/R = \{[x]_R | x \in \Omega\}$ in which $[x]_R = \{y | y \in \Omega, xRy\}$ is an equivalence class consisting $x. \Omega/R$ can generate a topological space, denoted as (Ω, τ_R). and Ω/R is a topology base of τ_R, each subset of τ_R is both open and close [5].

Definition 2.3 [33]. Let $S = (\Omega, AT, f)$ be an information system. Suppose that $X \subseteq \Omega$, $R_1, R_2, \cdots, R_q$ be q equivalence relations on Ω, the lower approximation $\underline{\sum_{i=1}^{q} R_i}(X)$ and the upper approximation $\overline{\sum_{i=1}^{q} R_i}(X)$ of X with respect to $R_1, R_2, \cdots, R_q$ are defined as follows, respectively,

$$\underline{\sum_{i=1}^{q}} R_i(X) = \{x \in \Omega \mid \vee([x]_{R_i} \subseteq X), i \leq q\}; \tag{1}$$

$$\overline{\sum_{i=1}^{q}} R_i(X) = \{x \in \Omega \mid \wedge([x]_{R_i} \cap X \neq \emptyset), i \leq q\}. \tag{2}$$

From the above expressions, the operator '$\vee$' is a disjunctive operator which here indicates that in multiple independent granular structures, one needs only at least one granular structure to satisfy with the inclusion condition between an equivalence class and a target concept. The expression (2) is the upper approximation of the optimistic multigranulation rough set that can be also defined by the complement of the lower approximation, which has been proved in [23]. the operator '$\wedge$' in expression (2) is a conjunctive operator whose meaning is that in multiple independent granular structures, one needs all granular structures to satisfy with non-empty for joint operator between an equivalence class and a target concept. And $\underline{\sum_{i=1}^{q} R_i}(X) \subseteq X \subseteq \overline{\sum_{i=1}^{q} R_i}(X)$. So we can label multigranulation rough set $X = (\underline{\sum_{i=1}^{q} R_i}(X), \overline{\sum_{i=1}^{q} R_i}(X))$, accordingly, we call $(\Omega, R_1, R_2, \cdots, R_q)$ a multigranulation approximation space in the view of granular computing. From [33], we obtain the following interpretations:

- The lower approximation of a set X with respect to $\sum_{i=1}^{q} R_i$ is the set of all elements, which can certainly be classified as X using $\sum_{i=1}^{q} R_i$ (are certainly X in view of $\sum_{i=1}^{q} R_i$).
- The upper approximation of a set X with respect to $\sum_{i=1}^{q} R_i$ is the set of all elements, which can possibly be classified as X using $\sum_{i=1}^{q} R_i$ (are possibly X in view of $\sum_{i=1}^{q} R_i$).
- The boundary region of a set X with respect to $\sum_{i=1}^{q} R_i$ is the set of all elements, which can be classified neither as X nor as not-X using $\sum_{i=1}^{q} R_i$.

Let $\emptyset$ be an empty set, $\sim X$ the complement of X in U, we have the following properties of multigranulation rough sets [33].

(1ML)$\underline{\sum_{i=1}^{q} R_i}(U) = U$	(Co-normality)
(1MH)$\overline{\sum_{i=1}^{q} R_i}(U) = U$	(Co-normality)
(2ML)$\underline{\sum_{i=1}^{q} R_i}(\emptyset) = \emptyset$	(Normality)
(2MH)$\overline{\sum_{i=1}^{q} R_i}(\emptyset) = \emptyset$	(Normality)
(3ML)$\underline{\sum_{i=1}^{q} R_i}(X) \subseteq X$	(Contraction)
(3MH)$X \subseteq \overline{\sum_{i=1}^{q} R_i}(X)$	(Extension)
(4ML)$\underline{\sum_{i=1}^{q} R_i}(\bigcap_{j=1}^{n} X_j) \subseteq \bigcap_{j=1}^{n}(\underline{\sum_{j=1}^{n} R_i}(X_j))$	(Implication)
(4MH)$\overline{\sum_{i=1}^{q} R_i}(\bigcup_{j=1}^{n} X_j) \supseteq \bigcup_{j=1}^{n}(\overline{\sum_{j=1}^{n} R_i}(X_j))$	(Implication)
(5ML)$\underline{\sum_{i=1}^{q} R_i}(\bigcup_{j=1}^{n} X_j) \supseteq \bigcup_{j=1}^{n}(\underline{\sum_{j=1}^{n} R_i}(X_j))$	(Implication)
(5MH)$\overline{\sum_{i=1}^{q} R_i}(\bigcap_{j=1}^{n} X_j) \subseteq \bigcap_{j=1}^{n}(\overline{\sum_{j=1}^{n} R_i}(X_j))$	(Implication)
(6ML)$\underline{\sum_{i=1}^{q} R_i}(\underline{\sum_{i=1}^{q} R_i}(X)) = \underline{\sum_{i=1}^{q} R_i}(X)$	(Idempotency)
(6MH)$\overline{\sum_{i=1}^{q} R_i}(\overline{\sum_{i=1}^{q} R_i}(X)) = \overline{\sum_{i=1}^{q} R_i}(X)$	(Idempotency)
(7ML)$\underline{\sum_{i=1}^{q} R_i}(\sim X)) = \sim \overline{\sum_{i=1}^{q} R_i}(X)$	(Duality)
(7MH)$\overline{\sum_{i=1}^{q} R_i}(\sim X) = \sim \underline{\sum_{i=1}^{q} R_i}(X)$	(Duality)
(8ML)$X \subseteq Y \Rightarrow \underline{\sum_{i=1}^{q} A_i}(X) \subseteq \underline{\sum_{i=1}^{q} R_i}(Y)$	(Monotone)
(8MH)$X \subseteq Y \Rightarrow \overline{\sum_{i=1}^{q} R_i}(X) \subseteq \overline{\sum_{i=1}^{q} R_i}(Y)$	(Monotone)
(9ML)$\forall K \in U/R_i, i \in \{1, 2, \cdots, q\}, \underline{\sum_{i=1}^{q} R_i}(K) = K$	(Granularity)
(9MH)$\forall K \in U/R_i, i \in \{1, 2, \cdots, q\}, \overline{\sum_{i=1}^{q} R_i}(K) = K$	(Granularity)
(10ML)$\underline{\sum_{i=1}^{q} R_i}(X) = \bigcup_{i=1}^{q}(\underline{R_i}(X))$	(Relation based Addition)
(10MH)$\overline{\sum_{i=1}^{q} R_i}(X) = \bigcap_{i=1}^{q}(\overline{R_i}(X))$	(Relation based Multiplication)

