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Preorder Information Based Attributes' Weights Learning in Multi-attribute Decision Making

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Abstract. Choquet integral, as an adequate aggregation operator, extends the weighted mean operator by considering interactions among attributes. Choquet integral has been widely used in many real multi-attribute decision making. Weights (fuzzy measures) of attribute sets directly affect the decision results in multi-attribute decision making. In this paper, we aim to propose an objective method based on granular computing for determining the weights of the attribute sets. To address this issue, we first analyze the implied preorder relations under four evaluation forms and construct the corresponding preorder granular structures. Then, we define fuzzy measure of an attribute set by the similarity degree between a special preorder pairs. Finally, we employ two numerical examples for illustrating the feasibility and effectiveness of the proposed method. It is deserved to point out that the weight of each attribute subset can be learned from a given data set by the proposed method, not but be given subjectively by the decision maker. This idea provides a new perspective for multi-attribute decision making.

Keywords: preorder relation, granular computing, similarity degree, Choquet integral, multi-attribute decision making

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1. Introduction

Granular computing (GrC), which is a term coined jointly by Zadeh and Lin [22, 23, 49, 50], plays a fundamental role in information granulation of human reasoning. Since then a rapid development and a fast growing interest of GrC have been witnessed [18, 19, 20, 28, 30, 32, 47, 48]. Granular computing is often loosely regarded as an umbrella term to cover theories, methodologies, techniques, and tools that make use of granules in complex problem solving [46]. Several granular computing models such as computing with words, rough set theory and quotient space theory have been successfully used in many fields, especially in artificial intelligence. Granule, granulation and granularity are regarded as the three primitive notions of GrC. A granule is a clump of objects drawn together by indistinguishability, similarity and proximity of functionality. Granulation of an object leads to a collection of granules. The granularity is the measurement of the granulation degree of objects [32]. Yao [47] thought that the framework of GrC is based on three perspectives: philosophy, methodology and information processing paradigm .

Multi-attribute decision making (MADM) is a kind of important decision making problem. One of the tasks in MADM is to find the most desirable alternative(s) from a group of feasible alternatives with respect to a finite set of attributes [15, 27]. MADM has become a hot research topic over the last three decades, and has been extensively applied to various areas such as society, economics, management, and others [7, 13, 15, 26, 27, 35]. GrC could be a new perspective for solving multi-attribute decision making problems, and several relative works have been addressed [8, 9, 13, 14, 24, 25, 29, 31, 35]. By replacing equivalence relations with dominance relations, Greco et al. [8, 9] generalized classical rough sets to dominance rough sets for analyzing multi-attribute decision making problems. Herrera et al. [13] used multi-granular linguistic information to solve MADM problems. Hu et al. [14] presented a fuzzy preference rough set model and concluded that the lower and upper approximations in their model can be understood as the pessimistic and optimistic decision in human reasoning. Liu et al. [24, 25] employed an attribute reduction approach to determine the weights of attributes. Qian et al. [29, 31] developed methods to rank objects with interval and set values based on a local dominance degree and a global dominance degree. Song et al. [35] defined an ordered mutual information to calculate the weight of each criterion and the directional distance index with weights for obtaining a total rank of all objects.

Many multi-attribute decision making researchers use the weighted mean operator to aggregate evaluation information. This aggregation process is based on the assumption that the attributes are independent of one another and their effects are viewed as additive. However, the interdependence and interaction among attributes are very common in many real multi-attribute decision making problems. Then the independence assumption is too strong to match decision behaviors in the real world. To overcome this limitation, Choquet [4] introduced a useful tool called Choquet integral to model not only the importance of each attribute but also the importance of each coalition of attributes. The importance of a family of attributes may not be the sum of the importance of each attribute and it can be smaller or greater, due respectively to redundancy or synergy among the attributes. Under the framework of the Choquet integral, fuzzy measures can be used to describe interactions among attributes and also model the relative importance of attributes. Recently, the Choquet integral has been studied and applied widely in multi-attribute decision making [2, 5, 6, 7, 11, 17, 34, 36, 43, 44]. Ashayeri et al. [2] applied the Choquet integral operator in supply chain partners and configuration selection problem. Chen et al. [5] developed an identification procedure for calculating the $\lambda-$fuzzy measures by using sampling design and genetic algorithms. Demirel et al. [6] showed a successful application of the multi-attribute Choquet integral to a real warehouse location selection problem of a big Turkish logistic firm. Fan [7] indicated that the Choquet integral should be studied further in multi-attribute decision making. Grabisch [11] introduced an iterative method to identify the fuzzy measure. Kojadinovic [17] proposed an alternative unsupervised identification method based on the estimation of the fuzzy measure coefficients by means of information-theoretic functionals. Sekita [34] studied on the $\lambda-$ fuzzy measure identification. Xu [43] and Tan [36] applied the Choquet integral to multi-criteria inteval-valued intuitionistic fuzzy decision making and group decision making problems. Yang et al. [44] studied the decision making problem in which the evaluation values are linguistic arguments and developed some new aggregation operators by using the Choquet integral.

Clearly, the definition of the fuzzy measure is required before using the Choquet integral as an aggregation operator. Some researchers [2, 6, 11, 36, 43, 44] gave the fuzzy measures of the attribute sets directly, which requires the knowledge of subjective estimates for the alternative set. The others [5, 10, 17, 34] used complex computing process to calculate the fuzzy measures of attribute sets. While if the number of attributes m is large, it is rather unrealistic to assume that the $2^m - 2$ fuzzy measures on the attribute set can be provided by the decision maker subjectively. In addition, the evaluation values of attributes and their distribution characteristic have been greatly ignored in the latter methods such as [5, 10, 34]. Kojadinovic [17] estimated the fuzzy measures by the information contents of attribute sets under the utility function expression. In the weighted mean method, Liu et al. [24, 25] defined the importance degree and weight of each attribute by integrating attribute reduction of rough set theory and information entropy method in Data Mining. These weight acquisition methods considered the judgment information and its distribution characteristic and they are data-driven objective computation methods. However, some decision makers would like to express the preference information under different attributes in different forms such as utility function, multiplicative preference relation, fuzzy preference relation and ordinal ranking in MADM. Then it is a very interesting work to compute the fuzzy measures of attribute sets in different formats. Motivated by the works of Kojadinovic and Liu, the main aim of our research is to determine the fuzzy measures of the attribute sets objectively by granular computing for solving the interdependent multi-attribute decision making problems in which the preference information is expressed in different forms.