By the above discussions and similar to Definition 1 of [48], we can define an interior and closure operators of multigranulation rough set X from a topological point of view.

3 Topological approach to multigranulation rough sets

In this section, in order to better apply the multigranulation rough set theory into complex data analysis, we shall investigate an interesting and natural research topic of studying multigranulation rough set theory via topology theory.

Definition 3.1 Let $(\Omega, \tau_1), (\Omega, \tau_2), \cdots, (\Omega, \tau_q)$ be q topological spaces induced by equivalence relations $R_1, R_2, \cdots, R_q$, respectively, and $X \subseteq \Omega$. Then we define *mint* and *mcl* operators of X with respect to Γ, where $\Gamma = \{\tau_1, \tau_2, \cdots, \tau_q\}$, respectively, as follows:

$$mint(X) = \bigcup\{A \in \tau_i | \vee (A \subseteq X), i \leq q\}; \tag{3}$$

$$mcl(X) = \bigcup\{A \in \tau_i | \wedge (A \cap X \neq \emptyset), i \leq q\}. \tag{4}$$

The area of topological uncertainty or boundary (*mbn*)is defined as

$mbn(X) = mcl(X) \setminus mint(X).$

Theorem 3.1 *Let* $(\Omega, \tau_1), (\Omega, \tau_2), \cdots, (\Omega, \tau_q)$ *be* q *topological spaces induced by equivalence relations* $R_1, R_2, \cdots, R_q$, *respectively, and* $X \subseteq \Omega$. *We have* $mint(X) = \underline{\sum_{i=1}^{q} R_i}(X)$, $mcl(X) = \overline{\sum_{i=1}^{q} R_i}(X)$.

Proof By Definition 3.1 and the definitions of lower and upper approximations of X in MGRS they can be easily proved.

According to the above propositions of multigranulation rough sets, we easily obtain the following results.

Proposition 3.1 *Let* $(\Omega, \tau_1), (\Omega, \tau_2), \cdots, (\Omega, \tau_q)$ *be* q *topological spaces induced by equivalence relations* $R_1, R_2, \cdots, R_q$, *respectively, and* $X, Y \subseteq \Omega$. *Then, with respect to the operators mint, we have*

(1) $mint(\Omega) = \Omega$,
(2) $mint\ (\emptyset) = \emptyset$,
(3) $mint(X) \subseteq X$,
(4) $X \subseteq Y \Rightarrow mint(X) \subseteq mint(Y)$,
(5) $mint(mint(X)) = mint(X)$.

Similarly, with respect to the operators mcl, we have the following results:

(1) $mcl(\Omega) = \Omega$,
(2) $mcl\ (\emptyset) = \emptyset$,
(3) $X \subseteq mcl(X)$,
(4) $X \subseteq Y \Rightarrow mcl(X) \subseteq mcl(Y)$,
(5) $mcl(mcl(X)) = mcl(X)$.

Theorem 3.2 *Let $(\Omega, \tau_1), (\Omega, \tau_2), \cdots, (\Omega, \tau_q)$ be q topological spaces induced by equivalence relations $R_1, R_2, \cdots, R_q$, respectively, and $X, Y \subseteq \Omega$. Then, $mint(X \cap Y) = mint(X) \cap mint(Y)$ holds if and only if the multigranulation rough set model is equivalent to the Pawlak's single granulation rough set model.*

Proof It can be proved by employing the result of Proposition 10 in [20].

Theorem 3.3 *Let $(\Omega, \tau_1), (\Omega, \tau_2), \cdots, (\Omega, \tau_q)$ be q topological spaces induced by equivalence relations $R_1, R_2, \cdots, R_q$, respectively, and $X, Y \subseteq \Omega$. Then, $mcl(X \cup Y) = mcl(X) \cup mcl(Y)$ holds if and only if the multigranulation rough set model is equivalent to the Pawlak's single granulation rough set model.*

Proof It can be proved by employing the result of Proposition 10 in [20].

From the above discussions, we can get the following results.

Theorem 3.4 *Let $(\Omega, \tau_1), (\Omega, \tau_2), \cdots, (\Omega, \tau_q)$ be q topological spaces induced by equivalence relations $R_1, R_2, \cdots, R_q$, respectively, and $X, Y \subseteq \Omega$. Then, mint and mcl are interior and closure operators, respectively, if and only if the multigranulation rough set model is equivalent to the Pawlak's single granulation rough set model.*

Definition 3.2 (Natural mapping). Let R be an equivalence relation, the family of $\{[x] \mid x \in \Omega\}$ is a quotient set on Ω, denoted by Ω/R. A mapping $f : \Omega \to \Omega/R$ satisfies $f(x) = [x], x \in \Omega$, we call f a natural mapping on Ω.