The rest of this paper is organized as follows. The model of multi-attribute decision making problem is reviewed and the preorder granular structures of the four preference forms: utility function, multiplicative preference relation, fuzzy preference relation and ordinal ranking are presented in Section 2. In Section 3, the similarity degree of preorder pairs is defined and the properties are also analyzed thoroughly; a fuzzy measure of an attribute set is given objectively based on the new similarity degree. Two illustrative examples are shown in Section 4. Finally, conclusions and future works are given in Section 5.

2. Preorder granular structure in multi-attribute decision making

Before going into detail, the model of multi-attribute decision making is reviewed in the following subsection.

2.1. The model of multi-attribute decision making

Multi-attribute decision making could be described by means of the following sets. A discrete set of n feasible alternatives: $U = \{u_1, \dots, u_i, \dots, u_n\}(n \geq 2)$; a finite set of attributes: $A = \{a_1, \dots, a_i, \dots, a_m\}$

$(m \geq 2)$. A decision maker evaluates the alternatives under the attributes in A. And some aggregation methods can be used to integrate the evaluation information. The decision maker's preferences over the alternative set might be expressed in the following one or more forms.

(1) *Utility function* [7, 43]

The preferences under an attribute a_k are given as a set of n utility values $V^k = \{v_{1k}, \dots, v_{ik}, \dots, v_{nk}\}$, where v_{ik} represents the utility evaluation of u_i with respect to the attribute a_k.

(2) *Multiplicative preference relation* [33]

The decision maker's preferences on U under a_k can be described by a positive preference relation $P^k = (p_{ij}^k)$, where p_{ij}^k indicates a ratio of the preference intensity of alternative u_i to u_j, i.e., it is interpreted as u_i is p_{ij}^k times as good as u_j under a_k. Saaty suggests that p_{ij}^k should be measured on a 1-9 scale: $p_{ij}^k = 1$ indicates indifference between u_i and u_j, $p_{ij}^k = 9$ indicates that u_i is unanimously preferred to u_j, and $p_{ij}^k \in \{2, ..., 8\}$ indicates intermediate evaluations. It is usual to assume the multiplicative reciprocity property $p_{ij}^k \cdot p_{ji}^k = 1$ $(\forall i, j)$ and $p_{ii}^k = 1$.

(3) *Fuzzy preference relation* [12]

The decision maker's preferences on U under a_k can also be described by a fuzzy preference relation $Q^k = (q_{ij}^k)$, where q_{ij}^k $(0 \leq q_{ij}^k \leq 1)$ denotes the preference degree or intensity of u_i over u_j: $q_{ij}^k = \frac{1}{2}$ indicates indifference between u_i and u_j, and $q_{ij}^k > \frac{1}{2}$ indicates that u_i is preferred to u_j. Generally, it is assumed that $q_{ij}^k + q_{ji}^k = 1$ $(\forall i, j)$ and $q_{ii}^k = \frac{1}{2}$.

(4) *Ordinal ranking of the alternatives* [3]

The decision maker gives the preferences on U under an attribute a_k as an individual preference ranking $O^k = \{o^k(u_1), \dots, o^k(u_i), \dots, o^k(u_n)\}$, where $o^k(u_i)$ is the rank or priority assigned to alternative u_i. This expression dates back at least to Borda's "method of marks". $o^k(u_i) = o^k(u_j)$ $(i \neq j)$ is allowed, that is to say, some alternatives are tied in some places. Let $U = \{u_1, u_2, u_3, u_4\}$. $O = \{1, 4, 2.5, 2.5\}$ is a preference ranking under an attribute a. The ranking 2.5 indicates that alternative u_3 and u_4 are tied for the second.

Example 2.1. A customer is going to buy a car. Five cars are to be evaluated and denoted by u_1, u_2, u_3, u_4 and u_5. The following attributes are considered: a_1- breaking performance, a_2- fuel economy (L/100km), a_3- comfortable level, a_4- operating stability. Let $A = \{a_1, a_2, a_3, a_4\}$. The preferences under the breaking performance may be given in the ranking form as $O^1 = \{o^1(u_1), o^1(u_2), o^1(u_3), o^1(u_4), o^1(u_5)\} = \{2, 3, 1, 5, 4\}$; the preferences under the fuel economy are in the utility function form as $V^2 = \{v_{12}, v_{22}, v_{32}, v_{42}, v_{52}\} = \{10, 11, 9, 8, 9\}$; the preferences under the comfortable level are in the multiplicative preference relation form as

$$P^3 = \begin{pmatrix} 1 & \frac{1}{2} & 3 & 4 & 1 \\ 2 & 1 & 6 & 8 & 2 \\ \frac{1}{3} & \frac{1}{6} & 1 & \frac{4}{3} & \frac{1}{3} \\ \frac{1}{4} & \frac{1}{8} & \frac{3}{4} & 1 & \frac{1}{4} \\ 1 & \frac{1}{2} & 3 & 4 & 1 \end{pmatrix};$$

the preferences under the operating stability are conveyed in the fuzzy preference relation form as

$$Q^4 = \begin{pmatrix} 0.5 & 0.55 & 0.7 & 0.95 & 0.5 \\ 0.45 & 0.5 & 0.65 & 0.9 & 0.45 \\ 0.3 & 0.35 & 0.5 & 0.75 & 0.3 \\ 0.05 & 0.1 & 0.25 & 0.5 & 0.05 \\ 0.5 & 0.55 & 0.7 & 0.95 & 0.5 \end{pmatrix}.$$

The resolution of a multi-attribute decision making problem consists of obtaining a set of solution alternatives by integrating the different preferences under the given attribute set. In the integration process, different weights of the attribute sets directly influence the Choquet integral decision result. Consequently, it is necessary to obtain the attributes' rational weights. In the following section, we are going to propose a new data-driven method which is based on granular computing to compute the weights of attribute sets. In what follows, the two key GrC notions granules and granulation are defined in MADM with different evaluation forms.

2.2. Preoder granular structures in the four evaluation forms

Constructing a granular structure is a vital work of GrC. So we should analyze and define granular structures in multi-attribute decision making first.

Definition 2.2. [16] A preorder is a binary relation P over a set U which is reflexive and transitive, i.e., for any u in U, $(u, u) \in P$ (reflexivity); if $(u, v) \in P$ and $(v, w) \in P$, then $(u, w) \in P$ (transitivity).

In this paper, a preorder P is symbolled by "$\succeq_P$", then $(u, v) \in P$ denoted by "$u \succeq_P v$". We call $[u]^{\succeq_P} = \{v \in A | v \succeq_P u\}$ the non-inferior granule of u. $U/\succeq_P = \{[u]^{\succeq_P} | u \in U\}$ is the granulation of U induced by P. In what follows, we construct the granulations of U in the above four evaluation forms.

(1) *Utility function*

Let $V^k = \{v_{1k}, ..., v_{ik}, ..., v_{nk}\}$ be an utility preference provided by a decision maker and v_{ik} represents the utility evaluation. For a profit attribute, let $\succeq_{V^k} = \{(u_i, u_j) | v_{ik} \geq v_{jk} | u_i, u_j \in U\}$; for a cost attribute, let $\succeq_{V^k} = \{(u_i, u_j) | v_{ik} \leq v_{jk} | u_i, u_j \in U\}$. In the following, we take the profit attribute for example, for any $u_i \in U$, $v_{ik} \geq v_{ik}$, $(u_i, u_i) \in \succeq_{V^k}$, so $\succeq_{V^k}$ is reflexive. If $(u_i, u_j) \in \succeq_{V^k}$ and $(u_j, u_l) \in \succeq_{V^k}$, we have $v_{ik} \geq v_{jk}$ and $v_{ik} \geq v_{lk}$, then $v_{ik} \geq v_{lk}$, so $(u_i, u_l) \in \succeq_{V^k}$, thus $\succeq_{V^k}$ is transitive. Therefore, we conclude that $\succeq_{V^k}$ is a preorder. Then, the granulation of U induced by $\succeq_{V^k}$ can be formed as $U/\succeq_{V^k} = \{[u_i]^{\succeq_{V^k}} | u_i \in U\}$.

(2) *Multiplicative preference relation*

In a decision making process, a consistent multiplicative preference relation $P^k = (p_{ij}^k)$ which satisfies the condition: $p_{ij}^k \cdot p_{jl}^k = p_{il}^k, (\forall i, j, l = 1, 2, ..., n)$ is desired and if a multiplicative preference relation is not consistent, some ready-made methods are given to transform it to be consistent [33]. Let P^k be a consistent multiplicative preference relation and $\succeq_{P^k}^1 = \{(u_i, u_j) | p_{ij}^k \geq 1, u_i, u_j \in U\}$. For any $u_i \in U$, we have $p_{ii}^k = 1$, $(u_i, u_i) \in \succeq_{P^k}^1$, so $\succeq_{P^k}^1$ is reflexive. If $(u_i, u_j) \in \succeq_{P^k}^1, (u_j, u_l) \in \succeq_{P^k}^1$, $p_{ij}^k \geq 1$ and $p_{jl}^k \geq 1$, $p_{il}^k = p_{ij}^k \cdot p_{jl}^k \geq 1$, so $(u_i, u_l) \in \succeq_{P^k}^1$, which means that $\succeq_{P^k}^1$ is transitive. It can be concluded that $\succeq^1_{P^k}$ is a preorder. Therefore, the granulation of U induced by $\succeq^1_{P^k}$ can be formed as $U/\succeq^1_{P^k}=\{[u_i]^{\succeq^1_{P^k}}|u_i\in U\}$.

(3) *Fuzzy preference relation*

In a decision making process, a consistent fuzzy preference relation $Q^k=(q^k_{ij})$ which satisfies the condition: $q^k_{ij}+q^k_{jl}-q^k_{il}=\frac{1}{2},(\forall i,j,k=1,2,...,n)$ is desired and if a fuzzy preference relation is not consistent, there are some ready-made methods to transform it to be consistent [12]. Let Q^k be a consistent fuzzy preference relation and $\succeq^{\frac{1}{2}}_{Q^k}=\{(u_i,u_j)|q^k_{ij}\geq\frac{1}{2},u_i,u_j\in U\}$. For any $u_i\in U$, we have $q^k_{ii}=\frac{1}{2}$, $(u_i,u_i)\in\succeq^{\frac{1}{2}}_{Q^k}$, so $\succeq^{\frac{1}{2}}_{Q^k}$ is reflexive. If $(u_i,u_j)\in\succeq^{\frac{1}{2}}_{Q^k},(u_j,u_l)\in\succeq^{\frac{1}{2}}_{Q^k}$, $q^k_{ij}\geq\frac{1}{2}$ and $q^k_{jl}\geq\frac{1}{2}$, $q^k_{il}=q^k_{ij}+q^k_{jl}-\frac{1}{2}\geq\frac{1}{2}$, so $(u_i,u_l)\in\succeq^{\frac{1}{2}}_{Q^k}$, which means that $\succeq^{\frac{1}{2}}_{Q^k}$ is transitive. It can be concluded that $\succeq^{\frac{1}{2}}_{Q^k}$ is a preorder. Therefore, the corresponding granulation of U can be formed as $U/\succeq^{\frac{1}{2}}_{Q^k}=\{[u_i]^{\succeq^{\frac{1}{2}}_{Q^k}}|u_i\in A\}$.

(4) *Ordinal ranking of the alternatives*

Given an individual preference ranking $O^k=\{o^k(u_1),\ldots,o^k(u_i),\ldots,o^k(u_n)\}$. Let $\succeq_{O^k}=\{(u_i,u_j)|o^k(u_i)\leq o^k(u_j)\}$. Similar to Case (1), it is easy to prove that $\succeq_{O^k}$ is a preorder. Then, the granular structure $U/\succeq_{O^k}$ induced by O^k is constructed as $U/\succeq_{O^k}=\{[u_i]^{\succeq_{O^k}}|u_i\in U\}$.

Example 2.3. (Continued from Example 2.1) We construct the corresponding preorders and extract the preorder granular structures under different attributes.

The preorder determined by the evaluation information under a_1 and the corresponding preorder granular structure are given as

$$\begin{aligned}\succeq_{O^1}=\{&(u_3,u_1),(u_3,u_2),(u_3,u_3),(u_3,u_4),(u_3,u_5),(u_1,u_1),(u_1,u_2),(u_1,u_4),\\&(u_1,u_5),(u_2,u_2),(u_2,u_4),(u_2,u_5),(u_5,u_4),(u_5,u_5),(u_4,u_4)\}\end{aligned}$$

and

$$U/\succeq_{O^1}=\{\{u_1,u_3\},\{u_1,u_2,u_3\},\{u_3\},\{u_1,u_2,u_3,u_4,u_5\},\{u_1,u_2,u_3,u_5\}\},$$

respectively.