Definition 3.3 (Intersection operator). Let R_1 and R_2 be equivalence relations on a finite universe Ω, f_1 and f_2 natural mappings. Then we define a intersection mapping $F_\cap : \Omega \to 2^\Omega$ satisfies $F_\cap(x) = f_1(x) \cap f_2(x)$.

Further, if τ_1, τ_2 are two topologies induced by R_1 and R_2, then we can define $\tau_1 \sqcap \tau_2 = \{F_\cap(x) \mid x \in \Omega\}$.

Theorem 3.5 *$\tau_1 \sqcap \tau_2$ is a topology.*

Proof Suppose Ω be a finite universe, R_1 and R_2 two equivalence relations, and $\tau_1 = \{\emptyset, \Omega, [x_{i1}]_{R_1}, [x_{i2}]_{R_1}, \cdots, [x_{ik}]_{R_1}\}$, $\tau_2 = \{\emptyset, \Omega, [x_{j1}]_{R_2}, [x_{j2}]_{R_2}, \cdots, [x_{jl}]_{R_2}\}$ induced by R_1 and $R_2, k, l \leq |\Omega|$, where $|\cdot|$ is cardinality of Ω.

(1) Based on the definition of $\tau_1 \sqcap \tau_2$, obviously $\emptyset \in \tau_1 \sqcap \tau_2$, $\Omega \in \tau_1 \sqcap \tau_2$.
(2) Assume that $X, Y \in \tau_1 \sqcap \tau_2$, then there exists two equivalence classes $[x]_{R_1} \in \tau_1$, $[x']_{R_2} \in \tau_2$ such that $X \subset [x]_{R_1}$, $Y \subset [x']_{R_2}$. Hence $X \cap Y \in [x]_{R_1} \cap [x']_{R_2} \in \tau_1 \sqcap \tau_2$.
(3) Let $\tau \in \tau_1 \sqcap \tau_2$, suppose that $\bigcup_{X \in \tau} X \notin \tau_1 \sqcap \tau_2$. Then there at least exists an element $x \in X \in \tau$, we have an equivalence class $[x]$ consisting x in $\tau_1 \sqcap \tau_2$ such that $[x] \notin \tau_1 \sqcap \tau_2$. Note that $[x] = ([x]_{R_1} \cap [x]_{R_2) \in \tau_1 \sqcap \tau_2}$ holds, a contraction!

Therefore, $\tau_1 \sqcap \tau_2$ is still a topology.

Similarly, we can prove that the intersection of the finite topologies is topology, i.e. $\sqcap_{i=1}^m \tau_i$ is a topology with respect to $\tau_1, \tau_2, \cdots, \tau_q$, denoted by $\sqcap_{i=1}^m \tau_i = \Gamma_M$.

Definition 3.4 (Multigranulation topological rough space) Let $(\Omega, \tau_1), (\Omega, \tau_2), \cdots, (\Omega, \tau_q)$ be q topological spaces induced by equivalence relations $R_1, R_2, \cdots, R_q$, respectively. An intersection operation $F_\cap$: $\Omega \to 2^\Omega$. Then $(\Omega, \sqcap_{i=1}^q \tau_i)$ is called a multigranulation topological rough space, denoted as $(\Omega, \sqcap_{i=1}^q \tau_i) = (\Omega, \Gamma_M)$, written by (Ω, Γ) for simplicity.

Definition 3.5 (The partial relation between two multigranulation topological rough space) Let τ_1, τ_2 be two topologies on Ω, if for any $X \in \tau_1$, there exists $Y \in \tau_2$ such that $X \subseteq Y$. Then we call τ_1 finer than τ_2, denoted by $\tau_1 \leq^\tau \tau_2$. If τ_1 is strictly finer than τ_2, denoted by $\tau_1 <^\tau \tau_2$. If and only if $X = Y$, then $\tau_1 = \tau_2$. Similarly, let Γ_1, Γ_2 be two multigranulation topological rough spaces on Ω, if for any $\tau_1 \in \Gamma_1$, there exists $\tau_2 \in \Gamma_2$ such that $\tau_1 \leq^\tau \tau_2$, then we call Γ_1 than Γ_2, denoted by $\Gamma_1 \leq^\Gamma \Gamma_2$. If Γ_1 is strictly finer than Γ_2, denoted by $\Gamma_1 <^\Gamma \Gamma_2$. If and only if $\tau_1 = \tau_1$, then $\Gamma_1 = \Gamma_2$.

Theorem 3.6 *Let $\tau_1, \tau_2, \cdots, \tau_q$ be q topologies on Ω induced by equivalence relations $R_1, R_2, \cdots, R_q$, respectively. If $\tau_1 <^\tau \tau_2 <^\tau \cdots <^\tau \tau_q$, then $\Gamma = \tau_1$.*

Actually, from the above definition, we know (Ω, Γ) is finer than each topology on Ω.