The preorder extracted from the evaluation information under a_2 and the corresponding preorder granular structure are shown as

$$\begin{aligned}\succeq_{V^2}=\{&(u_4,u_1),(u_4,u_2),(u_4,u_3),(u_4,u_4),(u_4,u_5),(u_3,u_1),(u_3,u_2),(u_3,u_3),\\&(u_3,u_5),(u_5,u_1),(u_5,u_2),(u_5,u_3),(u_5,u_5),(u_1,u_1),(u_1,u_2),(u_2,u_2)\}\end{aligned}$$

and

$$U/\succeq_{V^2}=\{\{u_1,u_3,u_4,u_5\},\{u_1,u_2,u_3,u_4,u_5\},\{u_3,u_4,u_5\},\{u_4\},\{u_3,u_4,u_5\}\}.$$

The preorder extracted from the preference information under a_3 and the corresponding preorder granular structure are given as

$$\begin{aligned}\succeq^1_{P^3}=\{&(u_1,u_1),(u_1,u_3),(u_1,u_4),(u_1,u_5),(u_2,u_1),(u_2,u_2),(u_2,u_3),(u_2,u_4),\\&(u_2,u_5),(u_3,u_3),(u_3,u_4),(u_4,u_4),(u_5,u_1),(u_5,u_3),(u_5,u_4),(u_5,u_5)\}\end{aligned}$$

and

$$U/\succeq^1_{P^3}=\{\{u_1,u_2,u_5\},\{u_2\},\{u_1,u_2,u_3,u_4,u_5\},\{u_1,u_2,u_4,u_5\},\{u_1,u_2,u_5\}\}.$$

The last, the preorder extracted from the evaluation under a_4 and the corresponding preorder granular structure are displayed as

$$\begin{aligned}\succeq^{\frac{1}{2}}_{Q^4}=\{&(u_1,u_1),(u_1,u_2),(u_1,u_3),(u_1,u_4),(u_1,u_5),(u_2,u_2),(u_2,u_3),(u_2,u_4),\\&(u_2,u_5),(u_3,u_3),(u_3,u_4),(u_4,u_4),(u_5,u_1),(u_5,u_2),(u_5,u_3),(u_5,u_4),(u_5,u_5)\}\end{aligned}$$

and

$$U/\succeq^{\frac{1}{2}}_{Q^4}=\{\{u_1,u_5\},\{u_1,u_2,u_5\},\{u_1,u_2,u_3,u_5\},\{u_1,u_2,u_3,u_4,u_5\},\{u_1,u_5\}\}.$$

We have analyzed the preorder granular structures under a single attribute a_k in different evaluation forms and construct the corresponding granulations of U. The preorder granular structure under an attribute set A is presented based on the following proposition.

Proposition 2.4. Let $U=\{u_1,\dots,u_i,\dots,u_n\}$ be a set of the alternatives. If $\succeq_{P_1}$ and $\succeq_{P_2}$ are two preorders on U, then $\succeq_{P_1}\cap\succeq_{P_2}$ is a preorder on U.

Proof:

For any $u_i\in U$, $(u_i,u_i)\in\succeq_{P_1}$ and $(u_i,u_i)\in\succeq_{P_2}$, we get $(u_i,u_i)\in\succeq_{P_1}\cap\succeq_{P_2}$. Hence, $\succeq_{P_1}\cap\succeq_{P_2}$ is reflexive.

Let $(u_i,u_j)\in\succeq_{P_1}\cap\succeq_{P_2}$ and $(u_j,u_l)\in\succeq_{P_1}\cap\succeq_{P_2}$. $(u_i,u_j)\in\succeq_{P_1}$ and $(u_j,u_l)\in\succeq_{P_1}$, then $(u_i,u_l)\in\succeq_{P_1}$. Similarly, $(u_i,u_l)\in\succeq_{P_2}$. Thus, $(u_i,u_l)\in\succeq_{P_1}\cap\succeq_{P_2}$. We have that $\succeq_{P_1}\cap\succeq_{P_2}$ is transitive.

Therefore, $\succeq_{P_1}\cap\succeq_{P_2}$ is a preorder. □

Proposition 2.4 shows that the intersection of the finite preorders is also a preorder. $A=\{a_1,a_2,\dots,a_m\}$ is a nonempty attribute set. Let P^k be the preorder under the attribute a_k. Let $P^A=\bigcap_{k=1}^m P^k$. It is easy to prove that P^A is a preorder and P^A is called the preorder of the attribute set A.

Example 2.5. (Continued from Example 2.3) Based on the results in Example 2.3, we calculate the preorder under the attribute set A and lay out the granular structure as

$$\begin{aligned}\succeq_{P^A}&=\bigcap_{k=1}^4 P^4\\&=\succeq_{O^1}\bigcap\succeq_{V^2}\bigcap\succeq^1_{P^3}\bigcap\succeq^{\frac{1}{2}}_{Q^4}\\&=\{(u_1,u_1),(u_2,u_2),(u_3,u_3),(u_4,u_4),(u_5,u_5)\}\end{aligned}$$

and

$$U/\succeq_{P^A}=\{\{u_1\},\{u_2\},\{u_3\},\{u_4\},\{u_5\}\}.$$

3. Attributes' weights learning method based on preorder granular structure

The evaluation information under one attribute corresponds to a unique preorder, so we can analyze the relations of attributes and determine the weights by these preorders. In what follows, we investigate the importance of attributes via preorder granular computing method. One should note that the following preorder granular structures can be constructed from any one of the above four forms.

3.1. Preorder similarity degree

Similarity measure is fundamentally important in almost every scientific field. In this section, we will further study the similarity degree between a pair of preorders.

Definition 3.1. Let $U = \{u_1, \ldots, u_i, \ldots, u_n\}$ be a nonempty finite set and **P** be the set of all preorders on U. For any $\succeq_{P_1}, \succeq_{P_2} \in$ **P**, $U/\succeq_{P_1} = \{[u_1]^{\succeq_{P_1}}, \ldots, [u_i]^{\succeq_{P_1}}, \ldots, [u_n]^{\succeq_{P_1}}\}$, $U/\succeq_{P_2} = \{[u_1]^{\succeq_{P_2}}, \ldots, [u_i]^{\succeq_{P_2}}, \ldots, [u_n]^{\succeq_{P_2}}\}$. We define the similarity degree between $\succeq_{P_1}$ and $\succeq_{P_2}$ as

$$sim(\succeq_{P_1}, \succeq_{P_2}) = 1 - \frac{1}{n-1} \sum_{i=1}^{n} \frac{|[u_i]^{\succeq_{P_1}} \ominus [u_i]^{\succeq_{P_2}}|}{n}, \tag{1}$$

where $\ominus$ denotes the symmetric difference of the sets.

Proposition 3.2. Let $U = \{u_1, \ldots, u_i, \ldots, u_n\}$ be a nonempty finite set and **P** be the set of all preorders on U. For any $\succeq_{P_1}, \succeq_{P_2} \in$ **P**, $0 \leq sim(\succeq_{P_1}, \succeq_{P_2}) \leq 1$.