Corollary 3.1 *If $\tau_1 <^\tau \tau_2$, and β_1, β_2 are the topology base of τ_1, τ_2, respectively. Then $\beta_1 <^\tau \beta_2$.*

Corollary 3.2 *If $\Gamma_1 <^\Gamma \Gamma_2$, and β_{1M}, β_{2M} are their family of the topology base of Γ_1, Γ_2, respectively. Then $\beta_{1M} <^\Gamma \beta_{2M}$.*

Theorem 3.7 (Topology base of Γ_M) *Let β_i be the topology base of τ_i, then $\sqcap_{i=1}^{q}\beta_i$ is a topology base of multigranulation topological rough space Γ.*

Proof

(1) For any $x \in \Omega$, there exists $[x]_{R_i} \in \beta_i$ such that $x \in \cap[x]_{R_i}$. Note that $\cap[x]_{R_i} \in \sqcap_{i=1}^{q}\beta_i$. Hence, let $B = \cap[x]_{R_i} \in \sqcap_{i=1}^{q}\beta_i$, we have $x \in B$.
(2) For any $B_1, B_2 \in \sqcap_{i=1}^{q}\beta_i$, suppose $x \in B_1 \cap B_2$, but B_1 is a set which is intersection of $\cap_{i=1}^{q}[x]_{R\_i}$, B_2 is a set which is intersection of $\cap_{i=1}^{q}[y]_{R\_i}$, then $x \neq y$, otherwise $[x]_{R\_i} = [y]_{R\_i}$. Hence $B_1 \cap B_2 = \emptyset$. There exists $B_3 = \emptyset$, obviously, $\emptyset \subset \emptyset$ holds.

Therefore $\sqcap_{i=1}^{q}\beta_i$ is a topology base of multigranulation topological rough space Γ.

Corollary 3.3 *Let $\tau_1, \tau_2, \cdots, \tau_q$ be q topologies on Ω induced by equivalence relations $R_1, R_2, \cdots, R_q$ and β_i the topology base of τ_i* for $i \in \{1, 2, \cdots, q\}$. *Then $\sqcap_{i=1}^{q}\beta_i$ is a partition of Ω.*

Definition 3.5 Let (Ω, Γ) be a multigranulation topological rough space, β_M is a topology base of Γ, and $X \subseteq \Omega$. Then the interior operator is defined as

$$INT(X) = \{x \in X \mid \forall x \in A \in (\Gamma \setminus N), then A \subseteq X\}, \tag{5}$$

where $N = \bigcup\{Y | Y \subseteq X \subseteq \Omega\}$.

Definition 3.6 Let (Ω, Γ) be a multigranulation topological rough space, β_M a topology base of Γ, and $X \subseteq \Omega$. Then the closure operator is

$$CL(X) = \cup\{A \in \Gamma \mid A \cap X \neq \emptyset\}. \tag{6}$$

Example 3.1 Let $\Omega = \{x_1, x_2, x_3, x_4, x_5\}$ and $X = \{x_1, x_2, x_3, x_4\} \subseteq \Omega$. R_1 and R_2 are two equivalence relations on $\Omega. \Omega/R_1 = \{\{x_1, x_2, x_3\}, \{x_4, x_5\}\}$ corresponding to $f_1(x) = [x]_{R_1}, \Omega/R_2 = \{\{x_1\}, \{x_3\}, \{x_2, x_4, x_5\}\}$ corresponding to $f_2(x) = [x]_{R\_2}$. Then we have that $\underline{\sum_{i=1}^{2} R_i}(X) = \{x_1, x_2, x_3\}$, $\overline{\sum_{i=1}^{2} R_i}(X) = \{x_1, x_2, x_3, x_4, x_5\}$ and get two topologies $\tau_1 = \{\emptyset, \Omega, \{x_1, x_2, x_3\}, \{x_4, x_5\}\}, \tau_2 = \{\emptyset, \Omega, \{x_1\}, \{x_3\}, \{x_2, x_4, x_5\}\}. \beta_1 = \{\{x_1, x_2, x_3\}, \{x_4, x_5\}\}$ and $\beta_2 = \{\{x_1\}, \{x_3\}, \{x_2, x_4, x_5\}\}$ are topology bases of τ_1 and τ_2, respectively. By Definition 3.3, we have that $\Gamma = \tau_1 \sqcap \tau_2 = \{\emptyset, \Omega, \{x_1, x_2, x_3\}, \{x_4, x_5\}, \{x_1\}, \{x_3\}, \{x_2, x_4, x_5\}, \{x_1, x_3\}, \{x_1, x_2\}, \{x_2, x_3\}, \{x_2, x_3, x_4, x_5\}, \{x_1, x_2, x_4, x_5\}\}$. Then the topology base of $\beta_M = \{\{x_1\}, \{x_3\}, \{x_2\}, \{x_4, x_5\}\}$. Hence, $mint(X) = \{x_1, x_2, x_3\}$, $mcl(\mathrm{X}) = \{x_1, x_2, x_3, x_4, x_5\}$.

As a result of this example, we have the following propositions.

Proposition 3.1 *Let (Ω, Γ) be a multigranulation multigranulation topological rough space and $X \subseteq \Omega$. $\underline{\sum_{i=1}^{q} R_i}(X)$ and $\overline{\sum_{i=1}^{q} R_i}(X)$ are lower and upper approximations of X. Then we have*

(1) $\underline{\underline{\sum_{i=1}^{q} R_i}}X = INT(X)$,
(2) $\overline{\overline{\sum_{i=1}^{q} R_i}}X = CL(X)$.