Proof:

Since $\succeq_{P_1}$ and $\succeq_{P_2}$ are reflexive, for any $u_i \in U$, $\{u_i\} \subseteq [u_i]^{\succeq_{P_1}}$, $\{u_i\} \subseteq [u_i]^{\succeq_{P_2}}$, then $\emptyset \subseteq [u_i]^{\succeq_{P_1}} \ominus [u_i]^{\succeq_{P_2}} \subseteq U - \{u_i\}$, so $0 \leq |[u_i]^{\succeq_{P_1}} \ominus [u_i]^{\succeq_{P_2}}| \leq n-1$. Thus, $0 \leq \frac{|[u_i]^{\succeq_{P_1}} \ominus [u_i]^{\succeq_{P_2}}|}{n} \leq \frac{n-1}{n}$, $0 \leq \frac{1}{n-1} \sum_{i=1}^{n} \frac{|[u_i]^{\succeq_{P_1}} \ominus [u_i]^{\succeq_{P_2}}|}{n} \leq 1$, $0 \leq 1 - \frac{1}{n-1} \sum_{i=1}^{n} \frac{|[u_i]^{\succeq_{P_1}} \ominus [u_i]^{\succeq_{P_2}}|}{n} \leq 1$. Therefore,

$$0 \leq sim(\succeq_{P_1}, \succeq_{P_2}) \leq 1.$$ □

Proposition 3.3. Let $U = \{u_1, \ldots, u_i, \ldots, u_n\}$ be a nonempty finite set and **P** be the set of all preorders on U. For any $\succeq_{P_1}, \succeq_{P_2} \in$ **P**, $sim(\succeq_{P_1}, \succeq_{P_2}) = sim(\succeq_{P_2}, \succeq_{P_1})$.

Proof:

The set operator $\ominus$ is commutative, so the similarity measure is also commutative. □

Proposition 3.4. If $\succeq_{P_1} = \succeq_{P_2}$, then $sim(\succeq_{P_1}, \succeq_{P_2}) = 1$.

Proof:

Since $\succeq_{P_1} = \succeq_{P_2}$, for any $u_i \in U$, $[u_i]^{\succeq_{P_1}} = [u_i]^{\succeq_{P_2}}$, $|[u_i]^{\succeq_{P_1}} \ominus [u_i]^{\succeq_{P_2}}| = |\emptyset| = 0$. Thus,

$$\begin{aligned} sim(\succeq_{P_1}, \succeq_{P_2}) &= 1 - \tfrac{1}{n-1} \textstyle\sum_{i=1}^{n} \frac{|[u_i]^{\succeq_{P_1}} \ominus [u_i]^{\succeq_{P_2}}|}{n} \\ &= 1 - \tfrac{1}{n-1} \textstyle\sum_{i=1}^{n} \frac{0}{n} \\ &= 1 - 0 \\ &= 1. \end{aligned}$$ □

Proposition 3.5. Let $U = \{u_1, \ldots, u_i, \ldots, u_n\}$ be a nonempty finite set and **P** be the set of all preorders on U. If $\succeq_{P_1}$ is a total order and $\succeq_{P_1}^{-1} = \succeq_{P_2}$, then $sim(\succeq_{P_1}, \succeq_{P_2}) = 0$.

Proof:

Since $\succeq_{P_1}$ is a total order, without loss of generality, let $\succeq_{P_1} = \{u_1 \succeq_{P_1} u_2 \ldots \succeq_{P_1} u_i \ldots \succeq_{P_1} u_n\}$, then $\succeq_{P_2} = \succeq_{P_1}^{-1} = \{u_n \succeq_{P_2} u_{n-1} \ldots \succeq_{P_2} u_i \ldots \succeq_{P_2} u_1\}$. For any $u_i \in A$, $[u_i]^{\succeq_{P_1}} = \{u_1, u_2, \ldots, u_{i-1}, u_i\}$ and $[u_i]^{\succeq_{P_2}} = \{u_n, u_{n-1}, \ldots, u_{i+1}, u_i\}$, then $|[u_i]^{\succeq_{P_1}} \ominus [u_i]^{\succeq_{P_2}}| = |U - \{u_i\}| = n - 1$. Therefore,

$$\begin{aligned} sim(\succeq_{P_1}, \succeq_{P_2}) &= 1 - \tfrac{1}{n-1} \textstyle\sum_{i=1}^{n} \frac{|[u_i]^{\succeq_{P_1}} \ominus [u_i]^{\succeq_{P_2}}|}{n} \\ &= 1 - \tfrac{1}{n-1} \textstyle\sum_{i=1}^{n} \frac{n-1}{n} \\ &= 1 - 1 \\ &= 0. \end{aligned}$$ □

Proposition 3.5 shows that the similarity degree between two complete reverse preorders reaches the minimum value 0. It is similar to our intuitive feeling. In addition, we calculate the similarity measure between the two special preorders: the identity relation $I = \{(u_i, u_i)|u_i \in U\}$ and the universal relation $E = U \times U = \{(u_i, u_j)|u_i, u_j \in U\}$.

Proposition 3.6. Let $U = \{u_1, \ldots, u_i, \ldots, u_n\}$ be a nonempty finite set. I and E are the identify relation and the universal relation on U. Then, $sim(\succeq_I, \succeq_E) = 0$.

Proof:

Since $\succeq_I = \{(u_i, u_i)|u_i \in U\}$, $E = U \times U = \{(u_i, u_j)|u_i, u_j \in U\}$, $[u_i]^{\succeq_I} = \{u_i\}$, and $[u_i]^{\succeq_E} = U$. So $[u_i]^{\succeq_I} \ominus [u_i]^{\succeq_E} = \{u_i\} \ominus U = U - \{u_i\} = n - 1$. Thus,

$$\begin{aligned} sim(\succeq_I, \succeq_E) &= 1 - \tfrac{1}{n-1} \textstyle\sum_{i=1}^{n} \frac{[u_i]^{\succeq_I} \ominus [u_i]^{\succeq_E}|}{n} \\ &= 1 - \tfrac{1}{n-1} \textstyle\sum_{i=1}^{n} \frac{n-1}{n} \\ &= 1 - 1 \\ &= 0. \end{aligned}$$ □

In fact, the identity relation means that each element in U is distinct, and the universal relation tells that the elements in U are all the same. There exist pretty different logical meanings between these two preorders.

Proposition 3.7. Let $U = \{u_1, \ldots, u_i, \ldots, u_n\}$ be a nonempty finite set and **P** be the set of all preorders on U. If $\succeq_{P_1} \subseteq \succeq_{P_2} \subseteq \succeq_{P_3}$, then $sim(\succeq_{P_1}, \succeq_{P_2}) \geq sim(\succeq_{P_1}, \succeq_{P_3})$ and $sim(\succeq_{P_2}, \succeq_{P_3}) \geq sim(\succeq_{P_1}, \succeq_{P_3})$.