Proof

(1) For any $x \in \underline{\sum_{i=1}^{q} R_i}X$, there exists $[x]_R \subseteq X$. From Definition 3.4, $[x]_R \in \Gamma_M \setminus N$, where $N = \{Y | Y \subseteq X\}$. Hence $x \in INT(X)$, i. e., $\underline{\sum_{i=1}^{q} R_i}X \subseteq INT(X)$. On the other hand, for any $x \in INT(X)$, there exists $A \in \Gamma \setminus N$ such that $x \in A$ and $A \subset X$. Obviously, A is an equivalence class induced by R and $A = [x]_R$, i. e., $[x]_R \subseteq X$. So $x \in \underline{\sum_{i=1}^{q} R_i}X$. Hence $x \in INT(X) \subseteq \underline{\sum_{i=1}^{q} R_i}X$. Therefore, $\underline{\sum_{i=1}^{q} R_i}X = INT(X)$ holds.
(2) According to the definition of $\overline{\sum_{i=1}^{q} R_i}X$, for any R_i, $[x]_{R\_i} \cap X \neq \emptyset$ holds. To Γ, there exists some element V (a subset of Ω) in Γ such that $V \subseteq [x]_{R_i}$. So there exists two cases:
   (i) If $V \cap X = \emptyset$, then $V \not\subseteq CL(X)$. Hence $\overline{\sum_{i=1}^{q} R_i}X = CL(X)$ holds.
   (ii) If $V \cap X \neq \emptyset$, then $V \subseteq CL(X)$. Note that $V \subseteq [x]_{R_i}$, hence $\overline{\sum_{i=1}^{q} R_i}X = CL(X)$ also holds.

From the above Proposition, we find that a target concept's interior and closure operators in multigranulation topological rough space are also equal to its lower and upper approximations in the multigranulation rough rough sets, respectively.

Proposition 3.2 *Let $(\Omega, \Gamma$ be a multigranulation topological rough space and $X \subseteq Y \subset \Omega$. Then $INT(X) \subseteq INT(Y), CL(X) \subseteq CL(Y)$.*

Proof They can be proved similar to Theorem 3.2 in [34].

4 Measure of multigranulation topological rough space

4.1 Measure of multigranulation topological rough space

In this section, we introduce the uncertainty of multigranulation topological rough space.

Definition 4.1 (Granularity of a set) Let Ω be a finite nonempty universe. A function $m : 2^{\Omega} \to \Re$ is called a measure of the granularity of a set if it satisfies the following conditions: for all $A, B \in 2^{\Omega}$,

(M1) $m(A) \geq 0$,
(M2) $A \subset B \Rightarrow m(A) < m(B)$,

(M3) $A \sim_s B \Rightarrow m(A) = m(B)$.

Where $A \sim_s B \Leftrightarrow (\sim(A <_s B), \sim(B <_s A))$, "$<_s$" is the weak order that is an extension of "$\subset$" (see [47]).

Theorem 4.1 *Let $\Omega/R = \{A_1, A_2, \cdots, A_k\}$ be a partition of Ω, then we call*

$$m(A) = \frac{|\Omega|}{k}\left(1 - \frac{1}{k \cdot |A|}\right)$$

a measure of granularity of a set A, k is the number of blocks in Ω/R, denoted by $|\Omega/R| = k$.

Proof It is sufficient to show that m meets all the conditions in Definition 4.1.

(1) Obviously, $m(A) = \frac{|\Omega|}{k}(1 - \frac{1}{k \cdot |A|}) \geq 0$.
(2) If $A \subset B$, then $|A| < |B|$, then $m(A) - m(B) = \frac{|\Omega|}{k}(1 - \frac{1}{k \cdot |A|}) \quad - \frac{|\Omega|}{k}(1 - \frac{1}{k \cdot |B|}) = \frac{|\Omega|}{k}(1 - \frac{1}{k \cdot |A|} - 1 + \frac{1}{k \cdot |B|}) = \frac{|\Omega|}{k} (\frac{1}{k \cdot |B|} - \frac{1}{k \cdot |A|}) < 0$, i.e., $m(A) < m(B)$.
(3) If $\sim A \sim_s B$, then $|A| \leq |B|$ and $|A| \geq |B|$. Hence $m(A) = m(B)$.

Proposition 4.1 (Maximum) *Let Ω/R be a partition of Ω induced by an equivalence relation on Ω and $X \in \Omega/R$. The maximum granularity measure of X with respect to R is one. This value is achieved if and only if $k = 1, max(m(A)) = |\Omega|(1 - \frac{1}{|\Omega|})$.*

Proposition 4.2 (Minimum) *Let Ω/R be a partition of Ω induced by an equivalence relation on Ω and $X \in \Omega/R$. The minimum granularity measure of X with respect to R is one. This value is achieved if and only if $k = |\Omega|$, $min(m(A)) = (1 - \frac{1}{|\Omega|})$.*

Hence, $(1 - \frac{1}{|\Omega|}) \leq m(A) \leq |\Omega|(1 - \frac{1}{|\Omega|})$.

Example 4.1 (Continued from Example 3.1) To $\beta_1, m(B_{11}) = \frac{5}{2}(1 - \frac{1}{3 \cdot 2}) = \frac{25}{12}, m(B_{12}) = \frac{5}{2}(1 - \frac{1}{2 \cdot 2}) = \frac{15}{8}$, so $m(\beta_1) = (\frac{25}{12}, \frac{15}{8})^T$. To $\beta_2, m(B_{21}) = \frac{2}{9}$, $m(B_{22}) = \frac{2}{9}$, $m(B_{23}) = \frac{8}{27}$, so $m(\beta_2) = (\frac{2}{9}, \frac{2}{9}, \frac{8}{27})^T$. To $\beta_M, m(B_1) = \frac{4}{5}$, $m(B_2) = \frac{4}{5}, m(B_3) = \frac{9}{10}$, $m(B_4) = \frac{9}{10}, m(B_4) = \frac{9}{10}$, so $m(\beta_M) = (\frac{4}{5}, \frac{4}{5}, \frac{9}{10}, \frac{9}{10}, \frac{9}{10})^T$.

Definition 4.2 Let $T = \{\Gamma\}$ be a family of multigranulation topological rough spaces on Ω. A function $G : \Gamma \to \Re$ is called a measure of granularity of a partition if it satisfies the following conditions for all $\Gamma_1, \Gamma_2 \in T$,

(G1) $G(\Gamma) \geq 0$ (Nonnegativity)
(G2) $\Gamma_1 \subset \Gamma_2 \Rightarrow G(\Gamma_1) < G(\Gamma_2)$ (Monotonicity)
(G3) $\Gamma_1 = \Gamma_2 \Rightarrow G(\Gamma_1) = G(\Gamma_2)$ (Size invariance)

Considering a family set $\beta_M = \{B_1, B_2, \cdots, B_q\}$ of a finite non-empty universe Ω. One may associate it with a probability discussion [19], $P_{\beta_M} = (\frac{|B_1|}{|\Omega|}, \frac{|B_2|}{|\Omega|}, \cdots, \frac{|B_q|}{|\Omega|})$. Then we can define granularity of a multigranulation topological rough space as follows.

Theorem 4.2 (Topological granularity) *Let (Ω, Γ) be a multigranulation topological rough space, $m : 2^{\Omega} \to \Re$ a measure of the granularity of subsets of Ω, and $\beta_M = \{B_1, B_2, \cdots, B_q\}$ a topology base of Γ_M. Then a measure*

$$G_M(\Gamma) = \sum_{i=1}^{q} m(B_i) \cdot p(B_i).$$

is topological granularity of Γ, where $p(B_i) = \frac{|B_i|}{|\Omega|}$.

Proof It is sufficient to show that G_M satisfies all the conditions in Definition 4.2.

(1) Obviously, $G_M(\Gamma) \geq 0$ holds.
(2) Suppose $\Gamma_1 <^{\Gamma} \Gamma_2$, by Corollary 3.3, $\beta_{1M} \subset \beta_{2M}$ holds. This means that every equivalence class of β_{2M} is a union of one or more blocks of β_{1M} and at least one equivalence class of $\beta_M^{'}$ is the union of at least two blocks from β_{1M}. By the fact Ω is a finite universe, there exists a finite sequence of partitions $\beta_{1M} = \beta_{M1} \subset \beta_{M2} \subset \cdots \beta_{Ml} = \beta_{2M}$ such that exactly one block of β_{j+1} is the union of two equivalence classes from β_j for $j = 1, 2, \cdots, n-1$ and $n \geq 2$. We want to show that $G(\beta_j) < G(\beta_{j+1})$. Without loss of generality, suppose a equivalence class of β_{j+1} is obtained by the union of two equivalence classes B_{j1} and B_{j2} of β_j, that is, $\beta_j = \{B_{j1}, B_{j2}, \cdots, B_{jk}\}, k \geq 2$ and $\beta_{j+1} = \{B_{j1} \cup B_{j2}, \cdots, \cup B_{jk}\}$. According to the definition of $G_M(\Gamma)$ and monotonicity of m, we have:

$$\begin{aligned}
G_M(\beta_j) &= \sum_{i=1}^{k} m(B_{ji}) \cdot p(B_{ji}) \\
&= m(B_{j1}) \cdot p(B_{j1}) + m(B_{j2}) \cdot p(B_{j2}) \\
&\quad + \sum_{i=3}^{k} m(B_{ji}) \cdot p(B_{ji}) \\
&< m(B_{j1} \cup B_{j2}) \cdot p(B_{j1}) + m(B_{j2} \cup B_{j1}) \cdot p(B_{j2}) \\
&\quad + \sum_{i=3}^{k} m(B_{ji}) \cdot p(B_{ji}) \\
&= m(B_{j1} \cup B_{j2}) \cdot (p(B_{j1}) + p(B_{j2})) \\
&\quad + \sum_{i=3}^{k} m(B_{ji}) \cdot p(B_{ji}) \\
&= m(B_{j1} \cup B_{j2}) \cdot p(B_{j1} \cup B_{j2}) + \sum_{i=3}^{k} m(B_{ji}) \cdot p(B_{ji}) \\
&= G_M(\beta_{j+1}).
\end{aligned}$$

It immediately follows that $G(\Gamma_1) < G(\Gamma_2)$ holds.

(3) Suppose $\Gamma_1 = \Gamma_2$, based on Corollary 3.3, $\beta_{1M} \subset \beta_{2M}$ holds. And by Definition 3.4, $G(\Gamma_1) = G(\Gamma_2)$ holds.

Proposition 4.3 (Maximum). *Let (Ω, Γ) be a multigranulation topological rough space. The maximum topological granularity measure of τ with respect to Ω is one. This value is achieved if and only if $m = 1$, $max(G_M(\Gamma)) = |\Omega| - 1$.*

Proposition 4.4 (Minimum). *Let (Ω, Γ) be a multigranulation topological rough space. The maximum topological granularity of τ with respect to Ω is one. This value is achieved if and only if $m = |\Omega|$, $min(G_M(\Gamma)) = 1 - \frac{1}{|\Omega|}$.*

Hence, $1 - \frac{1}{|\Omega|} \leq G_M(\Gamma) \leq |\Omega| - 1$.

Theorem 4.3 *Let Γ_1, Γ_2 be two multigranulation topological rough space. If $\Gamma_1 <^{\tau} \Gamma_2$, then $G_M(\Gamma_1) < G_M(\Gamma_2)$.*

Theorem 4.4 *Let $\tau_1, \tau_2, \cdots, \tau_q$ be q topologies induced by equivalence relations $R_1, R_2, \cdots, R_q$, respectively. If $\tau_1 <^{\tau} \tau_2 <^{\tau} \cdots <^{\tau} \tau_q$, then $G_M(\Gamma) = G_M(\tau_q)$.*

Definition 4.3 (Topological entropy) Let (Ω, Γ) be a multigranulation topological rough space, $\beta_M = \{B_1, B_2, \cdots, B_q\}$ is a topology base of Γ. Then the topological entropy of Γ is defined as:

$$E_M(\Gamma) = 1 - \frac{1}{|\Omega|} \sum_{i=1}^{q} m(B_i) \cdot p(B_i),$$

where $p(B_i) = \frac{|B_i|}{|\Omega|}$.