Proof:

By the condition $\succeq_{P_1} \subseteq \succeq_{P_2} \subseteq \succeq_{P_3}$, we can easily get $[u_i]^{\succeq_{P_1}} \subseteq [u_i]^{\succeq_{P_2}} \subseteq [u_i]^{\succeq_{P_3}}$, then $([u_i]^{\succeq_{P_1}} \ominus [u_i]^{\succeq_{P_2}}) \subseteq (\succeq_{P_1} \ominus [u_i]^{\succeq_{P_3}})$, so $|[u_i]^{\succeq_{P_1}} \ominus [u_i]^{\succeq_{P_2}}| \leq |[u_i]^{\succeq_{P_1}} \ominus [u_i]^{\succeq_{P_3}}|$. Therefore,

$$\begin{aligned} sim(\succeq_{P_1}, \succeq_{P_2}) &= 1 - \tfrac{1}{n-1} \textstyle\sum_{i=1}^{n} \frac{|[u_i]^{\succeq_{P_1}} \ominus [u_i]^{\succeq_{P_2}}|}{n} \\ &\geq 1 - \tfrac{1}{n-1} \textstyle\sum_{i=1}^{n} \frac{|[u_i]^{\succeq_{P_1}} \ominus [u_i]^{\succeq_{P_3}}|}{n} \\ &= sim(\succeq_{P_1}, \succeq_{P_3}). \end{aligned}$$

The proof of $sim(\succeq_{P_2}, \succeq_{P_3}) \geq sim(\succeq_{P_1}, \succeq_{P_3})$ follows in a similar manner. □

The properties above show that the similarity degree proposed in this paper are reasonable and in what follows we will use the similarity degree to measure the importance of the attribute sets in multi-attribute decision making.

3.2. Weights of the attribute set

In the framework of the multi-attribute decision making, the interaction phenomena among attibutes can be reflected by a discrete fuzzy measure μ. $\mu(A')$ can also be interpreted as the importance of the subset $A' \subseteq A$.

Definition 3.8. [6, 11] A set function μ: $2^X \to [0, 1]$ is called a fuzzy measure if it satisfies the following properties:

(1) $\mu(\emptyset) = 0$;
(2) $\mu(X) = 1$;
(3) $\mu(A') \leq \mu(A)$ if $A' \subseteq A \subseteq X$.

The fuzzy measure of $\emptyset$ equals to 0 and the fuzzy measure of the whole attribute set reaches a maximum 1. The monotonicity of the fuzzy measure means that the importance of an attribute subset cannot decrease when a new attribute or some new attributes are added to it. The main characteristic of a fuzzy measure is the non-additivity, which enables it to represent flexible the various kinds of intersections among the attributes, ranging from redundancy (negative interaction) to synergy (positive interaction).

When using a fuzzy measure to model the importance of an attribute subset, the Choquet integral is a suitable aggregation function. We can rank alternatives according to the value of the Choquet integral in multi-attribute decision making.

Definition 3.9. [11, 43] Let $A = \{a_1, \ldots, a_i, \ldots, a_m\}$ be a set of attributes and f be a real-valued function on A, the Choquet integral of f with respect to a fuzzy measure μ on A is defined as

$$(C)\int f d\mu = \sum_{i=1}^{m} [f(a_{(i)}) - f(a_{i-1})]\mu(A_{(i)}) \tag{2}$$

or equally by

$$(C)\int f d\mu = \sum_{i=1}^{m} [\mu(A_{(i)} - \mu(A_{(i+1)})] f(a_{(i)}) \tag{3}$$

where the parentheses used for indices represent a permutation on A such that $f(a_{(1)}) \leq \ldots \leq f(a_{(m)})$, $f(a_{(0)}) = 0$, $A_{(i)} = \{a_{(i)}, \ldots, a_{(m)}\}$, and $A_{(m+1)} = \emptyset$.

The Choquet integral generalizes the WA and the OWA operators, and has good aggregation properties such as idempotency, boundedness, commutativity, monotonicity.

It is a very important and difficult work to determine the fuzzy measures of attribute sets before using the Choquet integral to solve a multi-attribute decision making problem. Many researchers suppose that the fuzzy measure was given subjectively in their models. While the judgments of the decision makers occasionally absolutely depend on their knowledge or experience, and to some extent, the subjective weights are tinctured with prejudice, so an objective weighting method desired. In fact, the similarity degree of the preorders corresponding to the attribute set is a valuable information to determine fuzzy measures of the attribute sets. In what follows, we use the similarity degree to depict the importance (weights) of the attribute and the attribute set.

Let $U = \{u_1, \ldots, u_i, \ldots, u_n\}$ be a feasible alternative set and $A = \{a_1, \ldots, a_i, \ldots, a_m\}$ be an attribute set. The preferences of the alternatives are evaluated under each attribute. The preorder under the attribute set $A'(A' \subseteq A)$ on U is defined as $\succeq_{P^{A'}} = \bigcap_{a \in A'}(\succeq_{P^a})$.

Definition 3.10. Let $U = \{u_1, \ldots, u_i, \ldots, u_n\}$ be a feasible alternative set, $A = \{a_1, \ldots, a_i, \ldots, a_m\}$ be an attribute set. We define a measure $\widetilde{\mu}$ of $A'(A' \subseteq A)$ as

$$\widetilde{\mu}(A') = sim(\succeq_{P^{A'}}, \succeq_{P^A}), \tag{4}$$

specially, we set $\widetilde{\mu}(\emptyset) = 0$.

Theorem 3.11. Let $U = \{u_1, \ldots, u_i, \ldots, u_n\}$ be a feasible alternative set, $A = \{a_1, \ldots, a_i, \ldots, a_m\}$ be an attribute set. The alternatives are evaluated under each attribute in the four different preference forms. Then, $\widetilde{\mu}$ defined above is a fuzzy measure on A.

Proof:

(1) According to the definition above, $\widetilde{\mu}(\emptyset) = 0$.

(2) By proposition 3.4, $\widetilde{\mu}(A) = sim(\succeq_{P^A}, \succeq_{P^A}) = 1$. So, $\widetilde{\mu}(A) = 1$ holds.

(3) Let $A_1, A_2 \subseteq A$, and $A_1 \subseteq A_2$. We easily have that $\succeq_{P^A} \subseteq \succeq_{P^{A_2}} \subseteq \succeq_{P^{A_1}}$, by Proposition 3.7, $sim(\succeq_{P^A}, \succeq_{P^{A_2}}) \geq sim(\succeq_{P^A}, \succeq_{P^{A_1}})$, i.e. $sim(\succeq_{P^{A_2}}, \succeq_{P^A}) \geq sim(\succeq_{P^{A_1}}, \succeq_{P^A})$. So $\widetilde{\mu}(A_1) \leq \widetilde{\mu}(A_2)$. Hence, $\widetilde{\mu}$ is a fuzzy measure on A.

This completes the proof. □

The method for determining the fuzzy measure of an attribute set is shown in Theorem 3.11. It is a data-driven method and the fuzzy measure can be calculated from the real evaluation information in whatever form. As analyzed in [37], the weights (fuzzy measure) of the attribute set are sometimes not consistent with the decision maker's subjective preferences, so the combination of objective method and subjective method might be appropriate for determining the weights of the attributes. In this study, we only consider the objective one.

4. Illustrative examples

In this section, we use two examples to illustrate the determination processes of the attribute sets' fuzzy measures. The first example involves a decision making problem with hybrid evaluation forms. The followed one shows not only the determination processes of the fuzzy measures, but also the computations of the Choquet integrals for a decision making problem with pure evaluation form as utility function.

Example 4.1. We continue to use Example 2.1 and the granular structures in Example 2.3 to illustrative how to compute the fuzzy measures of attributes. There are four attributes involved in this decision making problem. According to the definition and properties of the fuzzy measure, $\widetilde{\mu}(\emptyset) = 0$ and $\widetilde{\mu}(\{a_1, a_2, a_3, a_4\}) = 1$, then $2^4 - 2$ fuzzy measures need be determined by the proposed GrC method.

Let $A = \{a_1, a_2, a_3, a_4\}$. Take the calculation of $\widetilde{\mu}(\{a_2, a_3, a_4\})$ for example. The preorder granular structure of the whole attribute set $\{a_1, a_2, a_3, a_4\}$ has been calculated in Example 2.5 as

$$U/\succ_{P^A} = \{\{u_1\}, \{u_2\}, \{u_3\}, \{u_4\}, \{u_5\}\},$$

and we should calculate the preorder granular structure of $\{a_2, a_3, a_4\}$. Proposition 2.4 gives the method of constructing the preorder granular structure of an attribute set, so

$$\begin{aligned} U/\succeq_{P^{\{a_2,a_3,a_4\}}} &= \succeq_{P^{a_2}} \cap \succeq_{P^{a_3}} \cap \succeq_{P^{a_4}} \\ &= \succeq_{V^2} \cap \succeq_{P^3} \cap \succeq_{Q^4} \\ &= \{\{u_1, u_5\}, \{u_2\}, \{u_3, u_5\}, \{u_4\}, \{u_5\}\}. \end{aligned}$$

Then, by using Eq. (4), we have

$$\begin{aligned} \widetilde{\mu}(\{a_2, a_3, a_4\}) &= sim(\succeq_{P^{\{a_2,a_3,a_4\}}}, \succeq_{P^A}) \\ &= 1 - \tfrac{1}{5-1} \times (\tfrac{\{u_1,u_5\}\ominus\{u_1\}}{5} + \tfrac{\{u_2\}\ominus\{u_2\}}{5} + \tfrac{\{u_3,u_5\}\ominus\{u_3\}}{5} + \tfrac{\{u_4\}\ominus\{u_4\}}{5} + \tfrac{\{u_5\}\ominus\{u_5\}}{5}) \\ &= 1 - \tfrac{1}{4\times5} \times (1 + 1) \\ &= 0.9. \end{aligned}$$

Similarly, the following fuzzy measures can be calculated.

$$\begin{array}{llll} \widetilde{\mu}(\{a_1\}) = 0.5, & \widetilde{\mu}(\{a_2\}) = 0.45, & \widetilde{\mu}(\{a_3\}) = 0.45, & \widetilde{\mu}(\{a_4\}) = 0.45, \\ \widetilde{\mu}(\{a_1, a_2\}) = 0.75, & \widetilde{\mu}(\{a_1, a_3\}) = 0.75, & \widetilde{\mu}(\{a_1, a_4\}) = 0.75, & \widetilde{\mu}(\{a_2, a_3\}) = 0.85, \\ \widetilde{\mu}(\{a_2, a_4\}) = 0.8, & \widetilde{\mu}(\{a_3, a_4\}) = 0.6, & \widetilde{\mu}(\{a_1, a_2, a_3\}) = 1, & \widetilde{\mu}(\{a_1, a_2, a_4\}) = 0.95 \\ \widetilde{\mu}(\{a_1, a_3, a_4\}) = 0.8, & \widetilde{\mu}(\{a_2, a_3, a_4\}) = 0.9. & & \end{array}$$

From the fuzzy measure calculated above, one can find that there exist greatly redundancy between attribute a_3 and a_4, for the reason that $\widetilde{\mu}(\{a_3\}) + \widetilde{\mu}(\{a_4\})$ is much bigger than $\widetilde{\mu}(\{a_3, a_4\})$. Actually, a_3 (comfortable level) has a positive correlation with a_4 (operating stability).

Without the help of transformations between utility function and other evaluation forms, we cannot gain the comprehensive results of alternatives by using Eq. (2) or Eq. (3) for this example. Here, we only calculate the fuzzy measures of the attribute sets. It is one of our further works to aggregate the multi-attribute preference information with hybrid forms.

Example 4.2. In this example, we suppose a decision maker provides all the preferences in the utility function form. There are five students need to be ordered by the performances with respect to three subjects: a_1- Mathematics, a_2- Physics and a_3- Literature. They are evaluated on each subject on a scale of 0 to 100. The evaluations are presented in Table 1. We use the fuzzy measure $\widetilde{\mu}$ and the Choquet integral to calculate the comprehensive result.

Table 1. The evaluation of five students

	Mathematics	Physics	Literature
u_1	95	90	65
u_2	85	80	75
u_3	90	85	80
u_4	80	85	90
u_5	75	80	80

Firstly, the preorder granular structures of the attribute sets are computed as follows. We only give the representative ones, the rest of the attribute subsets are similar.

$$U/\succeq_{V^1}=\{\{u_1\},\{u_1,u_2,u_3\},\{u_1,u_3\},\{u_1,u_2,u_3,u_4\},\{u_1,u_2,u_3,u_4,u_5\}\},$$

$$U/\succeq_{V^2}=\{\{u_1\},\{u_1,u_2,u_3,u_4,u_5\},\{u_1,u_3,u_4\},\{u_1,u_3,u_4\},\{u_1,u_2,u_3,u_4,u_5\}\},$$

$$U/\succeq_{V^3}=\{\{u_1,u_2,u_3,u_4,u_5\},\{u_2,u_3,u_4,u_5\},\{u_3,u_4,u_5\},\{u_4\},\{u_3,u_4,u_5\}\},$$

$$U/\succeq_{V^{1,2}}=U/(\succeq_{V^1}\cap\succeq_{V^2})=\{\{u_1\},\{u_1,u_2,u_3\},\{u_1,u_3\},\{u_1,u_3,u_4\},\{u_1,u_2,u_3,u_4,u_5\}\},$$

$$\cdots\cdots$$

$$U/\succeq_{V^{1,2,3}}=U/(\succeq_{V^1}\cap\succeq_{V^2}\cap\succeq_{V^3})=\{\{u_1\},\{u_2,u_3\},\{u_3\},\{u_4\},\{u_3,u_4,u_5\}\}.$$