Proposition 4.5 (Maximum). *Let (Ω, Γ) be a multigranulation topological rough space. The maximum topological entropy of τ with respect to Ω is one. This value is achieved if and only if $q = |\Omega|$, $max(E_M(\Gamma)) = 1 - \frac{1}{|\Omega|} + \frac{1}{|\Omega|^2} = 1$.*

Proposition 4.6 (Minimum) *Let (Ω, Γ) be a multigranulation topological rough space. The maximum topological entropy measure of τ with respect to Ω is one. This value is achieved only if $q = 1$, $min(E_M(\Gamma)) = \frac{1}{|\Omega|}$.*

Hence, $\frac{1}{|\Omega|} \leq E_M(\Gamma) \leq 1 - \frac{1}{|\Omega|} + \frac{1}{|\Omega|^2}$.

Example 4.2 (Continued from Example 3.1) From Example 4.1, $P_\Gamma = \{\frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{2}{5}\}$, then we have $G_M(\Gamma) = \frac{22}{25}$ and $E_M(\Gamma) = \frac{103}{125}$.

4.2 Application of multigranulation topological rough space

Multigranulation rough set model is one of important extensions of Pawlak rough set model. One of the advantage of the former may be suitable to deal with the complex problem, such as multi-source information system where information comes from different sources. In such multi-source environment, the granular selection and the granulation selection are two key issues in process of the multigranulation rough data analysis. Granular selection theory is the same as covering reduct theory. Accordingly, we only investigate the granulation selection theory in this section.

In what follows, we give a real-life example to illustrate the application of the multigranulation rough set theory via topology theory. For example, when a doctor will incline to diagnose a disease of a patient more accurately, he always integrates multiple values of the patient's physical examination indicators from different hospitals where supply different examination indicators. These integrated information coming from different sources is called a multi-source information.

Example 4.3 Let $U = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$ be a universe of seven objects which are here regarded as patients. Suppose there are four hospitals (H_i, $i = \{1, 2, 3, 4\}$) providing us information regarding the attributes $\{a, b, c\}$, $\{a, b, d\}$, $\{a, b, e\}$, $\{f, g, h\}$, respectively. These attributes represent the patient's physical examination indicators. Table 1 depicts the information provided by the four hospitals. In Table 1, *a* denotes *HBsAg*, *b* denotes *RHBs*; *c* denotes *HBeAg*, *d* denotes *RHBe*, *e* denotes *RHBc*, *f* denotes denotes *CHO*, *g* denotes *TG*, and *h* denotes *TP* which are the eight examination indicators used to determine whether the patient has the disease of *Hepatitis B* or not. "+" and "-" represent two attribute values that the former indicates *positive* and the latter indicates *negative*.*"H"* , *"L"* and *"M"* represent three attribute values that *"H"* indicates *High*, *"L"* indicates *Low*, and *"M"* indicates *Middle*. The data in Table 1 come from the URL:http://www.forwardhealth.wi.gov/.../PEHIUserGuide.pdf.spage.

According to each subsystem where information was provided by each hospital can generate a granulation in the view of granular computing. Accordingly, this multi-source information system may generate three granulations.

Table 1 A multi-source information system

	H_1			H_2			H_3			H_4		
	a	*b*	*c*	*a*	*b*	*d*	*a*	*b*	*e*	*f*	*g*	*h*
1	+	−	+	+	+	−	+	−	+	*H*	*H*	*L*
2	+	+	+	−	−	+	−	+	+	*L*	*M*	*L*
3	+	−	−	−	−	+	−	−	+	*H*	*L*	*H*
4	−	+	−	+	−	+	−	−	+	*L*	*L*	*H*
5	−	−	−	−	−	+	−	−	+	*H*	*M*	*M*
6	−	+	−	−	−	−	−	−	−	*H*	*H*	*M*
7	−	−	−	−	−	−	−	−	+	M	M	L

However, these granulations are not equally significant or even some of those granulations are redundant that lead to too much cost for one patient. Seen by this way, it is necessary to delete some redundant granulations in the process of multigranulation rough data analysis.

Definition 4.2 (Significance of granularity) We say that τ_k is significant in Γ, if $E_\Gamma(\Omega, \sqcap_{i=1}^q \tau_i) \neq E_\Gamma(\Omega, \sqcap_{i=1, i\neq k}^q \tau_i)$. Whereas, τ_k is not significant in Γ, if $E_\Gamma(\Omega, \sqcap_{i=1}^q \tau_i) = zE_\Gamma(\Omega, \sqcap_{i=1, i\neq k}^q \tau_i)$.

To further study significance of τ_k, we introduce a quantitative measure for the significance as follows.

The significance measure of τ_k in Γ is defined as

$S_\Gamma(\tau_k) = E_\Gamma(\Omega, \sqcap_{i=1}^q \tau_i) \setminus E_\Gamma(\Omega, \sqcap_{i=1, i\neq k}^q \tau_i)$.

Example 4.4 (Continued from Example 3.1) From Example 4.1, we have $S_\Gamma(\tau_1) = \frac{-414}{3375} < 0$, $S_\Gamma(\tau_2) = \frac{6}{15}$.