Secondly, calculate the fuzzy measure on the attribute set $A=\{a_1=Mathematics, a_2=Physics,$ $a_3=Literature\}$ based on the similarity degree and Eq. (4).

$$\begin{aligned}\widetilde{\mu}(\{a_1\}) &= sim(\succeq_{V^1},\succeq_{V^{1,2.3}})\\ &= 1-\tfrac{1}{5-1}\textstyle\sum_{i=1}^{5}\frac{|[u_i]^{\succeq_{PV_1}}\ominus[u_i]^{\succeq_{PV_2}}|}{n}\\ &= 1-\tfrac{1}{4}\times(|\tfrac{\{u_1\}\ominus\{u_1\}|}{5}+\tfrac{|\{u_1,u_2,u_3\}\ominus\{u_2,u_3\}|}{5}+\tfrac{|\{u_1,u_3\}\ominus\{u_3\}|}{5}+\\ &\quad \tfrac{|\{u_1,u_2,u_3,u_4\}\ominus\{u_4\}|}{5}+\tfrac{|\{u_1,u_2,u_3,u_4,u_5\}\ominus\{u_3,u_4,u_5\}|}{5})\\ &= 1-\tfrac{1}{4}\times(\tfrac{0}{5}+\tfrac{1}{5}+\tfrac{1}{5}+\tfrac{3}{5}+\tfrac{2}{5})\\ &= 1-\tfrac{1}{4}\times\tfrac{7}{5}\\ &= 0.65.\end{aligned}$$

Similarly, we compute the fuzzy measures of the other subset of A: $\widetilde{\mu}(\{a_2\})=0.55$, $\widetilde{\mu}(\{a_3\})=0.6$, $\widetilde{\mu}(\{a_1,a_2\})=0.7$, $\widetilde{\mu}(\{a_1,a_3\})=1$, $\widetilde{\mu}(\{a_2,a_3\})=0.85$, $\widetilde{\mu}(\{a_1,a_2,a_3\})=1$.

Thirdly, with the fuzzy measure computed above, we calculate the Choquet integral of the five students by using Eq. (3).

$Ch(u_1,\widetilde{\mu})=95\times\widetilde{\mu}(\{a_1\})+90\times(\widetilde{\mu}\{a_1,a_2\}-\widetilde{\mu}\{a_1\})+65\times(\widetilde{\mu}\{a_1,a_2,a_3\}-\widetilde{\mu}\{a_1,a_2\})=85.75,$
$Ch(u_2,\widetilde{\mu})=85\times\widetilde{\mu}(\{a_1\})+80\times(\widetilde{\mu}\{a_1,a_2\}-\widetilde{\mu}\{a_1\})+75\times(\widetilde{\mu}\{a_1,a_2,a_3\}-\widetilde{\mu}\{a_1,a_2\})=81.75,$
$Ch(u_3,\widetilde{\mu})=90\times\widetilde{\mu}(\{a_1\})+85\times(\widetilde{\mu}\{a_1,a_2\}-\widetilde{\mu}\{a_1\})+80\times(\widetilde{\mu}\{a_1,a_2,a_3\}-\widetilde{\mu}\{a_1,a_2\})=86.75,$
$Ch(u_4,\widetilde{\mu})=90\times\widetilde{\mu}(\{a_3\})+85\times(\widetilde{\mu}\{a_3,a_2\}-\widetilde{\mu}\{a_3\})+80\times(\widetilde{\mu}\{a_3,a_2,a_1\}-\widetilde{\mu}\{a_3,a_2\})=87.25,$
$Ch(u_5,\widetilde{\mu})=80\times\widetilde{\mu}(\{a_3\})+80\times(\widetilde{\mu}\{a_3,a_2\}-\widetilde{\mu}\{a_3\})+75\times(\widetilde{\mu}\{a_3,a_2,a_1\}-\widetilde{\mu}\{a_3,a_2\})=79.25.$

Finally, we rank the students with respect to the comprehensive results calculated by the Choquet integral:

$$u_4\succ u_3\succ u_1\succ u_2\succ u_5.$$

According to the ranking of the five students, we conclude that u_4 comes first and u_5 the last. One may find that u_3 and u_4 with the same scores in Physics and with the reverse scores in Mathematics and Literature, while the Choquet integral of u_4 is bigger than u_3. It is because that the fuzzy measure of the set $\{a_1,a_2\}$ is much less than the sum of the fuzzy measures of a_1 and a_2 and the fuzzy measure of the set $\{a_2,a_3\}$ is slightly less than the sum of the weights of a_2 and a_3. That is to say there exists much more redundancy interaction between Mathematics and Physics than that between Physics and Literature. The

Choquet integral results reflect the interactions between attributes. If we use the weighted mean operator to aggregate the score, u_3 would be better than u_4 for that the individual weight of a_1 is better than that of a_3. In fact, the weighted mean operator does not consider the redundancy between attributes, the results are not rational and not accord with the judgements of us.

Both of Kojadinovic's mutual information method and the presented method are data-driven unsupervised weights acquisition methods. Different from Kojadinovic's method, the presented method has two main characteristics: one is that the different preference forms expressed by decision makers are considered, while Kojadinovic's method only deals with utility function expressed MADM problems; The other is that the fuzzy measures (weights) of attribute sets are derived from analyzing the dominance relations induced by the judgments under the corresponding attribute sets, while it is rather complex to estimate the probability distribution before computing the mutual information in Kojadinovic's method.

5. Conclusions and future works

In the present research, we have proposed a GrC based data-driven weights learning method for solving the MADM problems with different preference forms. The weight, i.e. fuzzy measure of attribute set is defined by the similarity degree of a special pair of preorder granular structures, which provides a new and objective way to determine the weight of an attribute subset under different preference forms.

The followings are the issues to be concerned in our further work: build the general flow of the Choquet integral based MADM with different preference forms; determine weights of attribute set by GrC method when the evaluation information is incomplete; explore the practical fuzzy measure learning method in order to overcome the "curse of dimensionality".

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