Definition 4.3 (Granulation selection) Let $\Gamma = \{\tau_1, \tau_2, \cdots, \tau_q\}$ be a family of topological spaces on Ω, if there exists a subset $\Gamma_i = \{\tau_{i1}, \tau_{i2}, \cdots, \tau_{ik}\} \subseteq \Gamma$, such that $E_\Gamma(\Omega, \sqcap_{i=1}^q \tau_i) = E_\Gamma(\Omega, \sqcap_{i=k}^q \tau_{ik})$, but $E_\Gamma(\Omega, \tau_{i1} \sqcap \tau_{i2} \sqcap \cdots \sqcap \tau_{ik} \sqcap \tau_{i(k+1)}) \neq E_\Gamma(\Omega, \tau_1 \sqcap \tau_2 \cdots \sqcap \tau_q)$, then we call Γ_i a granularity reduct of Γ.

Algorithm 4.1 (Granulation selection algorithm)

Input: A MSIDT $\mu = (U, (\{R_i\})_{i\in N})$, where MSIDT denotes a multi-source information decision table

Output: One reduct *reduct*.

Steps are shown as follows:

I1: reduct $\leftarrow \emptyset$; *//reduct* is the set to conserve the selected granularities

I2: For($i = j; j \leq |R_i|; j++$) Do

Compute $S_\Gamma(\tau_k)$, $k \leq |R_i|$;

Put τ_k into *reduct*, where $S_\Gamma(\tau_k) > 0$;// These granulations form the core of the given multi-source information decision table

While $E_\Gamma(\Omega, reduct) = E_\Gamma(\Omega, \sqcap_{i=1}^q \tau_i)$,

Break;

Endfor

Step *I2* is one of the key steps in this algorithm. The time complexity of computing is $O(m|R_i||U|^2)$, where m is the number of the attribute of each granularity R_i. $|R_i|$ is the number of granularities on U. $|U|$ is the number of the samples on U.

According the above algorithm, we can get one granularity selection $\{H_1, H_2, H_3\}$ which can determine whether one suffers from the desease of *Hepatitis B* from Table 1, which result is consistent to that determined by doctors in the real patient cases.

Example 4.5 Here, we employ another simple example to illustrate the effectiveness of the granularity selection algorithm. Table 2 depicts a complete target information system containing some information about an emporium investment project. *Locus, Investment* and *Population density* are the conditional attributes of the system, whereas *Decision* is the decision attribute. The attribute domains are as follows: $V_{Locus} = \{good, common, bad\}$, $V_{Investment} = \{high, low\}$, $V_{Population\ \ density} = \{big, small, medium\}$, $V_{Decision} = \{yes, no\}$. Each attribute in Table 2 here is regarded as a granulation as well as a source, hence, we call it a special multi-source information system.

Table 2 A complete target information system about emporium investment project

Project	Locus	Investment	Population density	Decision
x_1	Common	High	Big	Yes
x_2	Bad	High	Big	No
x_3	Bad	Low	Small	No
x_4	Bad	Low	Small	Yes
x_5	Bad	Low	Small	No

Based on Table 2, $\Omega = \{x_1, x_2, x_3, x_4, x_5\}$, suppose a target concept $X = \{x_2, x_4, x_5\} \subseteq \Omega$, by employing the multigranulation rough set theory and multigranulation topological rough space theory, we can get all topological spaces as follows:

$$\tau_1 = \{\emptyset, \Omega, \{x_1\}, \{x_2, x_3, x_4, x_5\}\},$$
$$\tau_2 = \{\emptyset, \Omega, \{x_1, x_2\}, \{x_3, x_4, x_5\}\},$$
$$\tau_3 = \{\emptyset, \Omega, \{x_1, x_2\}, \{x_3, x_4, x_5\}\},$$
$$\tau_4 = \{\emptyset, \Omega, \{x_1\}, \{x_2\}, \{x_3, x_4, x_5\}\},$$
$$\tau_5 = \{\emptyset, \Omega, \{x_1, x_2\}, \{x_3, x_4, x_5\}\},$$
$$\tau_6 = \{\emptyset, \Omega, \{x_1\}, \{x_2\}, \{x_3, x_4, x_5\}\}.$$

By Definition 3.4, then we can get a multigranulation topological rough space (Ω, Γ), where $\Gamma = \{\emptyset, \Omega, \{x_1\}, \{x_2\}, \{x_3, x_4, x_5\}, \{x_1, x_2\}, \{x_1, x_3, x_4, x_5\}, \{x_2, x_3, x_4, x_5\}\}$. By the granulation selection algorithm, we know τ_1, τ_2, τ_3, τ_4, τ_5 are redundant granular spaces, and $\{\tau_6\}$ is a granulation selection of Γ which preserves the invariance of topological entropy of X in the multigranulation topological rough space.

5 Conclusions and discussions

In this paper, we have presented an investigation of the topological method for multigranulation rough sets and addressed a series of concepts of the multigranulation topological rough space and its topological properties. Moreover, we have introduced topological granularity and topological entropy to show the uncertainty of multigranulation topological rough space from the topological view.

In particular, a granulation selection algorithm based on the invariance of topological entropy was preliminarily proposed to reduce redundant granular spaces in the multi-granulation rough data analysis.

Acknowledgments The authors would like to thank the anonymous reviewers and the editor for their constructive and valuable comments. This work is supported by grants from National Natural Science Foundation of China under Grant (No. 71031006), National Key Basic Research and Development Program of China (973) (No. 2013CB329404), Innovative Talents of Higher Learning Institutions of Shanxi, China (No. 20120301), and Education Committee of Fujian Province under Grant (Nos. JK2011031).

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